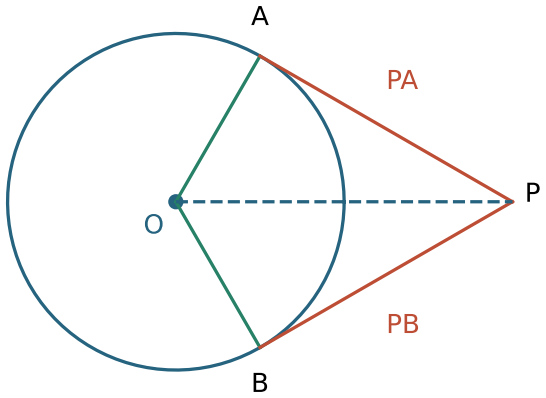
Exercise walk-through

4.4.5
Transformation Composition

AQA · GCSE · Mathematics

Figure: Congruence, similarity and circles (4)

Shared OP and equal radii → RHS → PA = PB



- Equal radii, shared OP and right angles give congruent triangles (RHS), so PA = PB
- Match each part of the diagram to the text.

Perimeter, area and volume

- Triangle: $\frac{1}{2} \times b \times h$, where h is the **perpendicular height** .
- Parallelogram: $b \times h$.
- Trapezium: $\frac{1}{2}(a+b)h$, where a and b are the parallel sides.
- Circle: $A = \pi r^2$, and the **circumference** is $C = 2\pi r = \pi d$.

$$\text{arc length} = \frac{\theta}{360} \times 2\pi r \qquad \text{sector area} = \frac{\theta}{360} \times \pi r^2$$

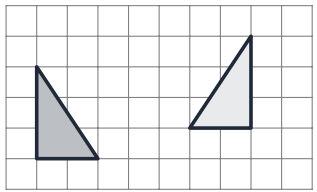
Pythagoras and trigonometry

- **Pythagoras' theorem.** In a right-angled triangle, $a^2 + b^2 = c^2$, where c is the **hypotenuse**, the longest side, opposite the right angle.
- A ladder whose foot is 3 m from a wall and whose top is 4 m up the wall is $\sqrt{9+16} = 5$ m long.
- **Trigonometric ratios.** Label the sides from the angle θ: opposite, **adjacent** (next to the angle) and **hypotenuse**.
- **Worked example 12.** In a right-angled triangle the hypotenuse is 23.7 cm and x is the side **adjacent** to an angle of $52°$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

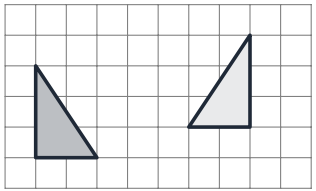
Exercise 2.1

2.1 *(H)* Reflect $(5, 2)$ in the line $x = 3$. Write down its image. [1]



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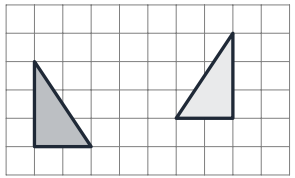


2.1

- $(5, 2)$ is 2 to the right of $x = 3$, so the image is 2 to the left: $(\mathbf{1}, \mathbf{2})$

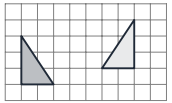
Setup for exercise 2.2

2.2 *(H)* Enlarge $(2, 3)$ by scale factor -1 from the origin. Write down its image. [1]

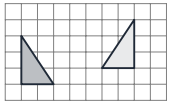


Exercise 2.2

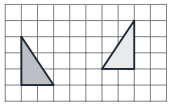
2.3 *(H)* Translate $(1, 2)$ by $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$, then by $\begin{pmatrix} -5 \\ 4 \end{pmatrix}$. Write down the final image and the single translation that does both. [2]



2.4 *(H)* Reflect $(4, 1)$ in the x-axis, then reflect the image in the y-axis. Write down the final image. [2]

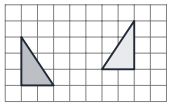


2.5 *(H)* Which of the points $(2, 2)$, $(3, 1)$ and $(0, 0)$ are invariant under reflection in the line $y = x$? [2]

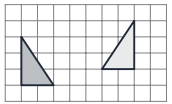


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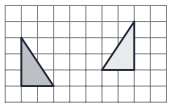
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2.2

- both signs change: $(\mathbf{-2}, \mathbf{-3})$

2.3

Full solution on next slide

Full solution 2.2

2.2

- both signs change: $(\mathbf{-2}, \mathbf{-3})$

2.3

- $(1 + 3 - 5, 2 - 1 + 4) = (\mathbf{-1}, \mathbf{5})$
- single translation $\begin{pmatrix} 3 - 5 \\ -1 + 4 \end{pmatrix} = \begin{pmatrix} \mathbf{-2} \\ \mathbf{3} \end{pmatrix}$

2.4

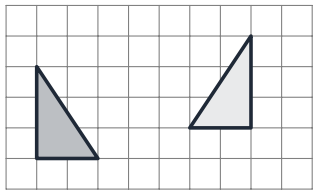
- in the x-axis: $(4, -1)$
- then in the y-axis: $(\mathbf{-4}, \mathbf{-1})$

2.5

- a point is invariant when it lies on the mirror line, where $y = x$
- $(\mathbf{2}, \mathbf{2})$ and $(\mathbf{0}, \mathbf{0})$; $(3, 1)$ moves to $(1, 3)$

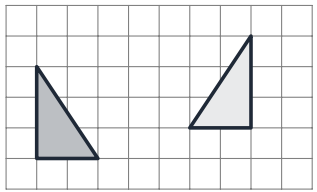
Exercise 3.1

3.1 *(H)* Explain why a rotation of 90° about the origin has exactly one invariant point. [1]



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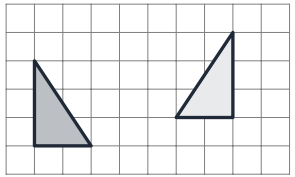


3.1

- every point except the centre moves round a circle; only the **centre**, the origin, stays where it is

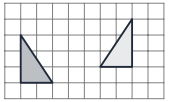
Setup for exercise 3.2

3.2 *(H)* Describe fully the single transformation equivalent to a reflection in the x-axis followed by a reflection in the y-axis. [2]

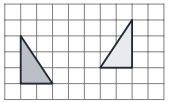


Exercise 3.2

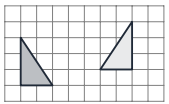
3.3 *(H)* Point $Q(2, 1)$ is rotated $90°$ anticlockwise about the origin and then translated by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Show that doing these in the reverse order gives a different image. [3]



3.4 *(H)* A shape is reflected in the line $x = 1$ and then in the line $x = 4$. Describe fully the single transformation that has the same effect. [3]

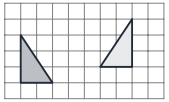


3.5 *(H)* A triangle has vertices $(2, 1)$, $(4, 1)$ and $(2, 2)$. It is enlarged by scale factor -2 from centre $(1, 1)$. Work out the vertices of the image and its area. [4]

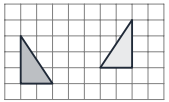


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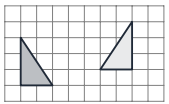
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3.2

- $(x, y) \to (x, -y) \to (-x, -y)$
- **rotation of** $180°$ **about the origin**

Full solution on next slide

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Full solution 3.2 (1/2)

3.2

- $(x, y) \to (x, -y) \to (-x, -y)$
- **rotation of $180°$ about the origin**

3.3

- rotate first: $(2, 1) \to (-1, 2)$, then translate: $(0, 2)$
- translate first: $(3, 1)$, then rotate: $(-1, 3)$
- $(0, 2) \neq (-1, 3)$, so the order matters

3.4

Full solution 3.2 (2/2)

- test the origin: in $x = 1$, $(0, 0) \to (2, 0)$; then in $x = 4$, $(2, 0) \to (6, 0)$
- every point moves 6 to the right: twice the 3-unit gap between the lines
- **translation by** $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$

3.5

- from $C(1, 1)$: $(2, 1)$ is $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, times -2 is $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$, giving $(-1, 1)$
- likewise $(4, 1) \to (1 - 6, 1) = (-5, 1)$ and $(2, 2) \to (1 - 2, 1 - 2) = (-1, -1)$

- vertices $(\mathbf{-1}, \mathbf{1})$, $(\mathbf{-5}, \mathbf{1})$, $(\mathbf{-1}, \mathbf{-1})$
- legs 4 and 2: area $= \frac{1}{2} \times 4 \times 2 = \mathbf{4}$ square units ($2^2 = 4$ times the original 1)