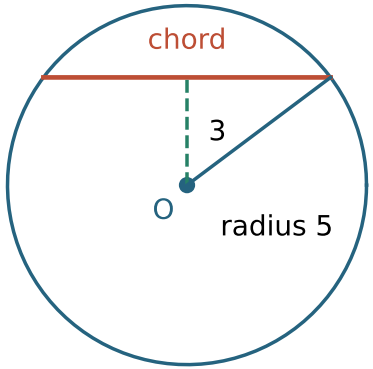
Exercise walk-through

4.3.3 Exact Trig

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Figure: Congruence, similarity and circles (2)

Circle of radius 5: distances use one consistent scale

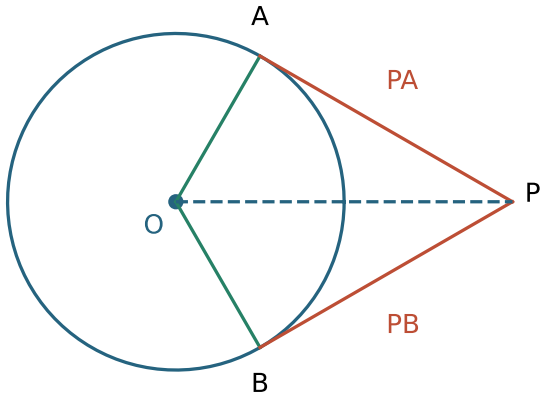


Perpendicular bisects chord: half-length 4

- Radius 5, distance 3: half the chord is $\sqrt{(5^2 - 3^2)} = 4$, so the chord is 8
- Match each part of the diagram to the text.

Figure: Congruence, similarity and circles (4)

Shared OP and equal radii → RHS → PA = PB



- Equal radii, shared OP and right angles give congruent triangles (RHS), so PA = PB
- Match each part of the diagram to the text.

Pythagoras and trigonometry

- **Pythagoras' theorem.** In a right-angled triangle, $a^2 + b^2 = c^2$, where c is the **hypotenuse**, the longest side, opposite the right angle.
- A ladder whose foot is 3 m from a wall and whose top is 4 m up the wall is $\sqrt{9+16} = 5$ m long.
- **Trigonometric ratios.** Label the sides from the angle θ: opposite, **adjacent** (next to the angle) and **hypotenuse**.
- **Worked example 12.** In a right-angled triangle the hypotenuse is 23.7 cm and x is the side **adjacent** to an angle of $52°$.

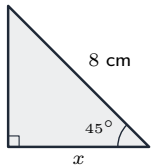
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Exercises 2.1·2.2

2.1 Write down the exact values of $\cos 60°$ and $\tan 45°$. [2]

$\cos 60°$ $\tan 45°$

2.2 Work out the exact length x. [2]



Exercises 2.1·2.2

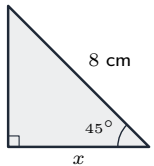
2.1 Write down the exact values of $\cos 60°$ and $\tan 45°$. [2]

$\cos 60°$ $\tan 45°$

2.1

- $\frac{1}{2}$
- 1

2.2 Work out the exact length x. [2]



2.2

- $x = 8\cos 45° = 8 \times \frac{\sqrt{2}}{2}$
- $4\sqrt{2}$ cm

Exercise 3.1

3.1 Without a calculator, work out $4\sin 30° + \sqrt{3}\tan 60°$. [3]

$4\sin 30° + \sqrt{3}\tan 60°$

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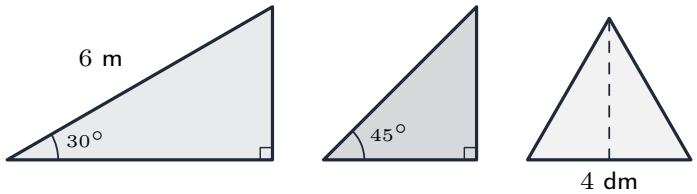
3.1

- $\sin 30° = \frac{1}{2}$ and $\tan 60° = \sqrt{3}$
- $2 + \sqrt{3} \times \sqrt{3} = 2 + 3$
- **5**

Exercise 3.2 (a)

3.2 Answer this question without a calculator. A ramp is 6 m long and rises at 30° to the ground.

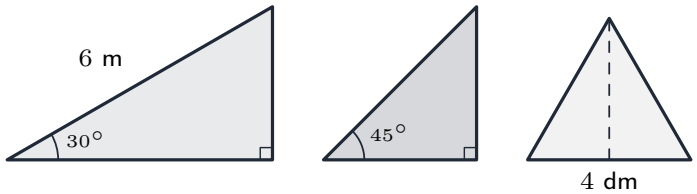
(a) Work out the exact height of the top of the ramp. [2]



Exercise 3.2 (a)

3.2 Answer this question without a calculator. A ramp is 6 m long and rises at 30° to the ground.

(a) Work out the exact height of the top of the ramp. [2]



(a) $6 \sin 30° = \mathbf{3}$ m

Exercise 3.2 (b)

3.2 Answer this question without a calculator. A ramp is 6 m long and rises at 30° to the ground.

(b) Work out the exact horizontal distance the ramp covers. [3]

Exercise 3.2 (b)

3.2 Answer this question without a calculator. A ramp is 6 m long and rises at 30° to the ground.

(b) Work out the exact horizontal distance the ramp covers. [3]

(b) $\cos 30° = \frac{\sqrt{3}}{2}$; $6 \times \frac{\sqrt{3}}{2} = \mathbf{3\sqrt{3}}$ m

Exercise 3.2 (c)

3.2 Answer this question without a calculator. A ramp is 6 m long and rises at 30° to the ground.

(c) A second ramp rises the same height at 45°. Work out its exact length. How much less horizontal distance does it cover than the first ramp? Give your answer in the form $a(\sqrt{3}-1)$. [4]

Exercise 3.2 (c)

3.2 Answer this question without a calculator. A ramp is 6 m long and rises at 30° to the ground.

(c) A second ramp rises the same height at 45°. Work out its exact length. How much less horizontal distance does it cover than the first ramp? Give your answer in the form $a(\sqrt{3}-1)$. [4]

(c) $\dfrac{3}{\sin 45^\circ} = \mathbf{3\sqrt{2}}$ m; horizontal distance 3 m, so $3\sqrt{3}-3 = \mathbf{3}(\sqrt{\mathbf{3}}-\mathbf{1})$ m

Exercise 3.2 (d)

3.2 Answer this question without a calculator. A ramp is 6 m long and rises at 30° to the ground.

(d) A warning sign is an equilateral triangle with sides of 4 dm. Use $\sin 60°$ to find its exact height. Find its exact area. Show that $\tan 60° = \sqrt{3}$ from your triangle. [5]

Exercise 3.2 (d)

3.2 Answer this question without a calculator. A ramp is 6 m long and rises at 30° to the ground.

(d) A warning sign is an equilateral triangle with sides of 4 dm. Use $\sin 60°$ to find its exact height. Find its exact area. Show that $\tan 60° = \sqrt{3}$ from your triangle. [5]

(d) height $4 \sin 60° = \mathbf{2\sqrt{3}}$ dm; area $\frac{1}{2} \times 4 \times 2\sqrt{3} = \mathbf{4\sqrt{3}}$ dm^2; $\tan 60° = \dfrac{2\sqrt{3}}{2} = \sqrt{3}$