Exercise walk-through

3.2.3
Iterative Growth

AQA · GCSE · Mathematics

Units and compound measures

- **Converting units.** Learn the metric links: 1 km = 1000 m, 1 m = 100 cm, 1 kg = 1000 g, 1 litre = 1000 cm^3.
- Time is not decimal: 2 hours 36 minutes is $2 + \frac{36}{60} = 2.6$ hours, not 2.36 hours.
- A compound unit combines two units, such as km/h or g/cm^3.
- **Density** tells you how much mass is packed into each unit of volume.
  Pressure is the force on each unit of area, measured in N/m^2.

$$\text{speed} = \frac{\text{distance}}{\text{time}} \qquad \text{density} = \frac{\text{mass}}{\text{volume}} \qquad \text{pressure} = \frac{\text{force}}{\text{area}}$$

Scale, similarity and rates of change

- **Scale drawings.** A map **scale** of $1 : 25\,000$ means 1 cm on the map is 25 000 cm in real life.
- A 4 cm line on the map is $4 \times 25\,000 = 100\,000$ cm, which is 1 km.
- **Length, area and volume.** If two shapes are similar with a length **scale** factor of k, the **area scale factor** is k^2 and the volume scale factor is k^3.
- A square with sides 3 times as long has 9 times the area.

Growth and decay

- **Simple interest** pays the same amount every year. **Compound interest** pays interest on the interest too, so the amount is multiplied by the same number every year:
- where P is the starting amount, r the percentage rate and n the number of years.
- For decay, such as a car losing value, use $1 - \frac{r}{100}$.
- **Worked example 13.** 1000 yuan is saved at 10% a year for 4 years.

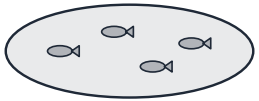
$$A = P\left(1 + \frac{r}{100}\right)^n$$

Exercises 2.1·2.2

2.1 Each year a balance grows by 5%. Then £100 is added. Write this as an iteration. Work out u_1 when $u_0 = 1000$. [2]

grow by 5%, then add £100 $u_0 = 1000$

2.2 A pond has 100 fish. Each year 20% of the fish are taken out, then 30 new fish are added. Work out the number of fish after 1 year and after 2 years. [2]



100 fish

Exercises 2.1·2.2

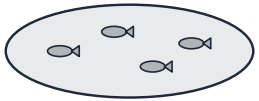
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grow by 5%, then add £100 $u_0 = 1000$

2.1

- $\mathbf{u_{n+1} = 1.05u_n + 100}$
- $u_1 = \mathbf{1150}$

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100 fish

2.2

- $0.8 \times 100 + 30 = \mathbf{110}$
- $0.8 \times 110 + 30 = \mathbf{118}$

Exercise 3.1

3.1 For the pond, $u_{n+1} = 0.8u_n + 30$. Find the fixed point. Explain what it tells you about the number of fish. [3]

$u_{n+1} = 0.8u_n + 30$

Exercise 3.1

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3.1

- $u = 0.8u + 30$
- $u = \mathbf{150}$
- over many years the number of fish gets closer and closer to 150

Exercise 3.2 (a)

3.2 Mei has ¥1000 to save. Plan A adds 10% interest each year, then Mei deposits ¥50. Plan B adds 4% interest each year, then Mei deposits ¥150.

(a) Write an iteration for each plan. Use a_n for plan A and b_n for plan B. [2]

plan	start	each year
A	¥1000	+10%, then +¥50
B	¥1000	+4%, then +¥150

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plan	start	each year
A	¥1000	+10%, then +¥50
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(a) $\mathbf{a_{n+1} = 1.1a_n + 50}$; $\mathbf{b_{n+1} = 1.04b_n + 150}$

Exercise 3.2 (b)

3.2 Mei has ¥1000 to save. Plan A adds 10% interest each year, then Mei deposits ¥50. Plan B adds 4% interest each year, then Mei deposits ¥150.

(b) Work out the balance in each plan after 2 years. [3]

Exercise 3.2 (b)

3.2 Mei has ¥1000 to save. Plan A adds 10% interest each year, then Mei deposits ¥50. Plan B adds 4% interest each year, then Mei deposits ¥150.

(b) Work out the balance in each plan after 2 years. [3]

(b) iterating; A: 1150, ¥**1315**; B: 1190, ¥**1387.60**

Exercise 3.2 (c)

3.2 Mei has ¥1000 to save. Plan A adds 10% interest each year, then Mei deposits ¥50. Plan B adds 4% interest each year, then Mei deposits ¥150.

(c) Which plan has more money after 5 years? By how much? [4]

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(c) A: 1496.50, 1696.15, 1915.77; B: 1593.10, 1806.83, 2029.10; after 5 years A ¥1915.77, B ¥2029.10; **plan B**, by ¥**113.33**

Exercise 3.2 (d)

3.2 Mei has ¥1000 to save. Plan A adds 10% interest each year, then Mei deposits ¥50. Plan B adds 4% interest each year, then Mei deposits ¥150.

(d) In which year does plan A first have more money than plan B? Explain why plan A catches up in the end. [5]

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(d) continuing both; year 8: A 2715.38, B 2750.70; year 9: A 3036.92, B 3010.73; **year** 9; plan A's interest is 10% of a growing balance, so it grows faster than the fixed extra ¥100 that plan B deposits