Exercise walk-through

4.1.2
Angle Reasoning

AQA · GCSE · Mathematics

Angles and polygons

- Angles at a point add up to $360°$; angles on a straight line add up to $180°$.
- Vertically opposite angles are equal.
- Where a line crosses two parallel lines, alternate angles are equal, **corresponding angles** are equal, and co-interior angles add up to $180°$. Write "alternate angles are equal", never "Z-angles".
- The angles in a triangle add up to $180°$.

Perimeter, area and volume

- Triangle: $\frac{1}{2} \times b \times h$, where h is the **perpendicular height** .
- Parallelogram: $b \times h$.
- Trapezium: $\frac{1}{2}(a+b)h$, where a and b are the parallel sides.
- Circle: $A = \pi r^2$, and the **circumference** is $C = 2\pi r = \pi d$.

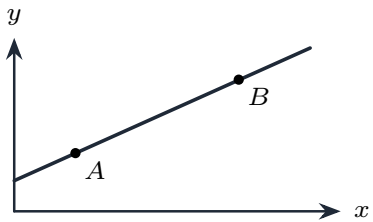
$$\text{arc length} = \frac{\theta}{360} \times 2\pi r \qquad \text{sector area} = \frac{\theta}{360} \times \pi r^2$$

Vectors and coordinate geometry (2)

- Add and subtract vectors by adding and subtracting their parts: with $\mathbf{a} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\mathbf{a} + \mathbf{b} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$ and $2\mathbf{a} - \mathbf{b} = \begin{pmatrix} 7 \\ -1 \end{pmatrix}$.
- **Vector proofs (H).** If one vector is a number times another, the lines are parallel.

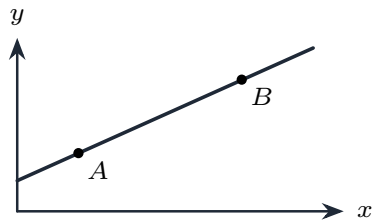
Exercise 2.1

2.1 Two angles on a straight line are $3x$ and x. Find x. [2]



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2.1

- angles on a straight line add to 180°: $3x + x = 180°$
- $4x = 180°$, so $\boldsymbol{x = 45°}$

Setup for exercise 2.2

2.2 Three angles of a quadrilateral are 85°, 110° and 72°. Find the fourth. [1]



Exercise 2.2

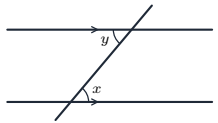
2.3 Work out the sum of the interior angles of a pentagon. [1]



2.4 Work out the exterior angle of a regular decagon (10 sides). [1]



2.5 A line crosses two parallel lines. One of the angles it makes is 65°. Its alternate angle is a and its co-interior angle is b. Find a and b, giving reasons. [3]



Exercise 2.2

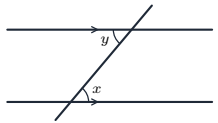
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2.2

- angles in a quadrilateral add to 360°: $360° - 85° - 110° - 72° = \mathbf{93}°$

Full solution on next slide

Full solution 2.2

2.2

- angles in a quadrilateral add to 360°: $360° - 85° - 110° - 72° = \mathbf{93}°$

2.3

- $(n-2) \times 180° = 3 \times 180° = \mathbf{540}°$

2.4

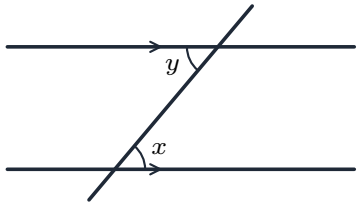
- $\dfrac{360°}{n} = \dfrac{360°}{10} = \mathbf{36}°$

2.5

- $a = \mathbf{65}°$: alternate angles are equal
- $b = 180° - 65° = \mathbf{115}°$: co-interior angles add to 180°

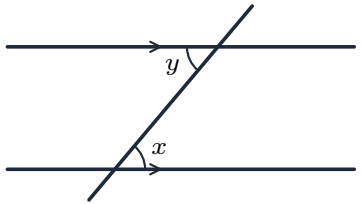
Exercise 3.1

3.1 Leo says the angles 70° and 110° on parallel lines are alternate, so they should be equal. Explain his mistake. [1]



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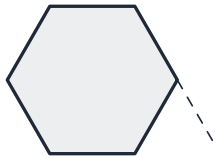


3.1

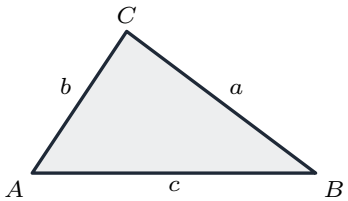
- angles that add to 180° on parallel lines are **co-interior**, not alternate; alternate angles are equal

Setup for exercise 3.2

3.2 A regular polygon has an exterior angle of $24°$. How many sides does it have? [2]

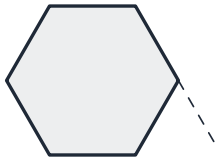


3.3 The angles of a triangle are x, $2x$ and $x + 40°$. Find the largest angle. [3]

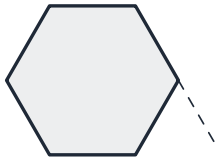


Exercise 3.2

3.4 The interior angle of a regular polygon is 156°. Work out the number of sides, and the sum of its interior angles. [3]

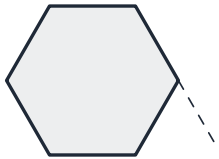


3.5 Can a regular polygon have an interior angle of 130°? Give a reason. [2]

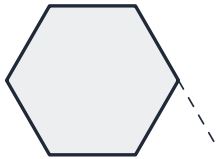


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Full solution on next slide

Full solution 3.2

3.2

- $\dfrac{360°}{n} = 24°$
- $n = 360 \div 24 =$ **15** sides

3.3

- angles in a triangle add to 180°: $x + 2x + x + 40° = 180°$
- $4x = 140°$, so $x = 35°$
- angles 35°, 70°, 75°: the largest is **75**°

3.4

- exterior angle $= 180° - 156° = 24°$, so $n = 360 \div 24 =$ **15**
- sum $= (15 - 2) \times 180° =$ **2340**°

3.5

- exterior angle would be $180° - 130° = 50°$, and $360 \div 50 = 7.2$
- **No**: the number of sides must be a whole number