Exercise walk-through

5.1.99

Independent transfer: Foundation then Higher

AQA · GCSE · Mathematics

The probability scale and outcomes

- The probability of an event is a number from 0 to 1 on the **probability scale** . 0 means impossible, 1 means certain, and 0.5 means an even chance.
- Write probabilities as fractions, decimals or percentages, never as "1 in 4" or "1 : 4".
- When every **outcome** is **equally likely** , the **theoretical probability** is
- For a fair six-sided die, $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$.

$$P(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{total number of outcomes}}$$

Conditional probability (H)

- A **conditional probability** is the probability of an event when you already know that another event has happened.
- $P(A \mid B)$ means "the probability of A, given B".
- Knowing B shrinks the group you count from to just the members of B:
- **Worked example 9 (H).** In the class of 30 (18 cycle, 12 swim, 7 both), $P(\text{swims} \mid \text{cycles}) = \frac{7}{18}$, because only the 18 cyclists count.

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

Sources for this lesson

- AQA, *GCSE Mathematics (8300) specification*, subject content section 3.5 (Probability).
- M. vos Savant, "Ask Marilyn", *Parade* (9 September 1990), and her follow-up columns of December 1990 and February 1991.
- AQA, *GCSE Mathematics* Higher Papers 2 and 3 (June 2024), the expected-frequency, dice and machine questions, adapted.

Exercises 2.1·2.2

2.1 The table shows the probabilities for a spinner. Find x. The spinner is spun 80 times. How many times would you expect green? [2]

colour	red	blue	green	yellow
probability	0.2	0.35	x	0.15

2.2 Two fair dice are rolled and the scores are added. Use the sample space to find $P(\text{total of } 8)$. [2]

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

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probability	0.2	0.35	x	0.15

2.1

- $x = 1 - 0.7 = \mathbf{0.3}$
- $0.3 \times 80 = \mathbf{24}$

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2.2

- 5 ways out of 36
- $\mathbf{\frac{5}{36}}$

Exercise 2.3

2.3 A drawing pin is dropped 50 times. It lands point up 34 times. Work out the relative frequency. Estimate how many times it lands point up in 400 drops. [2]



point up



point down

50 drops: 34 point up

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point up



point down

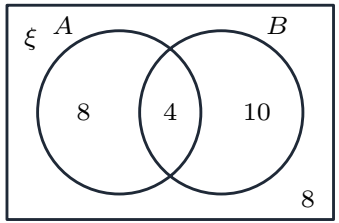
50 drops: 34 point up

2.3

- $\frac{34}{50} = \mathbf{0.68}$
- $0.68 \times 400 = \mathbf{272}$

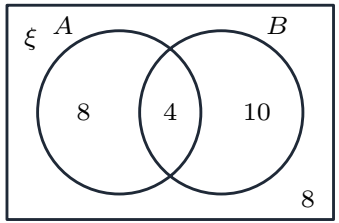
Exercise 2.4

2.4 The Venn diagram shows 30 students: A plays an instrument and B sings. Find $P(A \cup B)$ and $P(\text{not } B)$. [2]



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2.4

- $\frac{22}{30} = \mathbf{\frac{11}{15}}$
- $\frac{16}{30} = \mathbf{\frac{8}{15}}$

Exercise 3.1

3.1 $P(\text{bus late}) = 0.1$ and $P(\text{train late}) = 0.2$. The two are independent. Work out the probability that both are late, that neither is late, and that at least one is late. [3]

$P(\text{bus late}) = 0.1$ $P(\text{train late}) = 0.2$

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3.1

- $0.1 \times 0.2 = \mathbf{0.02}$
- $0.9 \times 0.8 = \mathbf{0.72}$
- $1 - 0.72 = \mathbf{0.28}$

Exercise 3.2 (a)

3.2 A bag has 4 red and 6 green counters. Ella takes a counter at random and does **not** put it back. Then she takes a second counter.

(a) Write down the probability that the first counter is red, and the probability that it is not red. [2]

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(a) $\mathbf{\frac{2}{5}}$; $\mathbf{\frac{3}{5}}$

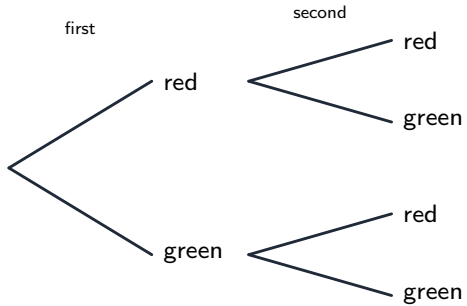
Exercise 3.2 (b)

3.2 A bag has 4 red and 6 green counters. Ella takes a counter at random and does **not** put it back. Then she takes a second counter.

(b) *(H)* Complete the tree diagram. [3]



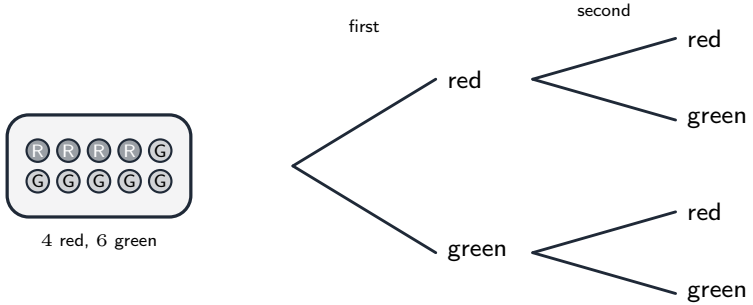
4 red, 6 green



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(b) *(H)* Complete the tree diagram. [3]



(b) first branches $\frac{4}{10}$ and $\frac{6}{10}$; after red: $\frac{3}{9}$ red, $\frac{6}{9}$ green; after green: $\frac{4}{9}$ red, $\frac{5}{9}$ green

Exercise 3.2 (c)

3.2 A bag has 4 red and 6 green counters. Ella takes a counter at random and does **not** put it back. Then she takes a second counter.

(c) *(H)* Work out the probability that both counters are the same colour, and the probability that they are different colours. [4]

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(c) *(H)* Work out the probability that both counters are the same colour, and the probability that they are different colours. [4]

(c) $\frac{4}{10} \times \frac{3}{9} + \frac{6}{10} \times \frac{5}{9} = \frac{42}{90} = \mathbf{\frac{7}{15}}$; different $\mathbf{\frac{8}{15}}$

Exercise 3.2 (d)

3.2 A bag has 4 red and 6 green counters. Ella takes a counter at random and does **not** put it back. Then she takes a second counter.

(d) *(H)* The two counters are the same colour. Work out the probability that they are both green. Ella does this 150 times. How many times would you expect two different colours? Work out the probability of at least one red counter. [5]

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(d) $\frac{30}{90} \div \frac{42}{90} = \mathbf{\frac{5}{7}}$; $150 \times \frac{8}{15} = \mathbf{80}$; $1 - \frac{30}{90} = \mathbf{\frac{2}{3}}$