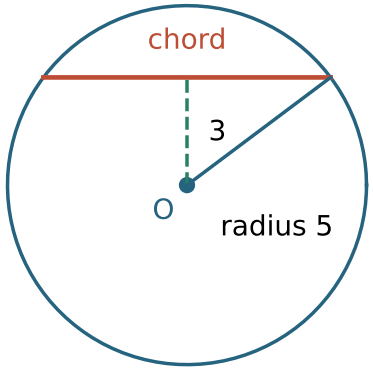
Exercise walk-through

# 4.5.3
# Circle Angle Proofs

AQA · GCSE · Mathematics

# Figure: Congruence, similarity and circles (2)

Circle of radius 5: distances use one consistent scale

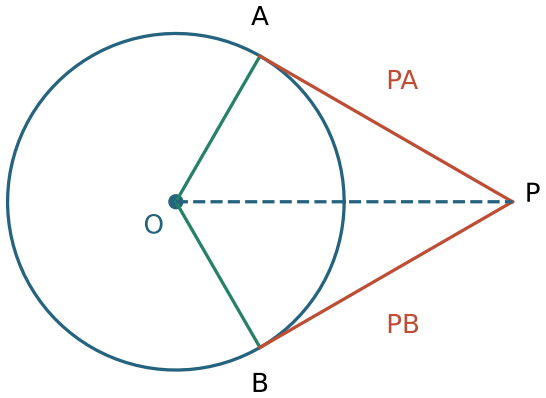



Perpendicular bisects chord: half-length 4

- Radius 5, distance 3: half the chord is $\sqrt{(5^2 - 3^2)} = 4$, so the chord is 8
- Match each part of the diagram to the text.

# Figure: Congruence, similarity and circles (4)

Shared OP and equal radii → RHS → PA = PB




- Equal radii, shared OP and right angles give congruent triangles (RHS), so PA = PB
- Match each part of the diagram to the text.

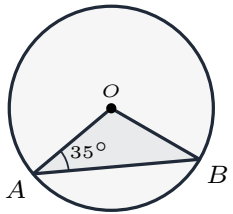
# Perimeter, area and volume

- Triangle: $\frac{1}{2} \times b \times h$, where $h$ is the **perpendicular height** .
- Parallelogram: $b \times h$.
- Trapezium: $\frac{1}{2}(a+b)h$, where $a$ and $b$ are the parallel sides.
- Circle: $A = \pi r^2$, and the **circumference** is $C = 2\pi r = \pi d$.

$$\text{arc length} = \frac{\theta}{360} \times 2\pi r \qquad \text{sector area} = \frac{\theta}{360} \times \pi r^2$$

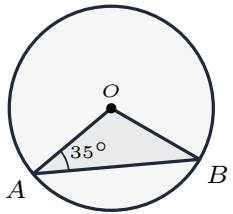
# Exercise 2.1

**2.1** $O$ is the centre and $A$, $B$ are on the circle. Angle $OAB = 35°$. Explain why triangle $OAB$ is isosceles. Work out angle $AOB$. [2]



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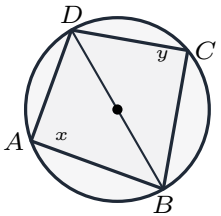



**2.1**

- $OA$ and $OB$ are radii, so they are equal
- $180 - 2 \times 35 = \mathbf{110}°$

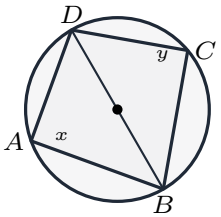
# Exercise 2.2

**2.2** $ABCD$ is a cyclic quadrilateral with centre $O$. Angle $A = x$ and angle $C = y$. The angles at the centre on the two arcs $BD$ are $2x$ and $2y$. Use this to prove that $x + y = 180°$. [2]



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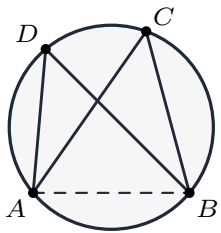



**2.2**

- angles around the point $O$: $2x + 2y = 360°$
- so $x + y = 180°$

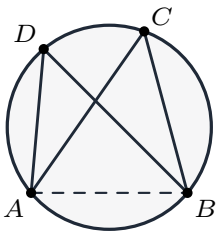
# Exercise 3.1

**3.1** Prove that angles in the same segment are equal. Use the points $A$, $B$ on the circle, and $C$, $D$ on the same side of $AB$. [3]



# Exercise 3.1

**3.1** Prove that angles in the same segment are equal. Use the points $A$, $B$ on the circle, and $C$, $D$ on the same side of $AB$. [3]




**3.1**

- angle $ACB = \frac{1}{2}$ angle $AOB$, angle at the centre
- angle $ADB = \frac{1}{2}$ angle $AOB$, for the same reason
- so angle $ACB =$ angle $ADB$

# Exercise 3.2 (a)

**3.2** $O$ is the centre of a circle. $TA$ is a tangent at $A$. $AB$ is a chord, and $C$ is on the major arc. Angle $TAB = x$.

(a) Explain why angle $OAT = 90°$. Write angle $OAB$ in terms of $x$. [2]

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**3.2** $O$ is the centre of a circle. $TA$ is a tangent at $A$. $AB$ is a chord, and $C$ is on the major arc. Angle $TAB = x$.

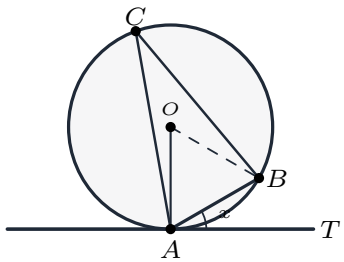
(a) Explain why angle $OAT = 90°$. Write angle $OAB$ in terms of $x$. [2]

(a) a tangent meets the radius at $90°$; angle $OAB = 90° - x$

# Exercise 3.2 (b)

**3.2** $O$ is the centre of a circle. $TA$ is a tangent at $A$. $AB$ is a chord, and $C$ is on the major arc. Angle $TAB = x$.

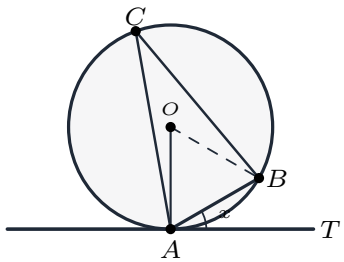
(b) Prove that angle $AOB = 2x$. [3]



# Exercise 3.2 (b)

**3.2** $O$ is the centre of a circle. $TA$ is a tangent at $A$. $AB$ is a chord, and $C$ is on the major arc. Angle $TAB = x$.

(b) Prove that angle $AOB = 2x$. [3]




(b) $OA = OB$, radii, so angle $OBA = 90° - x$; angle $AOB = 180 - 2(90 - x) = 2x$

# Exercise 3.2 (c)

**3.2** $O$ is the centre of a circle. $TA$ is a tangent at $A$. $AB$ is a chord, and $C$ is on the major arc. Angle $TAB = x$.

(c) Prove that angle $ACB = x$. Write down the name of the theorem you have proved. [4]

# Exercise 3.2 (c)

**3.2** $O$ is the centre of a circle. $TA$ is a tangent at $A$. $AB$ is a chord, and $C$ is on the major arc. Angle $TAB = x$.

(c) Prove that angle $ACB = x$. Write down the name of the theorem you have proved. [4]

(c) angle $ACB = \frac{1}{2}$ angle $AOB = x$; the angle at the centre is twice the angle at the circumference; **the alternate segment theorem**

# Exercise 3.2 (d)

**3.2** $O$ is the centre of a circle. $TA$ is a tangent at $A$. $AB$ is a chord, and $C$ is on the major arc. Angle $TAB = x$.

(d) Now $x = 58°$ and the radius is 5 cm. $D$ is a point on the minor arc $AB$. Work out angle $ADB$. Work out the length of the chord $AB$. [5]

# Exercise 3.2 (d)

**3.2** $O$ is the centre of a circle. $TA$ is a tangent at $A$. $AB$ is a chord, and $C$ is on the major arc. Angle $TAB = x$.

(d) Now $x = 58°$ and the radius is 5 cm. $D$ is a point on the minor arc $AB$. Work out angle $ADB$. Work out the length of the chord $AB$. [5]

(d) $ACBD$ is a cyclic quadrilateral: angle $ADB = 180 - 58 = \mathbf{122}°$; $AB = 2 \times 5 \times \sin 58°$ $= \mathbf{8.48}$ cm