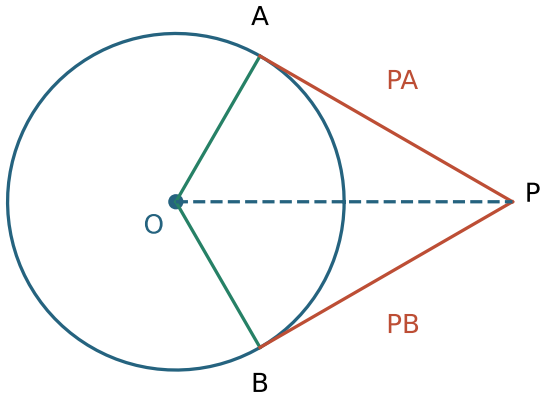
Exercise walk-through

4.3.5
Spatial Triangles

AQA · GCSE · Mathematics

Figure: Congruence, similarity and circles (4)

Shared OP and equal radii → RHS → PA = PB



- Equal radii, shared OP and right angles give congruent triangles (RHS), so PA = PB
- Match each part of the diagram to the text.

Pythagoras and trigonometry

- **Pythagoras' theorem.** In a right-angled triangle, $a^2 + b^2 = c^2$, where c is the **hypotenuse**, the longest side, opposite the right angle.
- A ladder whose foot is 3 m from a wall and whose top is 4 m up the wall is $\sqrt{9+16} = 5$ m long.
- **Trigonometric ratios.** Label the sides from the angle θ: opposite, **adjacent** (next to the angle) and **hypotenuse**.
- **Worked example 12.** In a right-angled triangle the hypotenuse is 23.7 cm and x is the side **adjacent** to an angle of $52°$.

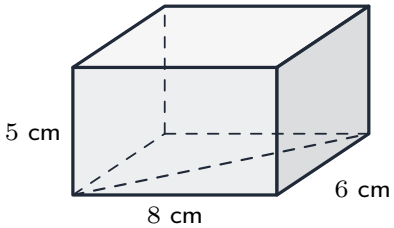
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Sources for this lesson

- AQA, *GCSE Mathematics (8300) specification*, subject content section 3.4 (Geometry and measures), and the GCSE formulae sheet.
- Cleomedes, *On the Circular Motions of the Celestial Bodies*, book 1, chapter 7, which records Eratosthenes' method.
- AQA, *GCSE Mathematics* Higher Papers 1 and 2 (June 2024), the trigonometry and vector questions, adapted.

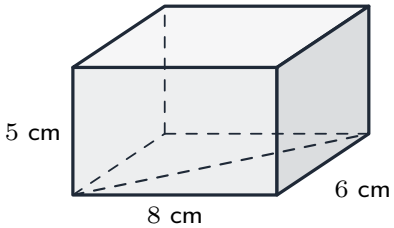
Exercise 2.1

2.1 A cuboid is 6 cm by 8 cm by 5 cm. Work out the diagonal of the 6 by 8 base and the space diagonal. [2]



Exercise 2.1

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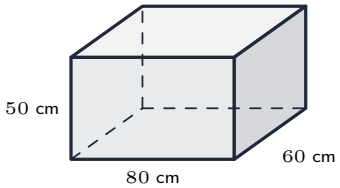


2.1

- base diagonal **10** cm
- $\sqrt{100 + 25} = \mathbf{11.2}$ cm

Exercise 2.2

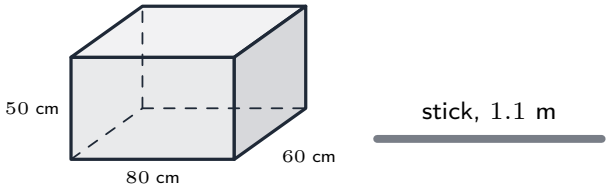
2.2 A box is 60 cm by 80 cm by 50 cm. Will a stick 1.1 m long fit inside the box? Show your working. [2]



stick, 1.1 m

Exercise 2.2

2.2 A box is 60 cm by 80 cm by 50 cm. Will a stick 1.1 m long fit inside the box? Show your working. [2]

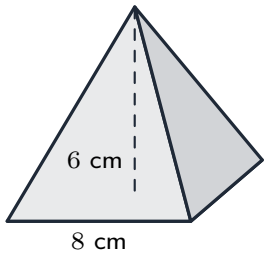


2.2

- $\sqrt{60^2 + 80^2 + 50^2} = 111.8$ cm
- $111.8 > 110$, so **yes**

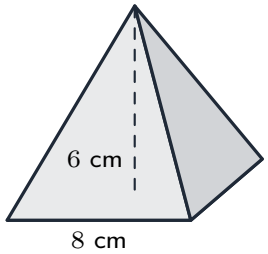
Exercise 3.1

3.1 A pyramid has a square base of side 8 cm. Its top is 6 cm above the centre of the base. Work out the length of a slant edge. [3]



Exercise 3.1

3.1 A pyramid has a square base of side 8 cm. Its top is 6 cm above the centre of the base. Work out the length of a slant edge. [3]



3.1

- half the base diagonal $= \frac{1}{2}\sqrt{128} = 5.66$
- $\sqrt{32 + 36}$
- **8.25** cm

Exercise 3.2 (a)

3.2 A room is a cuboid 5 m long, 4 m wide and 3 m high.

(a) Work out the length of the diagonal of the floor. [2]

Exercise 3.2 (a)

3.2 A room is a cuboid 5 m long, 4 m wide and 3 m high.

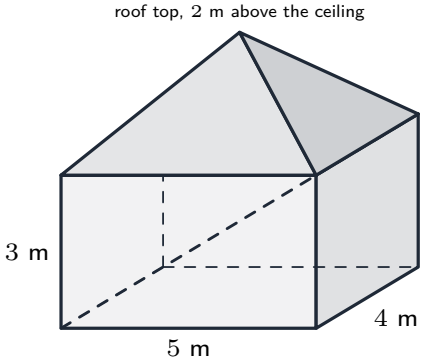
(a) Work out the length of the diagonal of the floor. [2]

(a) $\sqrt{5^2 + 4^2} = \sqrt{41} = \mathbf{6.40}$ m

Exercise 3.2 (b)

3.2 A room is a cuboid 5 m long, 4 m wide and 3 m high.

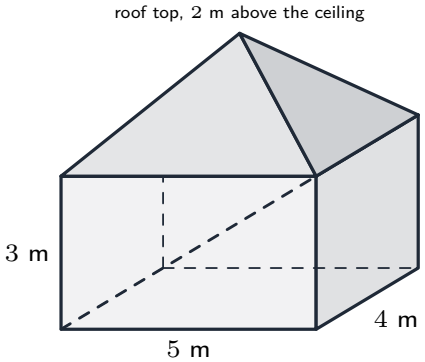
(b) Work out the length of the longest straight pole that fits in the room. [3]



Exercise 3.2 (b)

3.2 A room is a cuboid 5 m long, 4 m wide and 3 m high.

(b) Work out the length of the longest straight pole that fits in the room. [3]



(b) $\sqrt{41 + 3^2} = \sqrt{50} = \mathbf{7.07}$ m

Exercise 3.2 (c)

3.2 A room is a cuboid 5 m long, 4 m wide and 3 m high.

(c) Work out the angle between this pole and the floor, and the angle between the pole and a 5 m edge of the floor. [4]

Exercise 3.2 (c)

3.2 A room is a cuboid 5 m long, 4 m wide and 3 m high.

(c) Work out the angle between this pole and the floor, and the angle between the pole and a 5 m edge of the floor. [4]

(c) $\tan\theta = \frac{3}{\sqrt{41}}$; $\theta = \mathbf{25.1}°$; $\cos\phi = \frac{5}{\sqrt{50}}$; $\phi = \mathbf{45.0}°$

Exercise 3.2 (d)

3.2 A room is a cuboid 5 m long, 4 m wide and 3 m high.

(d) The roof is a pyramid on the room. Its top is 2 m above the centre of the ceiling. Work out the length of a sloping edge of the roof, and the angle between this edge and the ceiling. [5]

Exercise 3.2 (d)

3.2 A room is a cuboid 5 m long, 4 m wide and 3 m high.

(d) The roof is a pyramid on the room. Its top is 2 m above the centre of the ceiling. Work out the length of a sloping edge of the roof, and the angle between this edge and the ceiling. [5]

(d) half the ceiling diagonal $= \frac{1}{2}\sqrt{41} = 3.20$ m; $\sqrt{3.20^2 + 2^2} = \mathbf{3.77}$ m; $\tan^{-1}\left(\frac{2}{3.20}\right) = \mathbf{32.0}°$