Exercise walk-through

4.5.4
Circle Tangent Proofs

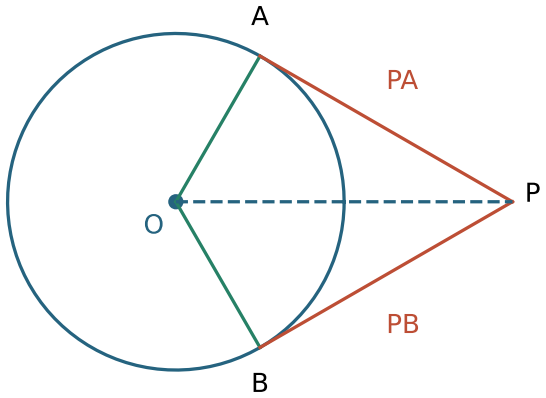
AQA · GCSE · Mathematics

Angles and polygons

- Angles at a point add up to $360°$; angles on a straight line add up to $180°$.
- Vertically opposite angles are equal.
- Where a line crosses two parallel lines, alternate angles are equal, **corresponding angles** are equal, and co-interior angles add up to $180°$. Write "alternate angles are equal", never "Z-angles".
- The angles in a triangle add up to $180°$.

Figure: Congruence, similarity and circles (4)

Shared OP and equal radii → RHS → PA = PB



- Equal radii, shared OP and right angles give congruent triangles (RHS), so PA = PB
- Match each part of the diagram to the text.

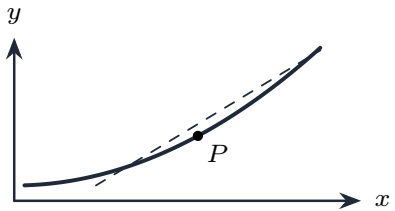
Pythagoras and trigonometry

- **Pythagoras' theorem.** In a right-angled triangle, $a^2 + b^2 = c^2$, where c is the **hypotenuse**, the longest side, opposite the right angle.
- A ladder whose foot is 3 m from a wall and whose top is 4 m up the wall is $\sqrt{9+16} = 5$ m long.
- **Trigonometric ratios.** Label the sides from the angle θ: opposite, **adjacent** (next to the angle) and **hypotenuse**.
- **Worked example 12.** In a right-angled triangle the hypotenuse is 23.7 cm and x is the side **adjacent** to an angle of $52°$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

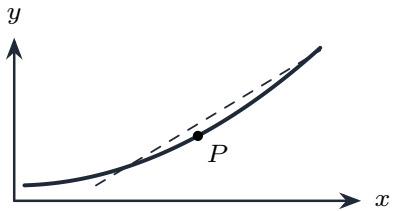
Exercise 2.1

2.1 *(H)* Write down the angle between a tangent and the radius at the point of contact. [1]



Exercise 2.1

2.1 *(H)* Write down the angle between a tangent and the radius at the point of contact. [1]

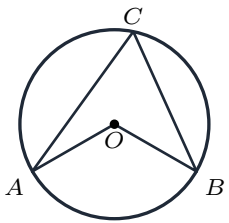


2.1

- **90°**

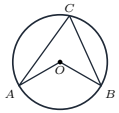
Setup for exercise 2.2

2.2 *(H)* Two tangents from P touch a circle at A and B. $PA = 7$ cm. Write down PB, with a reason. [1]

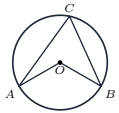


Exercise 2.2

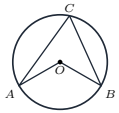
2.3 *(H)* The angle between a tangent and a chord is 48°. Write down the angle in the alternate segment. [1]



2.4 *(H)* A circle has centre O and radius 5 cm. P is 13 cm from O. Work out the length of a tangent from P. [2]

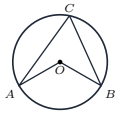


2.5 *(H)* Tangents from P touch a circle, centre O, at A and B. Angle $AOB = 110°$. Work out angle APB. [2]

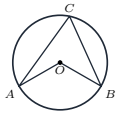


Exercise 2.2

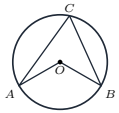
2.3 *(H)* The angle between a tangent and a chord is 48°. Write down the angle in the alternate segment. [1]



2.4 *(H)* A circle has centre O and radius 5 cm. P is 13 cm from O. Work out the length of a tangent from P. [2]



2.5 *(H)* Tangents from P touch a circle, centre O, at A and B. Angle $AOB = 110°$. Work out angle APB. [2]



2.2

- $PB =$ **7** cm: tangents from a point are equal

2.3

Full solution on next slide

exercises.lol exercises.com.cn

Full solution 2.2

2.2

- $PB = \mathbf{7}$ cm: tangents from a point are equal

2.3

- **48**°: alternate segment theorem

2.4

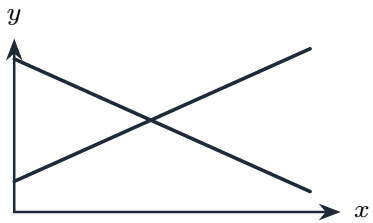
- $OAP = 90°$, so $PA^2 = OP^2 - OA^2 = 13^2 - 5^2 = 144$
- $PA = \mathbf{12}$ cm

2.5

- $OAP = OBP = 90°$ (tangent meets radius)
- $APB = 360° - 90° - 90° - 110° = \mathbf{70}°$ (angles in a quadrilateral)

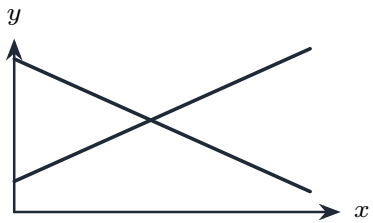
Exercise 3.1

3.1 *(H)* Zoe draws a tangent that meets the radius at 80°. Explain why her drawing cannot be right. [1]



Exercise 3.1

3.1 *(H)* Zoe draws a tangent that meets the radius at 80°. Explain why her drawing cannot be right. [1]

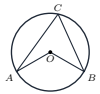


3.1

- a tangent always meets the radius at the point of contact at 90°, so 80° is impossible

Exercise 3.2

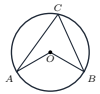
3.2 *(H)* Using the worked example, prove that OP bisects angle APB. [2]



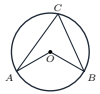
3.3 *(H)* Tangents at the two ends of a diameter are drawn. Prove that they are parallel. [2]



3.4 *(H)* A circle touches all three sides of triangle ABC, where $AB = 10$ cm, $BC = 8$ cm and $CA = 6$ cm. Work out the length of the tangent from each vertex. [3]

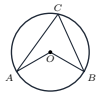


3.5 *(H)* A circle has centre O and radius 6 cm. Tangents from P, 10 cm from O, touch the circle at A and B. Work out PA, angle APB and the area of the kite $OAPB$. [4]



Exercise 3.2

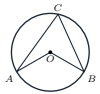
3.2 *(H)* Using the worked example, prove that OP bisects angle APB. [2]



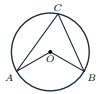
3.3 *(H)* Tangents at the two ends of a diameter are drawn. Prove that they are parallel. [2]



3.4 *(H)* A circle touches all three sides of triangle ABC, where $AB = 10$ cm, $BC = 8$ cm and $CA = 6$ cm. Work out the length of the tangent from each vertex. [3]



3.5 *(H)* A circle has centre O and radius 6 cm. Tangents from P, 10 cm from O, touch the circle at A and B. Work out PA, angle APB and the area of the kite $OAPB$. [4]



3.2

- triangles OAP and OBP are congruent (RHS)
- so the corresponding angles OPA and OPB are equal: OP bisects angle APB

3.3

Full solution on next slide

Full solution 3.2 (1/2)

3.2

- triangles OAP and OBP are congruent (RHS)
- so the corresponding angles OPA and OPB are equal: OP bisects angle APB

3.3

- each tangent meets the diameter at $90°$ (tangent meets radius)
- two lines both perpendicular to the same line make equal corresponding angles (co-interior angles $90° + 90° = 180°$), so they are parallel

3.4

Full solution 3.2 (2/2)

- let the tangent lengths from A, B, C be a, b, c (tangents from a point are equal): $a+b=10$, $b+c=8$, $a+c=6$
- adding: $2(a+b+c)=24$, so $a+b+c=12$
- $a=12-8=\mathbf{4}$ cm, $b=12-6=\mathbf{6}$ cm, $c=12-10=\mathbf{2}$ cm

3.5

- $PA=\sqrt{10^2-6^2}=\sqrt{64}=\mathbf{8}$ cm
- $\sin APO=\dfrac{\text{opp}}{\text{hyp}}=\dfrac{6}{10}$, so $APO=36.87°$
- $APB=2\times 36.87°=\mathbf{73.7}°$ (OP bisects the angle)
- area $=2\times\frac{1}{2}\times 6\times 8=\mathbf{48}$ cm^2 (two congruent right-angled triangles)