Exercise walk-through

3.3
Compound measures and rates of change (R6–R8, R13–R14)

AQA · GCSE · Mathematics

Units and compound measures

- **Converting units.** Learn the metric links: 1 km = 1000 m, 1 m = 100 cm, 1 kg = 1000 g, 1 litre = 1000 cm^3.
- Time is not decimal: 2 hours 36 minutes is $2 + \frac{36}{60} = 2.6$ hours, not 2.36 hours.
- A compound unit combines two units, such as km/h or g/cm^3.
- **Density** tells you how much mass is packed into each unit of volume.
  Pressure is the force on each unit of area, measured in N/m^2.

$$\text{speed} = \frac{\text{distance}}{\text{time}} \qquad \text{density} = \frac{\text{mass}}{\text{volume}} \qquad \text{pressure} = \frac{\text{force}}{\text{area}}$$

Growth and decay

- **Simple interest** pays the same amount every year. **Compound interest** pays interest on the interest too, so the amount is multiplied by the same number every year:
- where P is the starting amount, r the percentage rate and n the number of years.
- For decay, such as a car losing value, use $1 - \frac{r}{100}$.
- **Worked example 13.** 1000 yuan is saved at 10% a year for 4 years.

$$A = P\left(1 + \frac{r}{100}\right)^n$$

Sources for this lesson

- AQA, *GCSE Mathematics (8300) specification*, subject content section 3.3 (Ratio, proportion and rates of change).
- NASA, *Mars Climate Orbiter Mishap Investigation Board Phase I Report* (10 November 1999).
- AQA, *GCSE Mathematics* Higher Paper 2 (June 2024), the reverse-percentage and house-price questions, adapted.

Exercises 2.1·2.2

2.1 y is directly proportional to x. $y = 18$ when $x = 6$. Find y when $x = 11$. [2]

$y \propto x$ $y = 18$ when $x = 6$

2.2 8 pumps empty a tank in 9 hours. How long would 6 pumps take? [2]

8 pumps: 9 hours 6 pumps: ?

Exercises 2.1·2.2

2.1 y is directly proportional to x. $y = 18$ when $x = 6$. Find y when $x = 11$. [2]

$y \propto x$ $y = 18$ when $x = 6$

2.1

- $y = 3x$
- $y = \mathbf{33}$

2.2 8 pumps empty a tank in 9 hours. How long would 6 pumps take? [2]

8 pumps: 9 hours 6 pumps: ?

2.2

- $8 \times 9 = 72$ pump-hours
- $72 \div 6 = \mathbf{12}$ hours

Exercise 2.3

2.3 The graph shows the water in a tank as it empties. Work out the rate at which the water flows out. [2]



Exercise 2.3

2.3 The graph shows the water in a tank as it empties. Work out the rate at which the water flows out. [2]



2.3

- gradient $\dfrac{-500}{10}$
- **50** litres per minute

Exercises 2.4·3.1

2.4 A coach travels 150 km in 2 hours 30 minutes. Work out its average speed. [2]



150 km in 2 h 30 min

3.1 *(H)* y is inversely proportional to x^2. $y = 8$ when $x = 3$. Find a formula for y in terms of x. Find y when $x = 6$. [3]

$y \propto \frac{1}{x^2}$

$y = 8$ when $x = 3$

Exercises 2.4·3.1

2.4 A coach travels 150 km in 2 hours 30 minutes. Work out its average speed. [2]



150 km in 2 h 30 min

2.4

- $150 \div 2.5$
- **60** km/h

3.1 *(H)* y is inversely proportional to x^2. $y = 8$ when $x = 3$. Find a formula for y in terms of x. Find y when $x = 6$. [3]

$y \propto \frac{1}{x^2}$ $y = 8$ when $x = 3$

3.1

- $8 = \frac{k}{9}$, $k = 72$
- $\boldsymbol{y = \frac{72}{x^2}}$
- $y = \mathbf{2}$

Exercise 3.2 (a)

3.2 Water is pumped into tank A, then the pump stops, then some water drains out. Graph A shows the volume V litres after t minutes.

(a) Work out the rate of filling in the first 20 minutes. [2]

Exercise 3.2 (a)

3.2 Water is pumped into tank A, then the pump stops, then some water drains out. Graph A shows the volume V litres after t minutes.

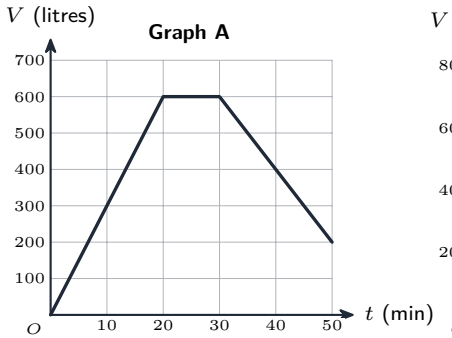
(a) Work out the rate of filling in the first 20 minutes. [2]

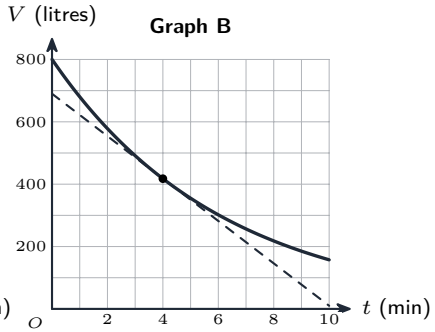
(a) $600 \div 20 = \mathbf{30}$ litres per minute

Exercise 3.2 (b)

3.2 Water is pumped into tank A, then the pump stops, then some water drains out. Graph A shows the volume V litres after t minutes.

(b) Describe what happens between $t = 20$ and $t = 30$. Work out the rate at which the water drains out after $t = 30$. [3]

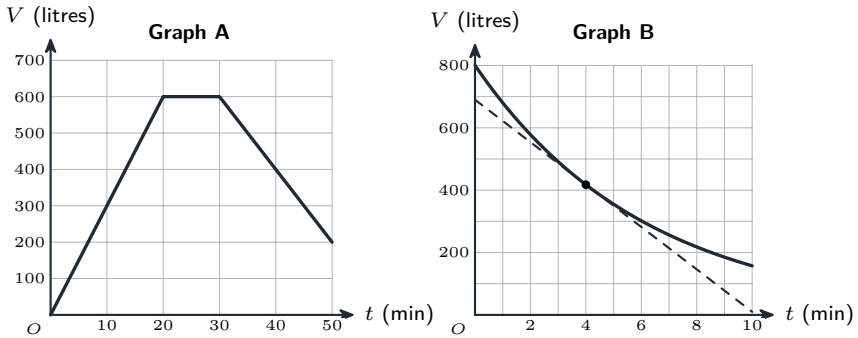




Exercise 3.2 (b)

3.2 Water is pumped into tank A, then the pump stops, then some water drains out. Graph A shows the volume V litres after t minutes.

(b) Describe what happens between $t = 20$ and $t = 30$. Work out the rate at which the water drains out after $t = 30$. [3]



(b) the pump is off, so the volume stays at 600 litres; $(600 - 200) \div 20 = \mathbf{20}$ litres per minute

Exercise 3.2 (c)

3.2 Water is pumped into tank A, then the pump stops, then some water drains out. Graph A shows the volume V litres after t minutes.

(c) The time T minutes to fill tank C is inversely proportional to the number of pumps p. With 3 pumps it takes 40 minutes. Find a formula for T. How long do 5 pumps take? What is the least number of pumps that fills the tank in less than 15 minutes? [4]

Exercise 3.2 (c)

3.2 Water is pumped into tank A, then the pump stops, then some water drains out. Graph A shows the volume V litres after t minutes.

(c) The time T minutes to fill tank C is inversely proportional to the number of pumps p. With 3 pumps it takes 40 minutes. Find a formula for T. How long do 5 pumps take? What is the least number of pumps that fills the tank in less than 15 minutes? [4]

(c) $k = 3 \times 40 = 120$, $\boldsymbol{T = \dfrac{120}{p}}$; **24** minutes; $\dfrac{120}{p} < 15$, $p > 8$; 9 **pumps**

Exercise 3.2 (d)

3.2 Water is pumped into tank A, then the pump stops, then some water drains out. Graph A shows the volume V litres after t minutes.

(d) *(H)* Graph B shows the water in tank B as it empties. The dashed tangent at $t = 4$ passes through $(0, 690)$ and $(10, 10)$. Estimate the rate at which water leaves at $t = 4$. Use the graph to work out the average rate for the first 4 minutes. [5]

Exercise 3.2 (d)

3.2 Water is pumped into tank A, then the pump stops, then some water drains out. Graph A shows the volume V litres after t minutes.

(d) *(H)* Graph B shows the water in tank B as it empties. The dashed tangent at $t = 4$ passes through $(0, 690)$ and $(10, 10)$. Estimate the rate at which water leaves at $t = 4$. Use the graph to work out the average rate for the first 4 minutes. [5]

(d) gradient $\dfrac{10 - 690}{10 - 0} = -68$; water leaves at about **68** litres per minute; $V \approx 420$ at $t = 4$, $\dfrac{800 - 420}{4} \approx$ **95** litres per minute