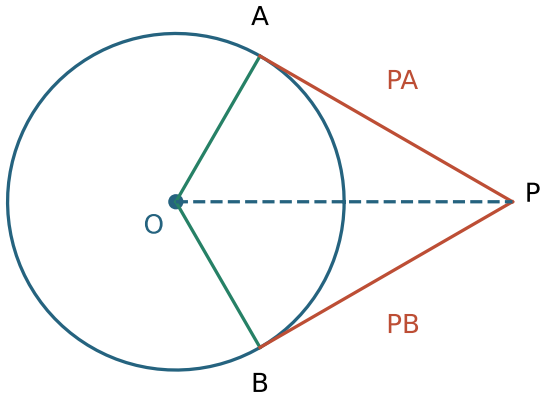
Exercise walk-through

4.3.4 General Triangles

AQA · GCSE · Mathematics

Figure: Congruence, similarity and circles (4)

Shared OP and equal radii → RHS → PA = PB



- Equal radii, shared OP and right angles give congruent triangles (RHS), so PA = PB
- Match each part of the diagram to the text.

Perimeter, area and volume

- Triangle: $\frac{1}{2} \times b \times h$, where h is the **perpendicular height** .
- Parallelogram: $b \times h$.
- Trapezium: $\frac{1}{2}(a+b)h$, where a and b are the parallel sides.
- Circle: $A = \pi r^2$, and the **circumference** is $C = 2\pi r = \pi d$.

$$\text{arc length} = \frac{\theta}{360} \times 2\pi r \qquad \text{sector area} = \frac{\theta}{360} \times \pi r^2$$

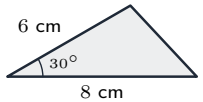
Pythagoras and trigonometry

- **Pythagoras' theorem.** In a right-angled triangle, $a^2 + b^2 = c^2$, where c is the **hypotenuse**, the longest side, opposite the right angle.
- A ladder whose foot is 3 m from a wall and whose top is 4 m up the wall is $\sqrt{9+16} = 5$ m long.
- **Trigonometric ratios.** Label the sides from the angle θ: opposite, **adjacent** (next to the angle) and **hypotenuse**.
- **Worked example 12.** In a right-angled triangle the hypotenuse is 23.7 cm and x is the side **adjacent** to an angle of $52°$.

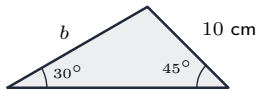
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Exercises 2.1·2.2

2.1 Work out the area of the triangle. [2]

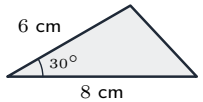


2.2 Work out the length b. [2]



Exercises 2.1·2.2

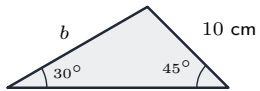
2.1 Work out the area of the triangle. [2]



2.1

- $\frac{1}{2} \times 6 \times 8 \times \sin 30°$
- **12** cm^2

2.2 Work out the length b. [2]

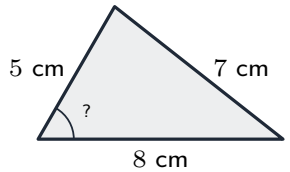


2.2

- $b = \dfrac{10 \sin 45°}{\sin 30°}$
- **14.1** cm

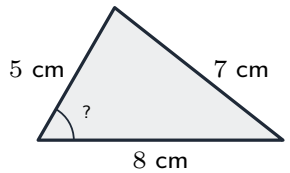
Exercise 3.1

3.1 A triangle has sides of 5 cm, 7 cm and 8 cm. Work out the angle opposite the 7 cm side. [3]



Exercise 3.1

3.1 A triangle has sides of 5 cm, 7 cm and 8 cm. Work out the angle opposite the 7 cm side. [3]



3.1

- $\cos\theta = \dfrac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8}$
- $= \frac{40}{80} = 0.5$
- $\mathbf{60^\circ}$

Exercise 3.2 (a)

3.2 A farmer's field is a triangle ABC. $AB = 120$ m, $AC = 90$ m and angle $BAC = 75°$.

(a) Work out the area of the field. [2]

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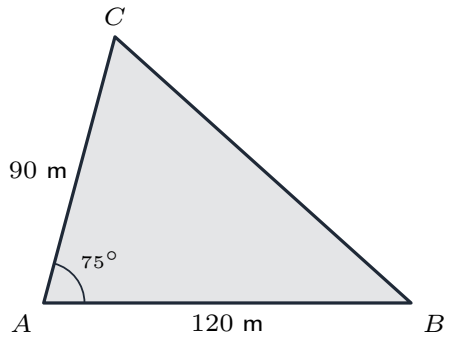
(a) Work out the area of the field. [2]

(a) $\frac{1}{2} \times 120 \times 90 \times \sin 75° = \mathbf{5216}$ m^2

Exercise 3.2 (b)

3.2 A farmer's field is a triangle ABC. $AB = 120$ m, $AC = 90$ m and angle $BAC = 75°$.

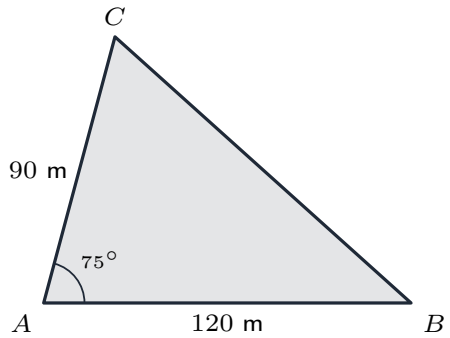
(b) Work out the length of BC. [3]



Exercise 3.2 (b)

3.2 A farmer's field is a triangle ABC. $AB = 120$ m, $AC = 90$ m and angle $BAC = 75°$.

(b) Work out the length of BC. [3]



(b) $BC^2 = 120^2 + 90^2 - 2 \times 120 \times 90 \cos 75° = 16\,910$; $BC = \mathbf{130.0}$ m

Exercise 3.2 (c)

3.2 A farmer's field is a triangle ABC. $AB = 120$ m, $AC = 90$ m and angle $BAC = 75°$.

(c) Work out angle ABC and angle ACB. [4]

Exercise 3.2 (c)

3.2 A farmer's field is a triangle ABC. $AB = 120$ m, $AC = 90$ m and angle $BAC = 75°$.

(c) Work out angle ABC and angle ACB. [4]

(c) $\dfrac{\sin B}{90} = \dfrac{\sin 75°}{130.0}$; $B = \mathbf{42.0}°$; $180 - 75 - 42.0 = \mathbf{63.0}°$

Exercise 3.2 (d)

3.2 A farmer's field is a triangle ABC. $AB = 120$ m, $AC = 90$ m and angle $BAC = 75°$.

(d) A fence goes all round the field. Fencing costs £12.50 per metre. Work out the cost of the fence. A straight path from A to the midpoint M of BC splits the field into two parts. Explain why the two parts have the same area, and write down this area. [5]

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(d) perimeter $120 + 90 + 130.0 = 340.0$ m; $340.04 \times 12.50 =$ £**4250.46**; both parts have the same height from A and equal bases $BM = MC$, so equal areas; **2608** m^2 each