Exercise walk-through

4.4.4 Coordinate Geometry

AQA · GCSE · Mathematics

Angles and polygons

- Angles at a point add up to $360°$; angles on a straight line add up to $180°$.
- Vertically opposite angles are equal.
- Where a line crosses two parallel lines, alternate angles are equal, **corresponding angles** are equal, and co-interior angles add up to $180°$. Write "alternate angles are equal", never "Z-angles".
- The angles in a triangle add up to $180°$.

Perimeter, area and volume

- Triangle: $\frac{1}{2} \times b \times h$, where h is the **perpendicular height** .
- Parallelogram: $b \times h$.
- Trapezium: $\frac{1}{2}(a+b)h$, where a and b are the parallel sides.
- Circle: $A = \pi r^2$, and the **circumference** is $C = 2\pi r = \pi d$.

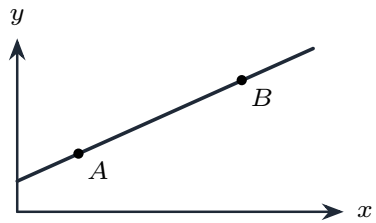
$$\text{arc length} = \frac{\theta}{360} \times 2\pi r \qquad \text{sector area} = \frac{\theta}{360} \times \pi r^2$$

Sources for this lesson

- AQA, *GCSE Mathematics (8300) specification*, subject content section 3.4 (Geometry and measures), and the GCSE formulae sheet.
- Cleomedes, *On the Circular Motions of the Celestial Bodies*, book 1, chapter 7, which records Eratosthenes' method.
- AQA, *GCSE Mathematics* Higher Papers 1 and 2 (June 2024), the trigonometry and vector questions, adapted.

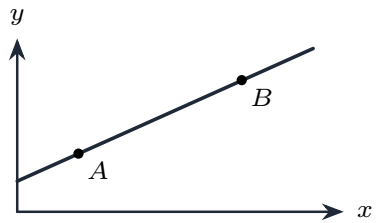
Exercise 2.1

2.1 Work out the midpoint of the line joining $(2, 7)$ and $(8, -1)$. [1]



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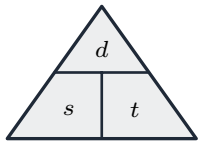


2.1

- $\left(\frac{2+8}{2}, \frac{7+(-1)}{2}\right) = (\mathbf{5}, \mathbf{3})$

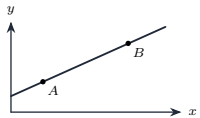
Setup for exercise 2.2

2.2 Work out the distance between $(-1, 3)$ and $(5, 11)$. [2]

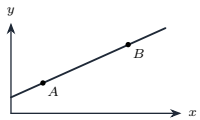


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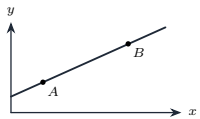
2.3 Work out the gradient of the line through $(1, 4)$ and $(4, 13)$. [1]



2.4 Find the equation of the line parallel to $y = 2x + 1$ that passes through $(3, 5)$. [2]

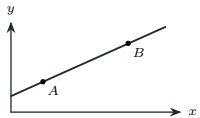


2.5 *(H)* Write down the gradient of a line perpendicular to $y = 3x - 2$. [1]

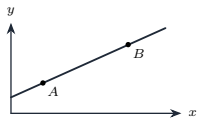


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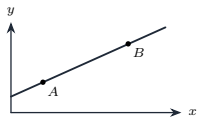
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2.2

- $d = \sqrt{(5-(-1))^2 + (11-3)^2} = \sqrt{36+64} = \sqrt{100} = \mathbf{10}$

2.3

Full solution on next slide

Full solution 2.2

2.2

- $d = \sqrt{(5-(-1))^2 + (11-3)^2} = \sqrt{36+64} = \sqrt{100} = \mathbf{10}$

2.3

- $m = \dfrac{13-4}{4-1} = \dfrac{9}{3} = \mathbf{3}$

2.4

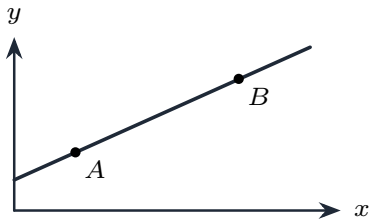
- parallel, so $m = 2$: $y - 5 = 2(x-3)$
- $\boldsymbol{y = 2x - 1}$

2.5

- $m_2 = -\dfrac{1}{3}$, so the gradient is $\boldsymbol{-\dfrac{1}{3}}$

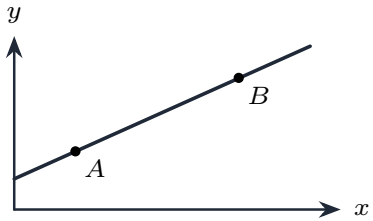
Exercise 3.1

3.1 Sam works out the gradient of the line through $(2, 1)$ and $(4, 7)$ as $\frac{2}{6} = \frac{1}{3}$. Explain his mistake and give the correct gradient. [2]



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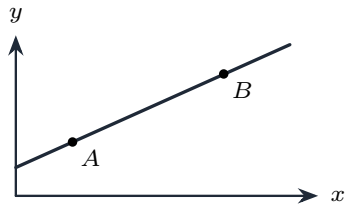


3.1

- he divided the change in x by the change in y; gradient is change in y over change in x
- $m = \frac{7-1}{4-2} = \frac{6}{2} = \mathbf{3}$

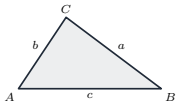
Setup for exercise 3.2

3.2 $M(4, 1)$ is the midpoint of AB, and A is $(1, -3)$. Work out the coordinates of B. [2]

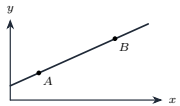


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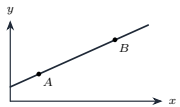
3.3 P is $(1, 1)$, Q is $(4, 5)$ and R is $(8, 2)$. Show that triangle PQR is isosceles and right-angled. [3]



3.4 *(H)* Find the equation of the line through $(0, 3)$ perpendicular to $2y = x + 4$, and where it crosses the x-axis. [3]

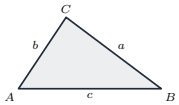


3.5 *(H)* A is $(2, 1)$ and B is $(6, 9)$. Find the equation of the perpendicular bisector of AB, in the form $y = mx + c$. [4]

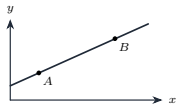


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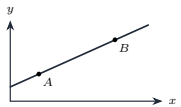
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3.2

- $\frac{1+x}{2} = 4$ and $\frac{-3+y}{2} = 1$
- $B = (\mathbf{7}, \mathbf{5})$

Full solution on next slide

Full solution 3.2 (1/2)

3.2

- $\frac{1+x}{2} = 4$ and $\frac{-3+y}{2} = 1$
- $B = (\mathbf{7}, \mathbf{5})$

3.3

- $PQ = \sqrt{3^2 + 4^2} = 5$ and $QR = \sqrt{4^2 + (-3)^2} = 5$, so $PQ = QR$: isosceles
- $PR^2 = 7^2 + 1^2 = 50 = 5^2 + 5^2 = PQ^2 + QR^2$, so by the converse of Pythagoras the angle at Q is $90°$

3.4

Full solution 3.2 (2/2)

- $2y = x + 4$ gives $y = \frac{1}{2}x + 2$, gradient $\frac{1}{2}$; perpendicular gradient $= -2$
- through $(0, 3)$: $\boldsymbol{y = -2x + 3}$
- $y = 0$: $2x = 3$, crossing at $(\mathbf{1.5}, \mathbf{0})$

3.5

- midpoint $= \left(\dfrac{2+6}{2}, \dfrac{1+9}{2}\right) = (4, 5)$
- gradient $AB = \dfrac{9-1}{6-2} = 2$, so the perpendicular gradient is $-\frac{1}{2}$

- $y - 5 = -\frac{1}{2}(x - 4)$
- $\boldsymbol{y = -\frac{1}{2}x + 7}$