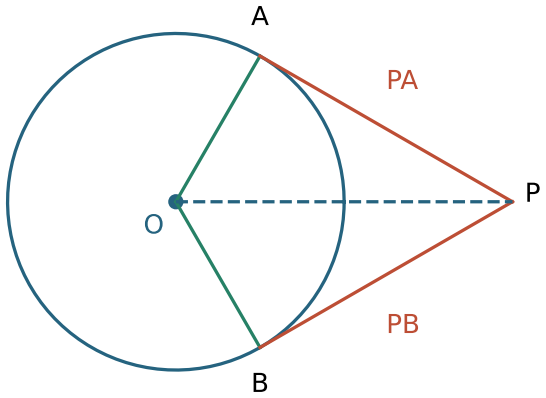
Exercise walk-through

4.5
Properties and congruence and similarity (G4–G5, G18)

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Figure: Congruence, similarity and circles (4)

Shared OP and equal radii → RHS → PA = PB



- Equal radii, shared OP and right angles give congruent triangles (RHS), so PA = PB
- Match each part of the diagram to the text.

Perimeter, area and volume

- Triangle: $\frac{1}{2} \times b \times h$, where h is the **perpendicular height** .
- Parallelogram: $b \times h$.
- Trapezium: $\frac{1}{2}(a+b)h$, where a and b are the parallel sides.
- Circle: $A = \pi r^2$, and the **circumference** is $C = 2\pi r = \pi d$.

$$\text{arc length} = \frac{\theta}{360} \times 2\pi r \qquad \text{sector area} = \frac{\theta}{360} \times \pi r^2$$

Pythagoras and trigonometry

- **Pythagoras' theorem.** In a right-angled triangle, $a^2 + b^2 = c^2$, where c is the **hypotenuse**, the longest side, opposite the right angle.
- A ladder whose foot is 3 m from a wall and whose top is 4 m up the wall is $\sqrt{9+16} = 5$ m long.
- **Trigonometric ratios.** Label the sides from the angle θ: opposite, **adjacent** (next to the angle) and **hypotenuse**.
- **Worked example 12.** In a right-angled triangle the hypotenuse is 23.7 cm and x is the side **adjacent** to an angle of $52°$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Exercises 2.1–2.3

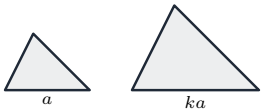
2.1 A quadrilateral has four equal sides and its diagonals bisect each other at right angles, but the diagonals are not equal in length. Name the quadrilateral. [2]



2.2 Write down the defining side property of a kite. [2]



2.3 Two triangles have three pairs of equal corresponding sides. Which congruence criterion applies? [2]



Exercises 2.1·2.3

2.1 A quadrilateral has four equal sides and its diagonals bisect each other at right angles, but the diagonals are not equal in length. Name the quadrilateral. [2]



2.1 A rhombus (a square would additionally need equal diagonals and right angles).

2.2 Write down the defining side property of a kite. [2]



2.2 Two pairs of adjacent sides equal (adjacent, not opposite).

2.3 Two triangles have three pairs of equal corresponding sides. Which congruence criterion applies? [2]

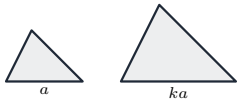


2.3 SSS.

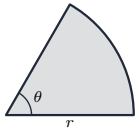
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Exercise 2.4

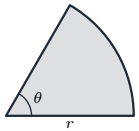
2.4 Explain why SSA is not a congruence criterion. [2]



2.5 A sector of radius 6 cm has angle 120°. Find its area in terms of π. [2]



2.6 A sector of radius 6 cm has area 9π cm^2. Find its angle. [3]

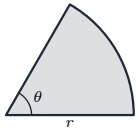


Exercise 2.4

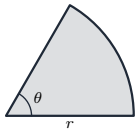
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2.4 Two different triangles can share two sides and a non-included angle – the given data does not fix the triangle, so congruence can fail.

2.5 $\frac{120}{360} \times \pi \times 6^2 = \frac{1}{3} \times 36\pi = 12\pi$ cm^2.

Full solution on next slide

Full solution 2.4

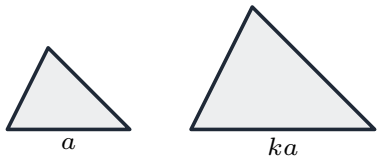
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2.5 $\frac{120}{360} \times \pi \times 6^2 = \frac{1}{3} \times 36\pi = 12\pi$ cm^2.

2.6 Fraction $= \dfrac{9\pi}{36\pi} = \frac{1}{4}$, so $\theta = \frac{1}{4} \times 360 = 90°$.

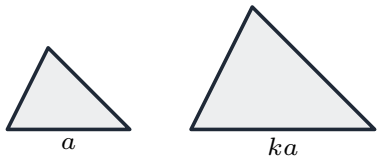
Exercise 3.1

3.1 *(June 2024 2H Q12, adapted.)* Two triangles have three pairs of equal corresponding sides. Write down the one-word congruence reason. [1]



Exercise 3.1

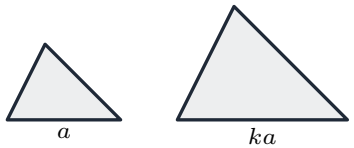
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3.1 SSS.

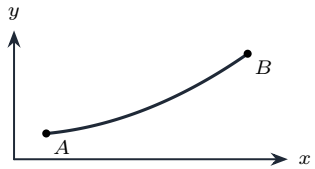
Setup for exercise 3.2

3.2 *(Original, proof.)* Triangle ABC is isosceles with $AB = AC$, and D is the midpoint of BC. Prove that the base angles ABC and BCA are equal. [4]

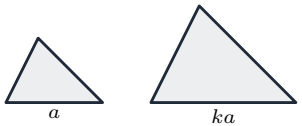


Exercise 3.2

3.3 *(Original.)* A sector of radius 6 cm has angle 90°. Find its arc length and its perimeter, in terms of π. [3]

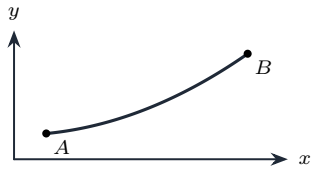


3.4 *(Original.)* An annulus has outer radius 5 cm and inner radius 3 cm. Find its area in terms of π. [2]

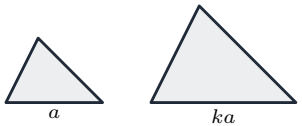


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3.2 Triangles ABD and ACD have $AB = AC$ (given), $BD = DC$ (D is the midpoint of BC) and AD shared, so they are congruent (SSS). In congruent triangles, matching angles are equal: $\angle ABD$ matches $\angle ACD$. Since D lies on BC, $\angle ABD = \angle ABC$ and

Full solution on next slide

Full solution 3.2

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3.3 Arc $= \frac{90}{360} \times 2\pi \times 6 = 3\pi$ cm; perimeter $= 3\pi + 6 + 6 = 3\pi + 12$ cm.

3.4 $\pi(5^2 - 3^2) = \pi(25 - 9) = 16\pi$ cm^2.

Local review guidance: names with the defining property, not appearance; congruence letters ordered so matching parts correspond; each of the three pairs stated with its reason (given, midpoint, common side) before the criterion; the $\frac{\theta}{360}$ fraction shown for every sector calculation. This is authored classroom guidance.