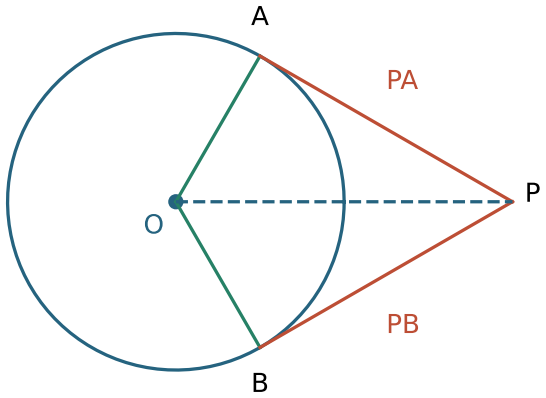
Exercise walk-through

4.3
Pythagoras, trigonometry and their graphs (G6, G20–G24)

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Figure: Congruence, similarity and circles (4)

Shared OP and equal radii → RHS → PA = PB



- Equal radii, shared OP and right angles give congruent triangles (RHS), so PA = PB
- Match each part of the diagram to the text.

Pythagoras and trigonometry

- **Pythagoras' theorem.** In a right-angled triangle, $a^2 + b^2 = c^2$, where c is the **hypotenuse**, the longest side, opposite the right angle.
- A ladder whose foot is 3 m from a wall and whose top is 4 m up the wall is $\sqrt{9+16} = 5$ m long.
- **Trigonometric ratios.** Label the sides from the angle θ: opposite, **adjacent** (next to the angle) and **hypotenuse**.
- **Worked example 12.** In a right-angled triangle the hypotenuse is 23.7 cm and x is the side **adjacent** to an angle of $52°$.

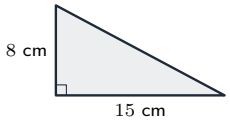
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Sources for this lesson

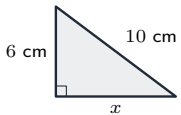
- AQA, *GCSE Mathematics (8300) specification*, subject content section 3.4 (Geometry and measures), and the GCSE formulae sheet.
- Cleomedes, *On the Circular Motions of the Celestial Bodies*, book 1, chapter 7, which records Eratosthenes' method.
- AQA, *GCSE Mathematics* Higher Papers 1 and 2 (June 2024), the trigonometry and vector questions, adapted.

Exercises 2.1·2.2

2.1 Work out the length of the hypotenuse. [2]

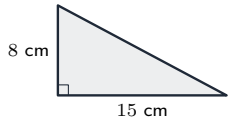


2.2 Work out the length x. [2]



Exercises 2.1·2.2

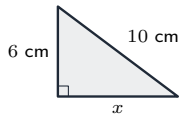
2.1 Work out the length of the hypotenuse. [2]



2.1

- $8^2 + 15^2 = 289$
- **17** cm

2.2 Work out the length x. [2]

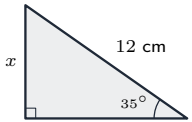


2.2

- $10^2 - 6^2 = 64$
- $x = \mathbf{8}$ cm

Exercises 2.3·2.4

2.3 Work out the length x. Give your answer to 3 significant figures. [2]

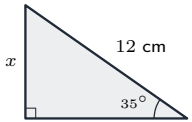


2.4 Work out $\mathbf{a} + \mathbf{b}$ and $2\mathbf{a} - \mathbf{b}$ as column vectors. [2]

$\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

Exercises 2.3·2.4

2.3 Work out the length x. Give your answer to 3 significant figures. [2]



2.3

- $x = 12 \sin 35°$
- **6.88** cm

2.4 Work out $\mathbf{a} + \mathbf{b}$ and $2\mathbf{a} - \mathbf{b}$ as column vectors. [2]

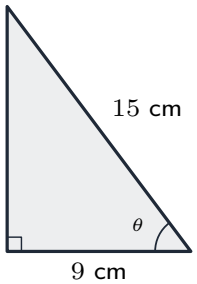
$\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

2.4

- $\mathbf{a} + \mathbf{b} = \begin{pmatrix} \mathbf{1} \\ \mathbf{3} \end{pmatrix}$
- $2\mathbf{a} - \mathbf{b} = \begin{pmatrix} \mathbf{8} \\ \mathbf{-6} \end{pmatrix}$

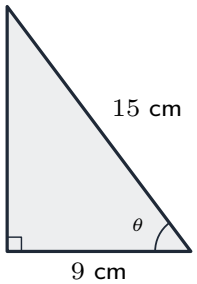
Exercise 3.1

3.1 Work out the angle θ. [3]



Exercise 3.1

3.1 Work out the angle θ. [3]



3.1

- $\cos\theta = \frac{9}{15}$
- $\theta = \cos^{-1}(0.6)$
- $\mathbf{53.1}^\circ$

Exercise 3.2 (a)

3.2 A window cleaner uses a ladder 6.5 m long. The foot of the ladder is 2.5 m from the wall. The ground is flat.

(a) How far up the wall does the ladder reach? [2]

Exercise 3.2 (a)

3.2 A window cleaner uses a ladder 6.5 m long. The foot of the ladder is 2.5 m from the wall. The ground is flat.

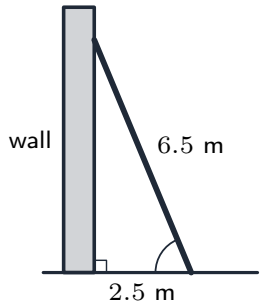
(a) How far up the wall does the ladder reach? [2]

(a) $\sqrt{6.5^2 - 2.5^2} = \mathbf{6}$ m

Exercise 3.2 (b)

3.2 A window cleaner uses a ladder 6.5 m long. The foot of the ladder is 2.5 m from the wall. The ground is flat.

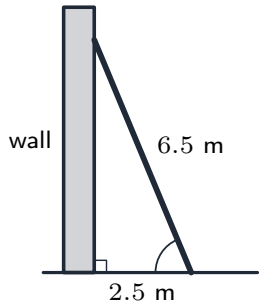
(b) Work out the angle between the ladder and the ground. A ladder is safe if this angle is between 70° and 80°. Is the ladder safe? [3]



Exercise 3.2 (b)

3.2 A window cleaner uses a ladder 6.5 m long. The foot of the ladder is 2.5 m from the wall. The ground is flat.

(b) Work out the angle between the ladder and the ground. A ladder is safe if this angle is between 70° and 80°. Is the ladder safe? [3]



(b) $\cos\theta = \frac{2.5}{6.5}$; $\theta = \mathbf{67.4°}$; **not safe**, as 67.4° is less than 70°

Setup for exercise 3.2

3.2 A window cleaner uses a ladder 6.5 m long. The foot of the ladder is 2.5 m from the wall. The ground is flat.

Exercise 3.2 (c)

(c) The window cleaner moves the foot of the ladder so that the angle is 75°. Work out how far the foot is from the wall and how far up the wall the ladder reaches. [4]

Exercise 3.2 (c)

(c) The window cleaner moves the foot of the ladder so that the angle is 75°. Work out how far the foot is from the wall and how far up the wall the ladder reaches. [4]

(c) $6.5\cos 75° = \mathbf{1.68}$ m; $6.5\sin 75° = \mathbf{6.28}$ m

Exercise 3.2 (d)

3.2 A window cleaner uses a ladder 6.5 m long. The foot of the ladder is 2.5 m from the wall. The ground is flat.

(d) A drone checks the roof. From the foot of the ladder it flies $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ m and then $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$ m. Write its total movement as one column vector. How far is the drone from where it started? Write down the vector that takes it straight back. [5]

Exercise 3.2 (d)

3.2 A window cleaner uses a ladder 6.5 m long. The foot of the ladder is 2.5 m from the wall. The ground is flat.

(d) A drone checks the roof. From the foot of the ladder it flies $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ m and then $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$ m. Write its total movement as one column vector. How far is the drone from where it started? Write down the vector that takes it straight back. [5]

(d) $\begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 5 \\ -1 \end{pmatrix} = \begin{pmatrix} \mathbf{8} \\ \mathbf{3} \end{pmatrix}$; $\sqrt{8^2 + 3^2} = \mathbf{8.54}$ m; $\begin{pmatrix} \mathbf{-8} \\ \mathbf{-3} \end{pmatrix}$