Exercise walk-through

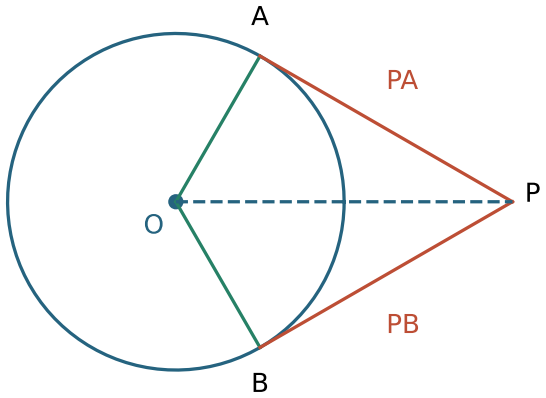
4.1.99

Independent transfer: Foundation then Higher

AQA · GCSE · Mathematics

Figure: Congruence, similarity and circles (4)

Shared OP and equal radii → RHS → PA = PB



- Equal radii, shared OP and right angles give congruent triangles (RHS), so PA = PB
- Match each part of the diagram to the text.

Pythagoras and trigonometry

- **Pythagoras' theorem.** In a right-angled triangle, $a^2 + b^2 = c^2$, where c is the **hypotenuse**, the longest side, opposite the right angle.
- A ladder whose foot is 3 m from a wall and whose top is 4 m up the wall is $\sqrt{9+16} = 5$ m long.
- **Trigonometric ratios.** Label the sides from the angle θ: opposite, **adjacent** (next to the angle) and **hypotenuse**.
- **Worked example 12.** In a right-angled triangle the hypotenuse is 23.7 cm and x is the side **adjacent** to an angle of $52°$.

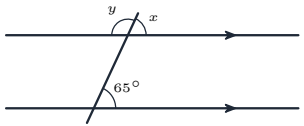
$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

Sources for this lesson

- AQA, *GCSE Mathematics (8300) specification*, subject content section 3.4 (Geometry and measures), and the GCSE formulae sheet.
- Cleomedes, *On the Circular Motions of the Celestial Bodies*, book 1, chapter 7, which records Eratosthenes' method.
- AQA, *GCSE Mathematics* Higher Papers 1 and 2 (June 2024), the trigonometry and vector questions, adapted.

Exercises 2.1·2.2

2.1 The two lines are parallel. Find the angles x and y. Give reasons. [2]

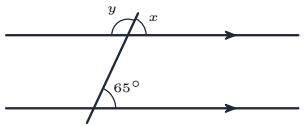


2.2 Work out the size of one interior angle of a regular octagon. [2]



Exercises 2.1·2.2

2.1 The two lines are parallel. Find the angles x and y. Give reasons. [2]



2.1

- $x = \mathbf{65}°$: corresponding angles are equal
- $y = 180 - 65 = \mathbf{115}°$: angles on a straight line

2.2 Work out the size of one interior angle of a regular octagon. [2]

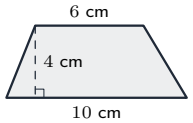


2.2

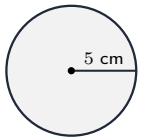
- exterior $360 \div 8 = 45°$
- interior $\mathbf{135}°$

Exercises 2.3·2.4

2.3 Work out the area of the trapezium. [2]

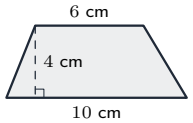


2.4 A circle has a radius of 5 cm. Work out its circumference and its area. [2]



Exercises 2.3·2.4

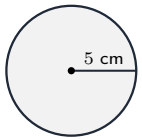
2.3 Work out the area of the trapezium. [2]



2.3

- $\frac{1}{2}(6+10) \times 4$
- **32** cm^2

2.4 A circle has a radius of 5 cm. Work out its circumference and its area. [2]

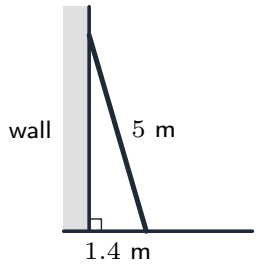


2.4

- $2\pi \times 5 = \mathbf{31.4}$ cm
- $\pi \times 5^2 = \mathbf{78.5}$ cm^2

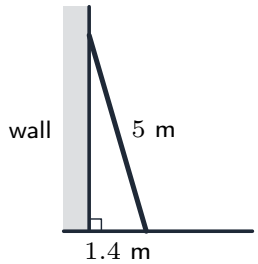
Exercise 3.1

3.1 A ladder 5 m long leans against a wall. Its foot is 1.4 m from the wall. How high up the wall does the ladder reach? [3]



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3.1

- $h^2 = 5^2 - 1.4^2$
- $= 25 - 1.96 = 23.04$
- $h = \mathbf{4.8}$ m

Exercise 3.2 (a)

3.2 A tent is a triangular prism 3 m long. Its front is an isosceles triangle with a base of 2.4 m and two sloping sides of 2 m.

(a) Work out the height of the tent. [2]

Exercise 3.2 (a)

3.2 A tent is a triangular prism 3 m long. Its front is an isosceles triangle with a base of 2.4 m and two sloping sides of 2 m.

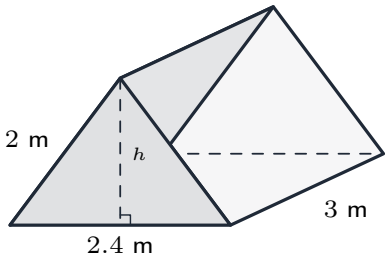
(a) Work out the height of the tent. [2]

(a) $\sqrt{2^2 - 1.2^2} = \mathbf{1.6}$ m

Exercise 3.2 (b)

3.2 A tent is a triangular prism 3 m long. Its front is an isosceles triangle with a base of 2.4 m and two sloping sides of 2 m.

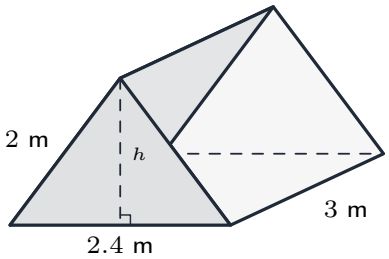
(b) Work out the volume of the tent. [3]



Exercise 3.2 (b)

3.2 A tent is a triangular prism 3 m long. Its front is an isosceles triangle with a base of 2.4 m and two sloping sides of 2 m.

(b) Work out the volume of the tent. [3]



(b) $\frac{1}{2} \times 2.4 \times 1.6 = 1.92$ m^2; $1.92 \times 3 =$ **5.76** m^3

Exercise 3.2 (c)

3.2 A tent is a triangular prism 3 m long. Its front is an isosceles triangle with a base of 2.4 m and two sloping sides of 2 m.

(c) The tent is made of fabric: two triangles, two sloping sides and a floor. Work out the area of fabric. Fabric costs £4.50 per m^2. Work out the cost. [4]

Exercise 3.2 (c)

3.2 A tent is a triangular prism 3 m long. Its front is an isosceles triangle with a base of 2.4 m and two sloping sides of 2 m.

(c) The tent is made of fabric: two triangles, two sloping sides and a floor. Work out the area of fabric. Fabric costs £4.50 per m^2. Work out the cost. [4]

(c) triangles 3.84, sides $2 \times 2 \times 3 = 12$, floor $2.4 \times 3 = 7.2$; $3.84 + 12 + 7.2 = \mathbf{23.04}$ m^2; $23.04 \times 4.50 = £\mathbf{103.68}$

Exercise 3.2 (d)

3.2 A tent is a triangular prism 3 m long. Its front is an isosceles triangle with a base of 2.4 m and two sloping sides of 2 m.

(d) *(H)* Work out the angle between a sloping side and the floor. A toy tent is similar to the real tent, with a scale of 1 : 20. Work out the length of the toy tent in cm and its volume in cm^3. [5]

Exercise 3.2 (d)

3.2 A tent is a triangular prism 3 m long. Its front is an isosceles triangle with a base of 2.4 m and two sloping sides of 2 m.

(d) *(H)* Work out the angle between a sloping side and the floor. A toy tent is similar to the real tent, with a scale of 1 : 20. Work out the length of the toy tent in cm and its volume in cm^3. [5]

(d) $\cos^{-1}\frac{1.2}{2} = \mathbf{53.1}°$; $300 \div 20 = \mathbf{15}$ cm; $5\,760\,000 \div 20^3 = \mathbf{720}$ cm^3