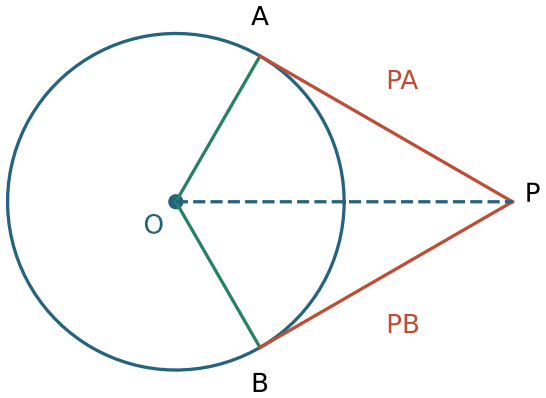
Exercise walk-through

4.2.6
Solid Measure

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Figure: Congruence, similarity and circles (4)

Shared OP and equal radii → RHS → PA = PB



- Equal radii, shared OP and right angles give congruent triangles (RHS), so PA = PB
- Match each part of the diagram to the text.

Perimeter, area and volume

- Triangle: $\frac{1}{2} \times b \times h$, where h is the **perpendicular height** .
- Parallelogram: $b \times h$.
- Trapezium: $\frac{1}{2}(a+b)h$, where a and b are the parallel sides.
- Circle: $A = \pi r^2$, and the **circumference** is $C = 2\pi r = \pi d$.

$$\text{arc length} = \frac{\theta}{360} \times 2\pi r \qquad \text{sector area} = \frac{\theta}{360} \times \pi r^2$$

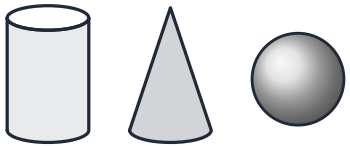
Pythagoras and trigonometry

- **Pythagoras' theorem.** In a right-angled triangle, $a^2 + b^2 = c^2$, where c is the **hypotenuse**, the longest side, opposite the right angle.
- A ladder whose foot is 3 m from a wall and whose top is 4 m up the wall is $\sqrt{9+16} = 5$ m long.
- **Trigonometric ratios.** Label the sides from the angle θ: opposite, **adjacent** (next to the angle) and **hypotenuse**.
- **Worked example 12.** In a right-angled triangle the hypotenuse is 23.7 cm and x is the side **adjacent** to an angle of $52°$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

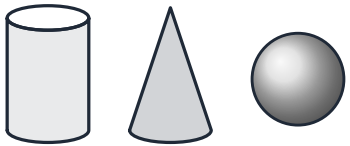
Exercise 2.1

2.1 Work out the volume of a sphere of radius 3 cm, in terms of π. [2]



Exercise 2.1

2.1 Work out the volume of a sphere of radius 3 cm, in terms of π. [2]



2.1

- $V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 27 = \mathbf{36}\pi \text{ cm}^3$

Setup for exercise 2.2

2.2 Work out the surface area of a sphere of radius 5 cm, in terms of π. [1]



Exercise 2.2

2.3 Work out the volume of a cone with radius 4 cm and height 9 cm, in terms of π. [2]



2.4 A square-based pyramid has base 6 cm by 6 cm and height 10 cm. Work out its volume. [2]



2.5 A cone has radius 5 cm and height 12 cm. Work out its slant height. [2]



Exercise 2.2

2.3 Work out the volume of a cone with radius 4 cm and height 9 cm, in terms of π. [2]



2.4 A square-based pyramid has base 6 cm by 6 cm and height 10 cm. Work out its volume. [2]



2.5 A cone has radius 5 cm and height 12 cm. Work out its slant height. [2]



2.2

- $4\pi r^2 = 4 \times \pi \times 25 = \mathbf{100}\pi$ cm^2

Full solution on next slide

Full solution 2.2

2.2

- $4\pi r^2 = 4 \times \pi \times 25 = \mathbf{100}\pi \text{ cm}^2$

2.3

- $V = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 16 \times 9 = \mathbf{48}\pi \text{ cm}^3$

2.4

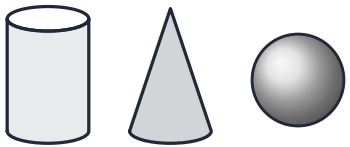
- $V = \frac{1}{3} \times 36 \times 10 = \mathbf{120} \text{ cm}^3$

2.5

- $l^2 = r^2 + h^2 = 5^2 + 12^2 = 169$
- $l = \mathbf{13} \text{ cm}$

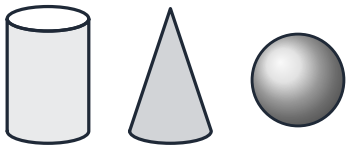
Exercise 3.1

3.1 Ali uses the slant height in $V = \frac{1}{3}\pi r^2 h$. Explain why this is wrong. [1]



Exercise 3.1

3.1 Ali uses the slant height in $V = \frac{1}{3}\pi r^2 h$. Explain why this is wrong. [1]



3.1

- h is the **perpendicular** height from the apex to the base; the slant height is longer, so the volume would be too large

Setup for exercise 3.2

3.2 Using **2.5**, work out the total surface area of the cone, in terms of π. [2]



3.3 A hemisphere has radius 6 cm. Work out its volume, to 3 significant figures. [2]



Exercise 3.2

3.4 A sphere has volume 288π cm^3. Work out its radius. [3]



3.5 *(H)* A cone of radius 9 cm and height 12 cm has its top 4 cm cut off parallel to its base. Work out the volume of the frustum in terms of π. [4]



Exercise 3.2

3.4 A sphere has volume 288π cm^3. Work out its radius. [3]



3.5 *(H)* A cone of radius 9 cm and height 12 cm has its top 4 cm cut off parallel to its base. Work out the volume of the frustum in terms of π. [4]



Full solution on next slide

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Full solution 3.2

3.2

- curved $\pi rl = \pi \times 5 \times 13 = 65\pi$; base $\pi r^2 = 25\pi$
- total = $\mathbf{90}\pi$ cm^2

3.3

- $V = \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3} \times \pi \times 216 = 144\pi = \mathbf{452}$ cm^3

3.4

- $\frac{4}{3}\pi r^3 = 288\pi$
- $r^3 = 288 \times \frac{3}{4} = 216$
- $r = \mathbf{6}$ cm

3.5

- small cone height 4 cm is $\frac{1}{3}$ of 12 cm, so its radius is $9 \div 3 = 3$ cm
- large: $\frac{1}{3}\pi \times 81 \times 12 = 324\pi$
- small: $\frac{1}{3}\pi \times 9 \times 4 = 12\pi$
- frustum = $324\pi - 12\pi = \mathbf{312}\pi$ cm^3