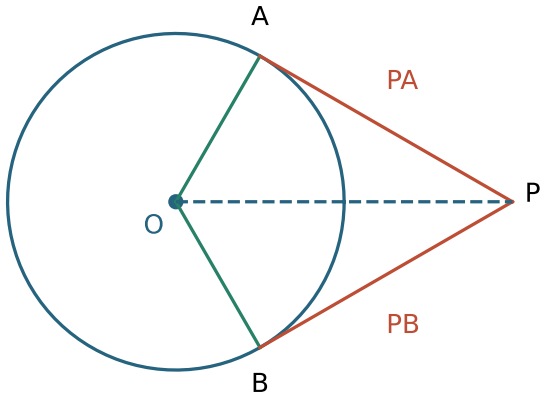
Exercise walk-through

4.5.2
Congruence Reasoning

AQA · GCSE · Mathematics

Figure: Congruence, similarity and circles (4)

Shared OP and equal radii → RHS → PA = PB



- Equal radii, shared OP and right angles give congruent triangles (RHS), so PA = PB
- Match each part of the diagram to the text.

Pythagoras and trigonometry

- **Pythagoras' theorem.** In a right-angled triangle, $a^2 + b^2 = c^2$, where c is the **hypotenuse**, the longest side, opposite the right angle.
- A ladder whose foot is 3 m from a wall and whose top is 4 m up the wall is $\sqrt{9+16} = 5$ m long.
- **Trigonometric ratios.** Label the sides from the angle θ: opposite, **adjacent** (next to the angle) and **hypotenuse**.
- **Worked example 12.** In a right-angled triangle the hypotenuse is 23.7 cm and x is the side **adjacent** to an angle of $52°$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Vectors and coordinate geometry (2)

- Add and subtract vectors by adding and subtracting their parts: with $\mathbf{a} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\mathbf{a} + \mathbf{b} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ and $2\mathbf{a} - \mathbf{b} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$.
- **Vector proofs (H).** If one vector is a number times another, the lines are parallel.

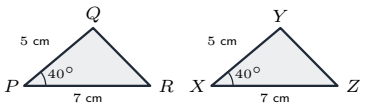
Exercises 2.1·2.2

2.1 For each card, say whether the facts prove the triangles congruent. If they do, give the condition. [2]

card 1: two pairs of equal angles and the side between them

card 2: three pairs of equal angles

2.2 In triangles PQR and XYZ, $PQ = XY = 5$ cm, angle P = angle $X = 40°$ and $PR = XZ = 7$ cm. $QR = 4.6$ cm. Give the condition. Write down the length of YZ. [2]



Exercises 2.1·2.2

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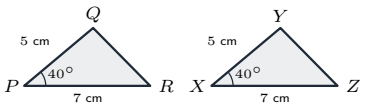
card 1: two pairs of equal angles and the side between them

card 2: three pairs of equal angles

2.1

- card 1: **yes, ASA**
- card 2: **no**, three equal angles only prove that they are similar

2.2 In triangles PQR and XYZ, $PQ = XY = 5$ cm, angle P = angle $X = 40°$ and $PR = XZ = 7$ cm. $QR = 4.6$ cm. Give the condition. Write down the length of YZ. [2]

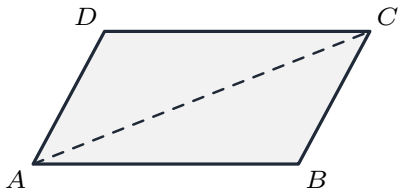


2.2

- **SAS**
- $YZ =$ **4.6** cm

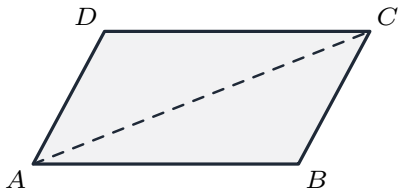
Exercise 3.1

3.1 $ABCD$ is a parallelogram with diagonal AC. Prove that triangles ABC and CDA are congruent. [3]



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3.1 $ABCD$ is a parallelogram with diagonal AC. Prove that triangles ABC and CDA are congruent. [3]



3.1

- $AB = CD$ and $BC = DA$: opposite sides of a parallelogram are equal
- AC is a side of both triangles
- so congruent by **SSS**

Exercise 3.2 (a)

3.2 $ABCD$ is a kite with $AB = AD$ and $CB = CD$. The diagonals meet at X.

(a) Prove that triangles ABC and ADC are congruent. [2]

Exercise 3.2 (a)

3.2 $ABCD$ is a kite with $AB = AD$ and $CB = CD$. The diagonals meet at X.

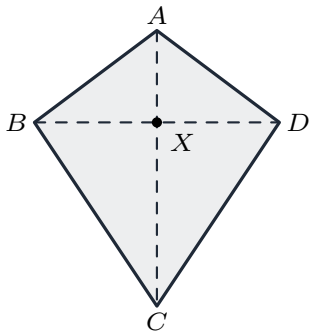
(a) Prove that triangles ABC and ADC are congruent. [2]

(a) $AB = AD$, $CB = CD$ (given) and AC is shared; so congruent by **SSS**

Exercise 3.2 (b)

3.2 $ABCD$ is a kite with $AB = AD$ and $CB = CD$. The diagonals meet at X.

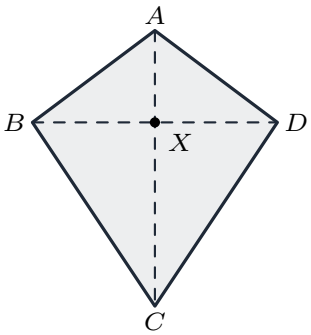
(b) Use part (a) to explain why angle ABC = angle ADC and why AC cuts angle BAD in half. [3]



Exercise 3.2 (b)

3.2 $ABCD$ is a kite with $AB = AD$ and $CB = CD$. The diagonals meet at X.

(b) Use part (a) to explain why angle ABC = angle ADC and why AC cuts angle BAD in half. [3]



(b) matching angles of congruent triangles are equal; so $ABC = ADC$; and angle BAC = angle DAC, so AC bisects angle BAD

Exercise 3.2 (c)

3.2 $ABCD$ is a kite with $AB = AD$ and $CB = CD$. The diagonals meet at X.

(c) Prove that triangles ABX and ADX are congruent. Use this to show that $BX = DX$ and that the diagonals cross at $90°$. [4]

Exercise 3.2 (c)

3.2 $ABCD$ is a kite with $AB = AD$ and $CB = CD$. The diagonals meet at X.

(c) Prove that triangles ABX and ADX are congruent. Use this to show that $BX = DX$ and that the diagonals cross at $90°$. [4]

(c) $AB = AD$, angle BAX = angle DAX, AX shared; congruent by **SAS**; so $BX = DX$; angles AXB and AXD are equal and add to $180°$, so each is $90°$

Exercise 3.2 (d)

3.2 $ABCD$ is a kite with $AB = AD$ and $CB = CD$. The diagonals meet at X.

(d) $AB = 5$ cm, $BD = 8$ cm and $AC = 9$ cm. Work out BX, AX, XC, the length BC and the area of the kite. [5]

Exercise 3.2 (d)

3.2 $ABCD$ is a kite with $AB = AD$ and $CB = CD$. The diagonals meet at X.

(d) $AB = 5$ cm, $BD = 8$ cm and $AC = 9$ cm. Work out BX, AX, XC, the length BC and the area of the kite. [5]

angles AXB and AXD are equal and add to $180°$, so each is $90°$

(d) $BX = \mathbf{4}$ cm; $AX = \sqrt{25 - 16} = \mathbf{3}$ cm; $XC = \mathbf{6}$ cm; $BC = \sqrt{16 + 36} = \mathbf{7.21}$ cm; area $\frac{1}{2} \times 9 \times 8 = \mathbf{36}$ cm^2