Exercise walk-through

# 3.1.4
# Similarity Scale

AQA · GCSE · Mathematics

# Units and compound measures

- **Converting units.** Learn the metric links: 1 km = 1000 m, 1 m = 100 cm, 1 kg = 1000 g, 1 litre = 1000 $\text{cm}^3$.
- Time is not decimal: 2 hours 36 minutes is $2 + \frac{36}{60} = 2.6$ hours, not 2.36 hours.
- A compound unit combines two units, such as km/h or $\text{g/cm}^3$.
- **Density** tells you how much mass is packed into each unit of volume.
  **Pressure** is the force on each unit of area, measured in $\text{N/m}^2$.

$$\text{speed} = \frac{\text{distance}}{\text{time}} \qquad \text{density} = \frac{\text{mass}}{\text{volume}} \qquad \text{pressure} = \frac{\text{force}}{\text{area}}$$

# Scale, similarity and rates of change

- **Scale drawings.** A map **scale** of $1 : 25\,000$ means 1 cm on the map is 25 000 cm in real life.
- A 4 cm line on the map is $4 \times 25\,000 = 100\,000$ cm, which is 1 km.
- **Length, area and volume.** If two shapes are similar with a length **scale** factor of $k$, the **area scale factor** is $k^2$ and the volume scale factor is $k^3$.
- A square with sides 3 times as long has 9 times the area.

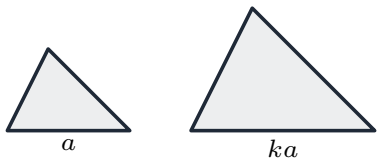
# Growth and decay

- **Simple interest** pays the same amount every year. **Compound interest** pays interest on the interest too, so the amount is multiplied by the same number every year:
- where $P$ is the starting amount, $r$ the percentage rate and $n$ the number of years.
- For decay, such as a car losing value, use $1 - \frac{r}{100}$.
- **Worked example 13.** 1000 yuan is saved at 10% a year for 4 years.

$$A = P\left(1 + \frac{r}{100}\right)^n$$

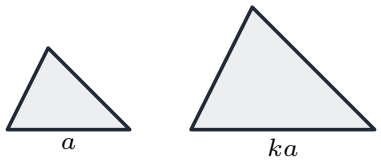
# Exercise 2.1

**2.1** Triangle B is an enlargement of triangle A. A side of 4 cm in A matches a side of 10 cm in B. Find the scale factor. [1]



# Exercise 2.1

**2.1** Triangle B is an enlargement of triangle A. A side of 4 cm in A matches a side of 10 cm in B. Find the scale factor. [1]




**2.1**

- $k = 10 \div 4 = \mathbf{2.5}$

# Exercise 2.2

**2.2** Using **2.1**, another side of A is 6 cm. Find the matching side of B. [1]




**2.3** Two similar rectangles have widths 5 cm and 15 cm. The smaller has length 8 cm. Find the length of the larger. [2]



**2.4** *(H)* A shape is enlarged by scale factor 4. Its area was 7 cm$^2$. Find the new area. [1]




**2.5** *(H)* Two similar cylinders have heights in the ratio 1 : 3. Write down the ratio of their volumes. [1]



# Exercise 2.2

**2.2** Using **2.1**, another side of A is 6 cm. Find the matching side of B. [1]




**2.3** Two similar rectangles have widths 5 cm and 15 cm. The smaller has length 8 cm. Find the length of the larger. [2]



**2.4** *(H)* A shape is enlarged by scale factor 4. Its area was 7 cm$^2$. Find the new area. [1]




**2.5** *(H)* Two similar cylinders have heights in the ratio 1 : 3. Write down the ratio of their volumes. [1]




**2.2**

- $6 \times 2.5 = \mathbf{15}$ cm

**2.3**

Full solution on next slide

# Full solution 2.2

**2.2**

- $6 \times 2.5 = \mathbf{15}$ cm

**2.3**

- $k = 15 \div 5 = 3$
- $8 \times 3 = \mathbf{24}$ cm

**2.4**

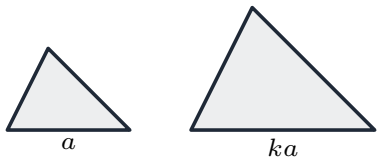
- area factor $4^2 = 16$: $7 \times 16 = \mathbf{112}$ cm$^2$

**2.5**

- cube the length ratio: $\mathbf{1} : \mathbf{27}$

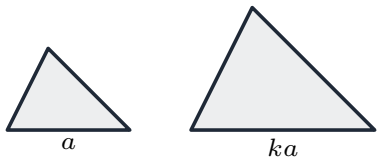
# Exercise 3.1

**3.1** Ali enlarges a rectangle of area 12 $\text{cm}^2$ by scale factor 2 and says the new area is 24 $\text{cm}^2$. Explain his mistake. [1]



# Exercise 3.1

**3.1** Ali enlarges a rectangle of area 12 cm$^2$ by scale factor 2 and says the new area is 24 cm$^2$. Explain his mistake. [1]

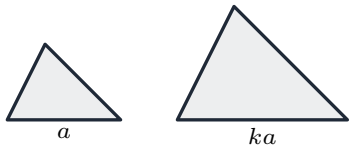



**3.1**

- area scales by $k^2 = 4$, not $k = 2$: the new area is $12 \times 4 = 48$ cm$^2$

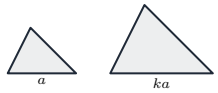
# Setup for exercise 3.2

**3.2** Triangles $PQR$ and $PST$ are similar, with $S$ on $PQ$ and $T$ on $PR$. $PS = 4$ cm, $PQ = 10$ cm and $ST = 3$ cm. Work out $QR$. [2]

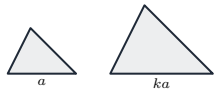


# Exercise 3.2

**3.3** *(H)* Two similar photo frames have areas 80 cm$^2$ and 180 cm$^2$. The smaller is 8 cm wide. How wide is the larger? [3]




**3.4** *(H)* Two similar statues have heights 30 cm and 45 cm. The smaller has mass 2.4 kg. Work out the mass of the larger, made of the same material. [3]

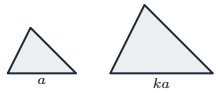



**3.5** *(H)* Two similar solids have surface areas in the ratio 9 : 25. The larger has volume 500 cm$^3$. Work out the volume of the smaller. [3]

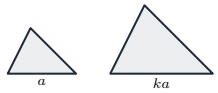


# Exercise 3.2

**3.3** *(H)* Two similar photo frames have areas 80 cm$^2$ and 180 cm$^2$. The smaller is 8 cm wide. How wide is the larger? [3]




**3.4** *(H)* Two similar statues have heights 30 cm and 45 cm. The smaller has mass 2.4 kg. Work out the mass of the larger, made of the same material. [3]




**3.5** *(H)* Two similar solids have surface areas in the ratio 9 : 25. The larger has volume 500 cm$^3$. Work out the volume of the smaller. [3]




**3.2**

- $k = PQ \div PS = 10 \div 4 = 2.5$
- $QR = 3 \times 2.5 = \mathbf{7.5}$ cm

Full solution on next slide

# Full solution 3.2

**3.2**

- $k = PQ \div PS = 10 \div 4 = 2.5$
- $QR = 3 \times 2.5 = \mathbf{7.5}$ cm

**3.3**

- area factor $= 180 \div 80 = 2.25$
- length factor $k = \sqrt{2.25} = 1.5$
- width $= 8 \times 1.5 = \mathbf{12}$ cm

**3.4**

- $k = 45 \div 30 = 1.5$
- mass scales like volume: $k^3 = 1.5^3 = 3.375$
- $2.4 \times 3.375 = \mathbf{8.1}$ kg

**3.5**

- length ratio $\sqrt{9} : \sqrt{25} = 3 : 5$
- volume ratio $27 : 125$
- $500 \times \dfrac{27}{125} = \mathbf{108}$ cm$^3$