Exercise walk-through

3.1.3 Proportion Models

AQA · GCSE · Mathematics

Scale, similarity and rates of change

- **Scale drawings.** A map **scale** of $1 : 25\,000$ means 1 cm on the map is 25 000 cm in real life.
- A 4 cm line on the map is $4 \times 25\,000 = 100\,000$ cm, which is 1 km.
- **Length, area and volume.** If two shapes are similar with a length **scale** factor of k, the **area scale factor** is k^2 and the volume scale factor is k^3.
- A square with sides 3 times as long has 9 times the area.

Growth and decay

- **Simple interest** pays the same amount every year. **Compound interest** pays interest on the interest too, so the amount is multiplied by the same number every year:
- where P is the starting amount, r the percentage rate and n the number of years.
- For decay, such as a car losing value, use $1 - \frac{r}{100}$.
- **Worked example 13.** 1000 yuan is saved at 10% a year for 4 years.

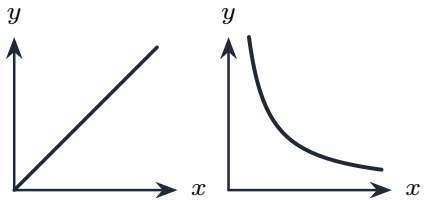
$$A = P\left(1 + \frac{r}{100}\right)^n$$

Sources for this lesson

- AQA, *GCSE Mathematics (8300) specification*, subject content section 3.3 (Ratio, proportion and rates of change).
- NASA, *Mars Climate Orbiter Mishap Investigation Board Phase I Report* (10 November 1999).
- AQA, *GCSE Mathematics* Higher Paper 2 (June 2024), the reverse-percentage and house-price questions, adapted.

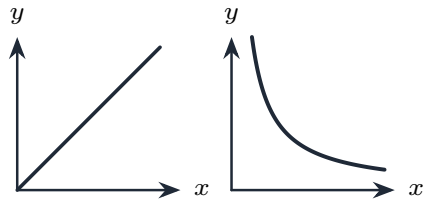
Exercise 2.1

2.1 5 pens cost ¥17.50. Work out the cost of 8 pens. [2]



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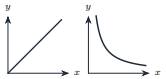


2.1

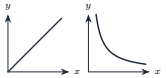
- one pen: $17.50 \div 5 = ¥3.50$
- $8 \times 3.50 = \mathbf{¥28}$

Exercise 2.2

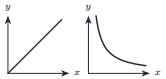
2.2 A recipe for 4 people uses 300 g of rice. How much rice is needed for 10 people? [2]



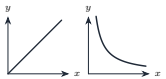
2.3 3 pumps empty a tank in 8 hours. How long would 4 pumps take? [2]



2.4 y is directly proportional to x. When $x = 5$, $y = 35$. Find y when $x = 8$. [2]

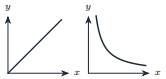


2.5 Which of these describe inverse proportion? (A) $y = 6x$ (B) $xy = 20$ (C) $y = \dfrac{9}{x}$ (D) $y = x + 4$ [1]

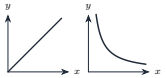


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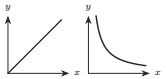
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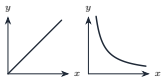
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2.2

- one person: $300 \div 4 = 75$ g
- $10 \times 75 = \mathbf{750}$ g

2.3

Full solution on next slide

Full solution 2.2

2.2

- one person: $300 \div 4 = 75$ g
- $10 \times 75 = \mathbf{750}$ g

2.3

- pump-hours: $3 \times 8 = 24$
- $24 \div 4 = \mathbf{6}$ hours

2.4

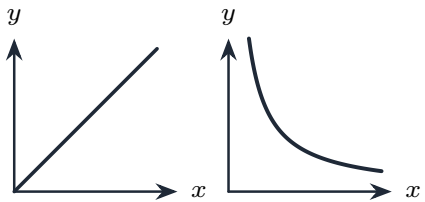
- $y = kx$: $35 = 5k$, so $k = 7$
- $y = 7 \times 8 = \mathbf{56}$

2.5

- **B** and **C**: both have xy constant

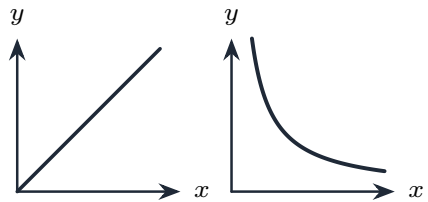
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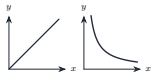


3.1

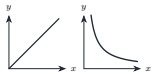
- more people finish **faster**: this is inverse proportion, so 4 people take $2 \times 6 \div 4 = 3$ hours, not 12

Exercise 3.2

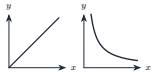
3.2 A car uses 6.4 litres of fuel for 80 km. How far can it travel on 50 litres? [2]



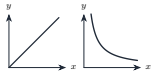
3.3 *(H)* y is directly proportional to x^2. When $x = 3$, $y = 36$. Find y when $x = 5$, and x when $y = 64$, given $x > 0$. [3]



3.4 *(H)* The time t hours for a journey is inversely proportional to the speed v km/h. At 60 km/h it takes 2.5 hours. Find the formula for t in terms of v, and the speed needed for 2 hours. [3]

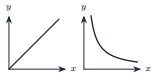


3.5 *(H)* F is inversely proportional to the square of d. When $d = 2$, $F = 45$. Find F when $d = 3$. [3]

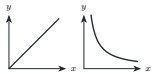


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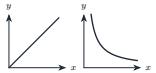
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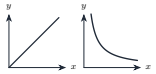
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3.2

- per litre: $80 \div 6.4 = 12.5$ km
- $50 \times 12.5 = \mathbf{625}$ km

3.3

- $y = kx^2$: $36 = 9k$, so $k = 4$

Full solution on next slide

Full solution 3.2

3.2

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3.3

- $y = kx^2$: $36 = 9k$, so $k = 4$
- $x = 5$: $y = 4 \times 25 = \mathbf{100}$
- $y = 64$: $x^2 = 16$, so $\boldsymbol{x = 4}$

3.4

- $t = \dfrac{k}{v}$: $2.5 = \dfrac{k}{60}$, so $k = 150$ and $\boldsymbol{t = \dfrac{150}{v}}$
- $2 = \dfrac{150}{v}$, so $v = \mathbf{75}$ km/h

3.5

- $F = \dfrac{k}{d^2}$: $45 = \dfrac{k}{4}$, so $k = 180$
- $F = \dfrac{180}{3^2} = \mathbf{20}$