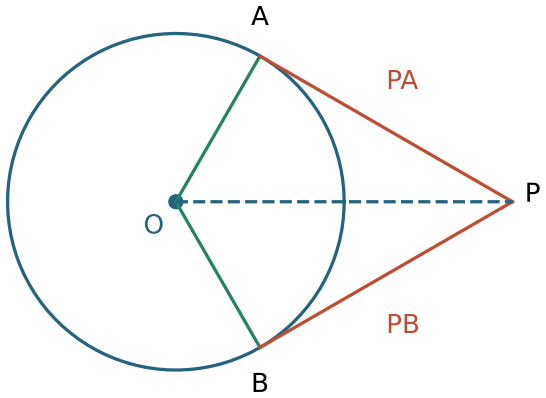
Exercise walk-through

4.4
Transformations, constructions, loci and vectors (G7–G10, G25)

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Figure: Congruence, similarity and circles (4)

Shared OP and equal radii → RHS → PA = PB



- Equal radii, shared OP and right angles give congruent triangles (RHS), so PA = PB
- Match each part of the diagram to the text.

Pythagoras and trigonometry

- **Pythagoras' theorem.** In a right-angled triangle, $a^2 + b^2 = c^2$, where c is the **hypotenuse**, the longest side, opposite the right angle.
- A ladder whose foot is 3 m from a wall and whose top is 4 m up the wall is $\sqrt{9+16} = 5$ m long.
- **Trigonometric ratios.** Label the sides from the angle θ: opposite, **adjacent** (next to the angle) and **hypotenuse**.
- **Worked example 12.** In a right-angled triangle the hypotenuse is 23.7 cm and x is the side **adjacent** to an angle of $52°$.

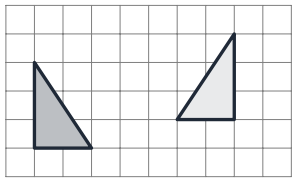
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Vectors and coordinate geometry (2)

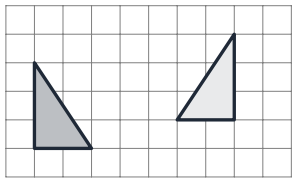
- Add and subtract vectors by adding and subtracting their parts: with $\mathbf{a} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\mathbf{a} + \mathbf{b} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ and $2\mathbf{a} - \mathbf{b} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$.
- **Vector proofs (H).** If one vector is a number times another, the lines are parallel.

Exercises 2.1·2.2

2.1 Reflect the point $(3, 2)$ in the y-axis. [2]

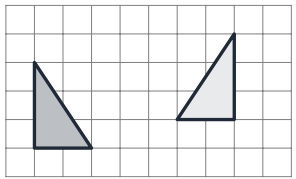


2.2 Rotate the point $(3, 2)$ by $90°$ anticlockwise about the origin. [2]



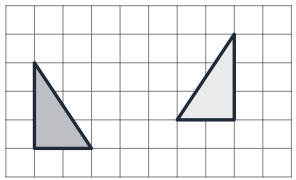
Exercises 2.1·2.2

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2.1 $(-3, 2)$.

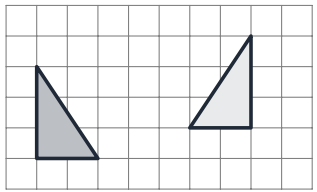
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2.2 $(-2, 3)$.

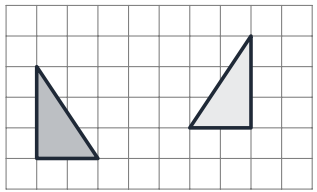
Exercise 2.3

2.3 *(Higher.)* Enlarge $P = (3, 2)$ about $C = (1, 1)$ with scale factor 2, and again with scale factor -2. [3]



Exercise 2.3

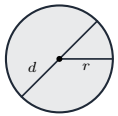
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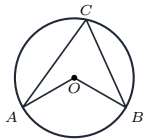
2.3 $k = 2$: $C + 2(P - C) = (1, 1) + 2(2, 1) = (5, 3)$. $k = -2$: $(1, 1) - 2(2, 1) = (-3, -1)$, opposite side of the centre.

Exercise 2.4

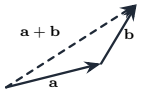
2.4 An arc subtends $100°$ at the centre. Find its angle at the circumference on the same arc. [2]



2.5 One angle of a cyclic quadrilateral is $112°$. Find its opposite angle. [2]

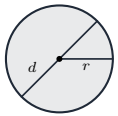


2.6 With $\mathbf{a} = (4, 1)$ and $\mathbf{b} = (1, 3)$, find $\mathbf{a} + \mathbf{b}$ and $2\mathbf{a} - \mathbf{b}$. [3]

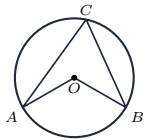


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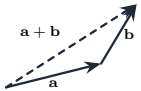
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2.4 $100 \div 2 = 50°$ – angle at the centre is twice the angle at the circumference.

2.5 $180 - 112 = 68°$ – opposite angles of a cyclic quadrilateral sum to $180°$.

2.6 $\mathbf{a} + \mathbf{b} = (5, 4)$; $2\mathbf{a} - \mathbf{b} = (8, 2) - (1, 3) = (7, -1)$.

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Full solution 2.4

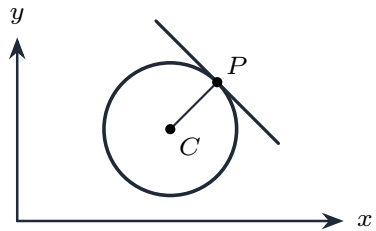
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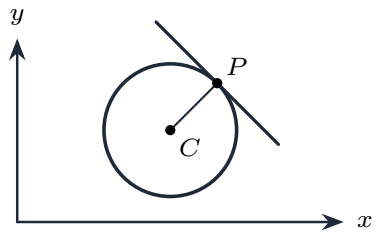
Exercise 3.1

3.1 *(June 2024 2H Q1, adapted.)* A circle has centre O. Name in one word each: the region between a chord and its arc; the region between two radii and their arc. [2]



Exercise 3.1

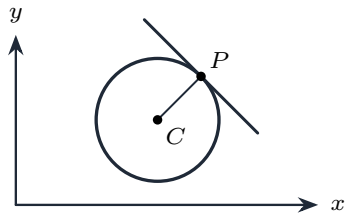
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3.1 The chord-and-arc region is the **segment**; the radii-and-arc region is the **sector**.

Setup for exercise 3.2

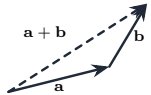
3.2 *(June 2024 1H Q22, adapted.)* Two tangents from an external point P touch a circle, centre O, at A and B. Angle $APB = 24°$. The point C lies on the major arc AB. Work out angle ACB. [4]



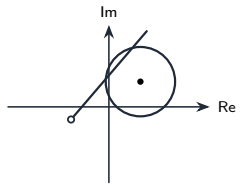
Exercise 3.2

3.3 *(June 2024 2H Q23, adapted.)* D, E and F are points with $\overrightarrow{CD} = 6\mathbf{a} + \mathbf{b}$, $\overrightarrow{CE} = 14\mathbf{a} - 3\mathbf{b}$ and $\overrightarrow{CF} = 16\mathbf{a} - 4\mathbf{b}$.

(a) Find $\overrightarrow{DE}$ and $\overrightarrow{EF}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. (b) Prove that DEF is a straight line. [4]



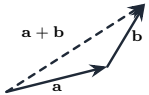
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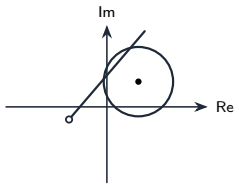
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3.2 Each tangent is perpendicular to its radius at the contact point, so quadrilateral $OAPB$ has two $90°$ angles: $\angle AOB = 360 - 90 - 90 - 24 = 156°$. The angle at the centre is twice the angle at the circumference on the same arc: $\angle ACB = 156 \div 2 = 78°$.

Full solution on next slide

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Full solution 3.2

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3.3 (a) $\overrightarrow{DE} = (14\mathbf{a} - 3\mathbf{b}) - (6\mathbf{a} + \mathbf{b}) = 8\mathbf{a} - 4\mathbf{b}$; $\overrightarrow{EF} = (16\mathbf{a} - 4\mathbf{b}) - (14\mathbf{a} - 3\mathbf{b}) = 2\mathbf{a} - \mathbf{b}$. (b) $\overrightarrow{DE} = 4(2\mathbf{a} - \mathbf{b}) = 4\,\overrightarrow{EF}$, so DE is parallel to EF; they share the point E, so DEF is a straight line.

3.4 The perpendicular bisector of AB; half of AB is 3 cm, so the distance is $\sqrt{5^2 - 3^2} = 4$ cm (a 3-4-5 triangle).

Local review guidance: full transformation descriptions – centre, factor, line, angle and direction; the quadrilateral angle sum $360-90-90-24$ then halving for the circumference angle in 3.2; $\overrightarrow{DE} = 4\overrightarrow{EF}$ stated WITH the shared point for the collinearity claim; locus named as a construction, not “the middle”. This is authored classroom guidance.