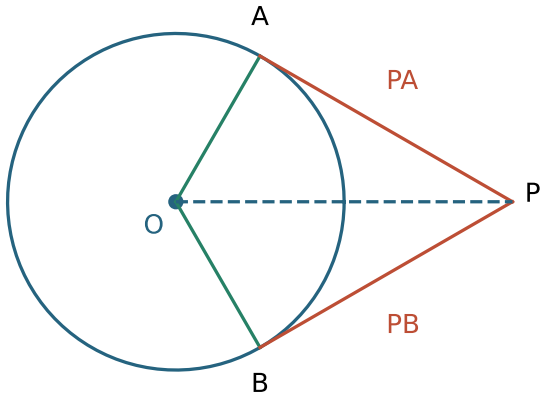
Exercise walk-through

4.4.2 Transformations

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Figure: Congruence, similarity and circles (4)

Shared OP and equal radii → RHS → PA = PB



- Equal radii, shared OP and right angles give congruent triangles (RHS), so PA = PB
- Match each part of the diagram to the text.

Perimeter, area and volume

- Triangle: $\frac{1}{2} \times b \times h$, where h is the **perpendicular height** .
- Parallelogram: $b \times h$.
- Trapezium: $\frac{1}{2}(a+b)h$, where a and b are the parallel sides.
- Circle: $A = \pi r^2$, and the **circumference** is $C = 2\pi r = \pi d$.

$$\text{arc length} = \frac{\theta}{360} \times 2\pi r \qquad \text{sector area} = \frac{\theta}{360} \times \pi r^2$$

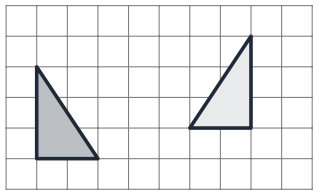
Pythagoras and trigonometry

- **Pythagoras' theorem.** In a right-angled triangle, $a^2 + b^2 = c^2$, where c is the **hypotenuse**, the longest side, opposite the right angle.
- A ladder whose foot is 3 m from a wall and whose top is 4 m up the wall is $\sqrt{9+16} = 5$ m long.
- **Trigonometric ratios.** Label the sides from the angle θ: opposite, **adjacent** (next to the angle) and **hypotenuse**.
- **Worked example 12.** In a right-angled triangle the hypotenuse is 23.7 cm and x is the side **adjacent** to an angle of $52°$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

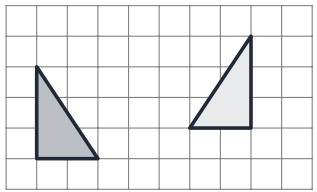
Exercise 2.1

2.1 Reflect the point $(3, 2)$ in the y-axis. Write down its image. [1]



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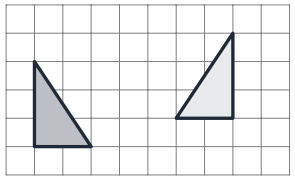


2.1

- the x-coordinate changes sign: $(\mathbf{-3}, \mathbf{2})$

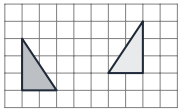
Setup for exercise 2.2

2.2 Reflect the point $(4, -1)$ in the line $y = x$. Write down its image. [1]

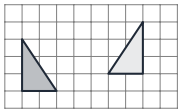


Exercise 2.2

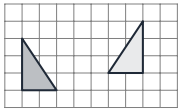
2.3 Rotate the point $(3, 2)$ through $90°$ anticlockwise about the origin. Write down its image. [1]



2.4 Translate the point $(2, 5)$ by $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$. Write down its image. [1]

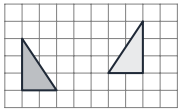


2.5 Enlarge the point $(4, 3)$ by scale factor 3 from centre $(2, 1)$. Write down its image. [2]

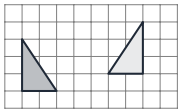


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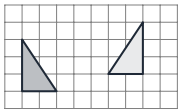
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2.2

- in $y = x$ the coordinates swap: $(\mathbf{-1}, \mathbf{4})$

2.3

Full solution on next slide

Full solution 2.2

2.2

- in $y = x$ the coordinates swap: $(\mathbf{-1}, \mathbf{4})$

2.3

- $90°$ anticlockwise about the origin: $(x, y) \to (-y, x)$, so $(\mathbf{-2}, \mathbf{3})$

2.4

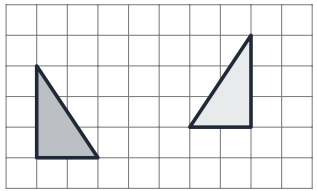
- $(2 - 4, 5 + 1) = (\mathbf{-2}, \mathbf{6})$

2.5

- from the centre: $\begin{pmatrix} 4-2 \\ 3-1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$, times 3 is $\begin{pmatrix} 6 \\ 6 \end{pmatrix}$
- image $= (2 + 6, 1 + 6) = (\mathbf{8}, \mathbf{7})$

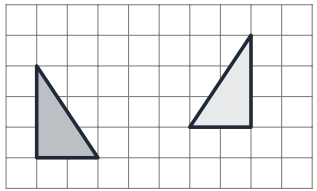
Exercise 3.1

3.1 Ben describes a transformation as “a rotation of 90°”. Explain why this is not a full description. [1]



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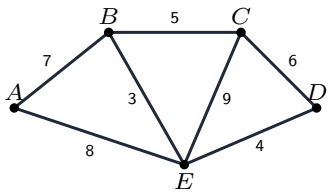


3.1

- a rotation also needs its **centre** and its **direction** (clockwise or anticlockwise)

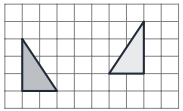
Setup for exercise 3.2

3.2 Triangle A has vertices $(1, 1)$, $(3, 1)$ and $(3, 2)$. Triangle B has vertices $(1, -1)$, $(3, -1)$ and $(3, -2)$. Describe fully the single transformation that maps A onto B. [2]

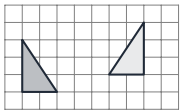


Exercise 3.2

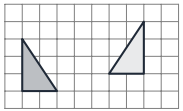
3.3 Rotate the point $(5, 1)$ through $180°$ about the centre $(2, 3)$. Write down its image. [2]



3.4 A triangle has vertices $(2, 2)$, $(4, 2)$ and $(2, 6)$. It is enlarged by scale factor $\frac{1}{2}$ from the origin. Write down the new vertices, and work out the area of the image. [3]

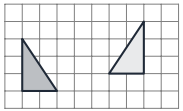


3.5 *(H)* An enlargement maps $A(2, 1)$ to $A'(5, 7)$ and $B(3, 1)$ to $B'(7, 7)$. Work out the scale factor and the centre of enlargement. [4]

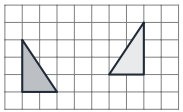


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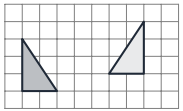
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3.2

- every y-coordinate changes sign and x stays the same
- **reflection in the x-axis** $(y = 0)$

Full solution on next slide

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Full solution 3.2

3.2

- every y-coordinate changes sign and x stays the same
- **reflection in the x-axis** $(y = 0)$

3.3

- from the centre: $\begin{pmatrix} 5-2 \\ 1-3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$; a half turn reverses it to $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$
- image $= (2-3, 3+2) = (\mathbf{-1}, \mathbf{5})$

3.4

- halve every coordinate: $(\mathbf{1}, \mathbf{1})$, $(\mathbf{2}, \mathbf{1})$, $(\mathbf{1}, \mathbf{3})$
- image legs are 1 and 2, so area $= \frac{1}{2} \times 1 \times 2 = \mathbf{1}$ square unit (a quarter of the original 4)

3.5

- $AB = 1$ and $A'B' = 2$, so the scale factor is **2**
- $A' = C + 2(A - C)$, so $C = 2A - A'$
- $C = (4-5, 2-7) = (\mathbf{-1}, \mathbf{-5})$
- check with B: $2B - B' = (6-7, 2-7) = (-1, -5)$, which agrees