Exercise walk-through

5.1.2 Experimental Probability

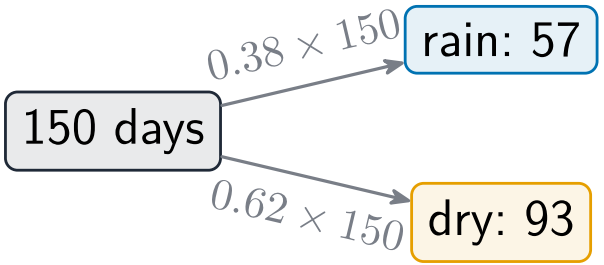
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Experiments and expected frequency

- When outcomes are not **equally likely**, such as a drawing pin landing point up, we estimate the probability by experiment.
- The **relative frequency** is
- A larger **sample size** gives a better estimate.
- Six heads in 10 tosses gives 0.6, but 502 heads in 1000 tosses gives 0.502, much closer to the true 0.5 of a fair coin.

$$\text{relative frequency} = \frac{\text{number of times the event happened}}{\text{number of trials}}$$

Figure: Experiments and expected frequency (2)



expected counts: a long-run average, not a guarantee

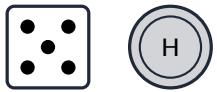
- A **frequency tree** with expected counts: 57 rainy and 93 dry days out of 150
- Match each part of the diagram to the text.

Key terms (2)

- **Expected frequency**: the number of times an event should happen on average: trials $\times$ probability.
- **Independent**: events where one does not change the probability of the other.
- **Sample space**: a list or table of all possible outcomes.
- **Conditional probability**: the probability of an event given that another event has happened.

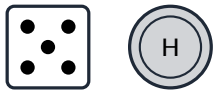
Exercise 2.1

2.1 A spinner lands on red 18 times in 60 spins. Work out the relative frequency of red. [1]



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2.1

- $\frac{18}{60} = \mathbf{0.3}$

Setup for exercise 2.2

2.2 Use your answer to **2.1** to estimate how many times the spinner lands on red in 200 spins. [1]





Exercise 2.2

2.3 In 50 trials, outcome A happened 12 times, B 25 times and C 13 times. Work out the relative frequency of B. [1]



2.4 A fair dice is rolled 120 times. How many sixes would you expect? [1]



2.5 The probability of winning a game is 0.15. 400 people play once each. How many winners would you expect? [2]



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2.2

- $200 \times 0.3 = \mathbf{60}$

Full solution on next slide

Full solution 2.2

2.2

- $200 \times 0.3 = \mathbf{60}$

2.3

- $\dfrac{25}{50} = \mathbf{0.5}$

2.4

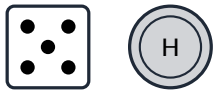
- $120 \times \frac{1}{6} = \mathbf{20}$

2.5

- $\text{expected number} = n \times p = 400 \times 0.15 = \mathbf{60}$

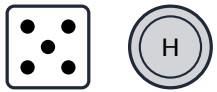
Exercise 3.1

3.1 Ali tosses a coin 10 times and gets 7 heads. He says the coin is biased. Comment on his conclusion. [1]



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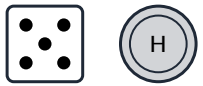


3.1

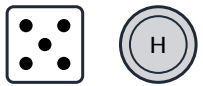
- 10 tosses is too few: 7 heads can happen by chance with a fair coin; he needs many more tosses

Setup for exercise 3.2

3.2 Amy spins a spinner 20 times and gets red 8 times. Ben spins the same spinner 200 times and gets red 62 times. Whose estimate of $P(\text{red})$ is more reliable? Give a reason and the estimate. [2]

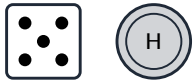


3.3 Combine Amy's and Ben's results from **3.2** to give the best estimate of $P(\text{red})$, and use it to estimate the number of reds in 1100 spins. [3]

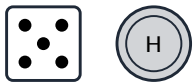


Exercise 3.2

3.4 A dice is rolled 300 times and lands on 6 seventy-five times. Is the dice fair? Use the expected number of sixes and the relative frequency in your answer. [3]

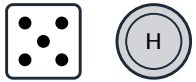


3.5 A game costs 50p to play and pays £2 to a winner. The probability of winning is 0.2. The game is played 100 times. Work out the expected profit for the person running the game. [4]

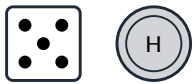


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Full solution on next slide

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Full solution 3.2 (1/2)

3.2

- **Ben's**: 200 spins is many more trials than 20
- estimate $= \dfrac{62}{200} = \mathbf{0.31}$

3.3

- total reds $8 + 62 = 70$ in $20 + 200 = 220$ spins: $\dfrac{70}{220} = 0.318\ldots$
- $1100 \times \dfrac{70}{220} = \mathbf{350}$

Full solution 3.2 (2/2)

3.4

- expected sixes if fair $= 300 \times \frac{1}{6} = 50$
- relative frequency $= \dfrac{75}{300} = 0.25$, well above $\frac{1}{6} \approx 0.167$
- 75 is far more than 50 over many rolls, so the dice is probably **biased** towards 6

3.5

- money taken $= 100 \times £0.50 = £50$
- expected winners $= 100 \times 0.2 = 20$
- expected payout $= 20 \times £2 = £40$
- expected profit $= £50 - £40 = £\mathbf{10}$