Exercise walk-through

5.1.1 Probability

AQA · GCSE · Mathematics

The probability scale and outcomes

- The probability of an event is a number from 0 to 1 on the **probability scale** . 0 means impossible, 1 means certain, and 0.5 means an even chance.
- Write probabilities as fractions, decimals or percentages, never as "1 in 4" or "1 : 4".
- When every **outcome** is **equally likely** , the **theoretical probability** is
- For a fair six-sided die, $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$.

$$P(\text{event}) = \frac{\text{number of outcomes in the event}}{\text{total number of outcomes}}$$

Experiments and expected frequency

- When outcomes are not **equally likely**, such as a drawing pin landing point up, we estimate the probability by experiment.
- The **relative frequency** is
- A larger **sample size** gives a better estimate.
- Six heads in 10 tosses gives 0.6, but 502 heads in 1000 tosses gives 0.502, much closer to the true 0.5 of a fair coin.

$$\text{relative frequency} = \frac{\text{number of times the event happened}}{\text{number of trials}}$$

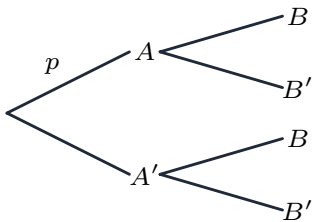
Conditional probability (H)

- A **conditional probability** is the probability of an event when you already know that another event has happened.
- $P(A \mid B)$ means "the probability of A, given B".
- Knowing B shrinks the group you count from to just the members of B:
- **Worked example 9 (H).** In the class of 30 (18 cycle, 12 swim, 7 both), $P(\text{swims} \mid \text{cycles}) = \frac{7}{18}$, because only the 18 cyclists count.

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

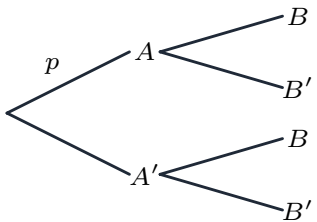
Exercise 2.1

2.1 A bag has 3 red and 2 blue counters. One is taken at random. Work out $P(\text{blue})$. [1]



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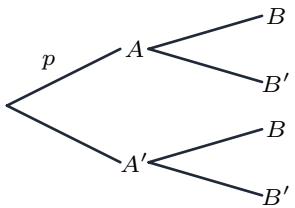


2.1

- $\dfrac{2}{5}$

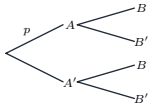
Setup for exercise 2.2

2.2 A fair dice is rolled. Work out $P(\text{not a } 6)$. [1]

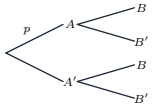


Exercise 2.2

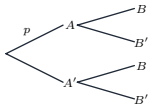
2.3 A spinner lands on red, blue, green or yellow. $P(\text{red}) = 0.2$, $P(\text{blue}) = 0.35$ and $P(\text{yellow}) = 0.15$. Work out $P(\text{green})$. [2]



2.4 From the bag in **2.1**, a counter is taken, replaced, and a second is taken. Work out $P(\text{both red})$. [2]

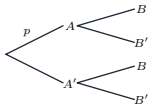


2.5 From the bag in **2.1**, two counters are taken without replacement. Work out $P(\text{both blue})$. [2]

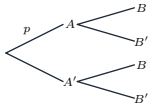


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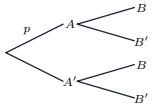
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2.2

- $1 - \frac{1}{6} = \mathbf{\frac{5}{6}}$

2.3

Full solution on next slide

Full solution 2.2

2.2

- $1 - \frac{1}{6} = \mathbf{\frac{5}{6}}$

2.3

- $0.2 + 0.35 + 0.15 = 0.7$
- $P(\text{green}) = 1 - 0.7 = \mathbf{0.3}$

2.4

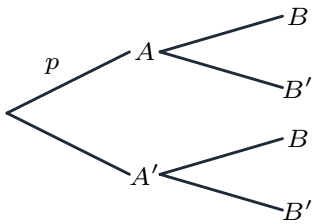
- with replacement both draws are $\frac{3}{5}$: $\frac{3}{5} \times \frac{3}{5} = \mathbf{\frac{9}{25}}$

2.5

- $\frac{2}{5} \times \frac{1}{4} = \mathbf{\frac{1}{10}}$

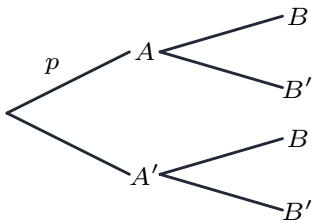
Exercise 3.1

3.1 Tom says the probability of two reds with replacement from the bag in **2.1** is $\frac{3}{5} + \frac{3}{5}$. Explain his mistake. [1]



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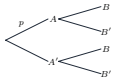
- red **and then** red means **multiply** along the branches: $\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$; his sum is more than 1

Exercise 3.2

3.2 A spinner lands on red with probability 0.4. It is spun twice. Work out the probability that it lands on red at least once. [3]



3.3 The probability of rain on any day is 0.3, independently. Work out the probability that it rains on exactly one of two days. [3]



3.4 A bag has 4 green and 6 yellow sweets. Two are taken without replacement. Work out the probability that they are the same colour. [3]



3.5 *(H)* A bag has n red and 3 blue counters. Two are taken without replacement, and $P(\text{both red}) = \frac{5}{14}$. Show that $3n^2 - 13n - 10 = 0$, and find n. [4]

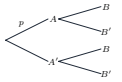


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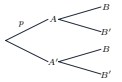
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3.2

- $P(\text{not red}) = 0.6$, so $P(\text{no red}) = 0.6 \times 0.6 = 0.36$
- $P(\text{at least one red}) = 1 - 0.36 = \mathbf{0.64}$

3.3

- rain then dry, or dry then rain: $0.3 \times 0.7 + 0.7 \times 0.3 = 0.21 + 0.21 = \mathbf{0.42}$

Full solution on next slide

Full solution 3.2

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3.4

- $P(GG) = \frac{4}{10} \times \frac{3}{9} = \frac{12}{90}$ and $P(YY) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90}$
- add the routes: $\frac{12}{90} + \frac{30}{90} = \frac{42}{90} = \boldsymbol{\frac{7}{15}}$

3.5

- $\dfrac{n}{n+3} \times \dfrac{n-1}{n+2} = \dfrac{5}{14}$
- $14n(n-1) = 5(n+3)(n+2)$, so $14n^2 - 14n = 5n^2 + 25n + 30$
- $9n^2 - 39n - 30 = 0$, and dividing by 3 gives $3n^2 - 13n - 10 = 0$
- $(3n+2)(n-5) = 0$, and n is a positive whole number, so $\boldsymbol{n = 5}$