Exercise walk-through

5.1.3
Sample Spaces

AQA · GCSE · Mathematics

The probability scale and outcomes

- The probability of an event is a number from 0 to 1 on the **probability scale** . 0 means impossible, 1 means certain, and 0.5 means an even chance.
- Write probabilities as fractions, decimals or percentages, never as "1 in 4" or "1 : 4".
- When every **outcome** is **equally likely** , the **theoretical probability** is
- For a fair six-sided die, $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$.

$$P(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{total number of outcomes}}$$

Experiments and expected frequency

- When outcomes are not **equally likely**, such as a drawing pin landing point up, we estimate the probability by experiment.
- The **relative frequency** is
- A larger **sample size** gives a better estimate.
- Six heads in 10 tosses gives 0.6, but 502 heads in 1000 tosses gives 0.502, much closer to the true 0.5 of a fair coin.

$$\text{relative frequency} = \frac{\text{number of times the event happened}}{\text{number of trials}}$$

Conditional probability (H)

- A **conditional probability** is the probability of an event when you already know that another event has happened.
- $P(A \mid B)$ means "the probability of A, given B".
- Knowing B shrinks the group you count from to just the members of B:
- **Worked example 9 (H).** In the class of 30 (18 cycle, 12 swim, 7 both), $P(\text{swims} \mid \text{cycles}) = \frac{7}{18}$, because only the 18 cyclists count.

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

Exercise 2.1

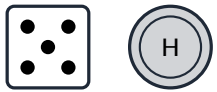
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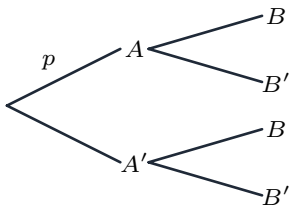


2.1

- $6 \times 6 = \mathbf{36}$

Setup for exercise 2.2

2.2 Two fair dice are rolled. Work out $P(\text{a double})$. [1]

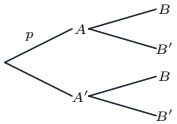


Exercise 2.2

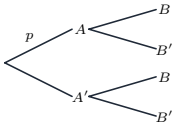
2.3 A coin is tossed and a fair dice is rolled. List the outcomes, and work out the probability of a head and an even number. [2]



2.4 Two fair dice are rolled. Work out $P(\text{total of at least 11})$. [2]



2.5 A fair spinner numbered 1, 2, 3 is spun twice and the scores are multiplied. Work out $P(\text{the product is even})$. [2]

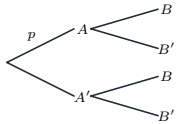


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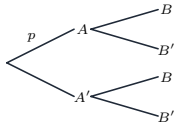
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2.2

- six doubles: $\frac{6}{36} = \mathbf{\frac{1}{6}}$

Full solution on next slide

Full solution 2.2

2.2

- six doubles: $\frac{6}{36} = \mathbf{\frac{1}{6}}$

2.3

- H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6: 12 outcomes
- H2, H4, H6: $\frac{3}{12} = \mathbf{\frac{1}{4}}$

2.4

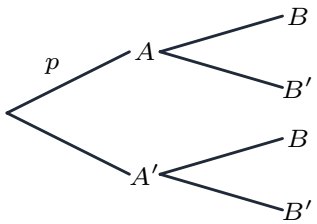
- $(5,6), (6,5), (6,6)$: 3 of the 36 cells
- $\frac{3}{36} = \mathbf{\frac{1}{12}}$

2.5

- 9 outcomes; even when at least one 2: $(1,2), (2,1), (2,2), (2,3), (3,2)$
- $\mathbf{\frac{5}{9}}$

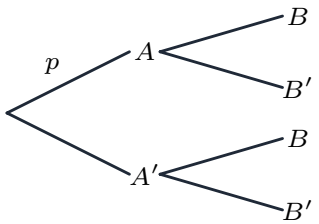
Exercise 3.1

3.1 Jo says there are 11 possible totals with two dice, from 2 to 12, so $P(\text{total} = 7) = \frac{1}{11}$. Explain her mistake. [1]



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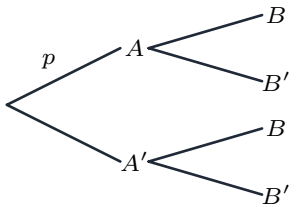


3.1

- the 11 totals are not equally likely: 7 comes from 6 of the 36 cells but 2 from only 1, so $P(7) = \frac{6}{36} = \frac{1}{6}$

Setup for exercise 3.2

3.2 A café offers 3 starters, 5 main courses and 2 desserts. How many different three-course meals are possible? [2]



Exercise 3.2

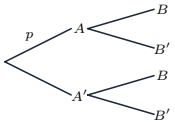
3.3 Two fair dice are rolled 180 times. Work out the expected number of times the total is 7. [2]



3.4 Two fair dice are rolled. Work out the probability that the scores differ by exactly 2. [3]



3.5 A 4-digit PIN uses the digits 0–9. Work out the number of PINs if digits may repeat, and the number if no digit may repeat. Then work out the probability of guessing a no-repeat PIN at the first attempt. [4]



Exercise 3.2

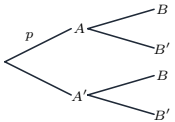
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3.2

- product rule: $3 \times 5 \times 2 = \mathbf{30}$ meals

3.3

Full solution on next slide

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Full solution 3.2

3.2

- product rule: $3 \times 5 \times 2 = \mathbf{30}$ meals

3.3

- $P(7) = \frac{1}{6}$, so expected number $= 180 \times \frac{1}{6} = \mathbf{30}$

3.4

- $(1,3), (3,1), (2,4), (4,2), (3,5), (5,3), (4,6), (6,4)$
- 8 of the 36 cells
- $\dfrac{8}{36} = \dfrac{\mathbf{2}}{\mathbf{9}}$

3.5

- repeats allowed: $10 \times 10 \times 10 \times 10 = \mathbf{10\,000}$
- no repeats: one fewer digit each time, $10 \times 9 \times 8 \times 7 = \mathbf{5040}$
- $P(\text{correct first guess}) = \dfrac{\mathbf{1}}{\mathbf{5040}}$