Exercise walk-through

3.1.1 Ratio Models

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Scale, similarity and rates of change

- **Scale drawings.** A map **scale** of $1 : 25\,000$ means 1 cm on the map is 25 000 cm in real life.
- A 4 cm line on the map is $4 \times 25\,000 = 100\,000$ cm, which is 1 km.
- **Length, area and volume.** If two shapes are similar with a length **scale** factor of k, the **area scale factor** is k^2 and the volume scale factor is k^3.
- A square with sides 3 times as long has 9 times the area.

Growth and decay

- **Simple interest** pays the same amount every year. **Compound interest** pays interest on the interest too, so the amount is multiplied by the same number every year:
- where P is the starting amount, r the percentage rate and n the number of years.
- For decay, such as a car losing value, use $1 - \frac{r}{100}$.
- **Worked example 13.** 1000 yuan is saved at 10% a year for 4 years.

$$A = P\left(1 + \frac{r}{100}\right)^n$$

Sources for this lesson

- AQA, *GCSE Mathematics (8300) specification*, subject content section 3.3 (Ratio, proportion and rates of change).
- NASA, *Mars Climate Orbiter Mishap Investigation Board Phase I Report* (10 November 1999).
- AQA, *GCSE Mathematics* Higher Paper 2 (June 2024), the reverse-percentage and house-price questions, adapted.

Exercise 2.1

2.1 Write 24 : 36 in its simplest form. [1]



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2.1

- divide by the HCF 12: **2** : **3**

Setup for exercise 2.2

2.2 Write 45 minutes : 2 hours in its simplest form. [2]



Exercise 2.2

2.3 Share ¥120 in the ratio 3 : 5. [2]



2.4 Write 5 : 8 in the form 1 : n. [1]



2.5 Red and blue beads are in the ratio 4 : 7. There are 28 red beads. How many blue beads are there? [2]



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2.2

- same units: 45 minutes : 120 minutes

Full solution on next slide

Full solution 2.2

2.2

- same units: 45 minutes : 120 minutes
- divide by 15: **3** : **8**

2.3

- $3 + 5 = 8$ shares; one share $= 120 \div 8 =$ ¥15
- **¥45** and **¥75**

2.4

- divide both parts by 5: **1** : **1.6**

2.5

- 4 shares $= 28$, so one share $= 7$
- blue $= 7 \times 7 =$ **49**

Exercise 3.1

3.1 Sam shares ¥60 in the ratio 1 : 2 and says the smaller share is ¥30. Explain his mistake and give the correct share. [2]



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3.1

- 1 : 2 means 3 shares, so the smaller share is $\frac{1}{3}$, not $\frac{1}{2}$
- $60 \div 3 = \mathbf{¥20}$

Setup for exercise 3.2

3.2 In a class the ratio of boys to girls is $4 : 5$. There are 3 more girls than boys. How many students are in the class? [2]



3.3 Concrete is cement, sand and gravel in the ratio $1 : 2 : 4$ by mass. Lin has 60 kg of sand and plenty of the others. What is the greatest mass of concrete she can make? [2]



3.4 $a : b = 2 : 3$ and $b : c = 4 : 5$. Find $a : c$ in its simplest form. [3]



Exercise 3.2

3.5 Jar A holds red and green sweets in the ratio 3 : 1; jar B holds them in the ratio 2 : 3. Each jar holds 60 sweets. All the sweets are mixed. Work out the ratio of red to green in the mixture, in its simplest form. [3]



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3.2

- difference $5 - 4 = 1$ share $= 3$ students
- 9 shares $= 9 \times 3 =$ **27** students

3.3

- sand is 2 shares $= 60$ kg, so one share $= 30$ kg
- total $7 \times 30 =$ **210** kg

Full solution on next slide

Full solution 3.2

3.2

- difference $5 - 4 = 1$ share $= 3$ students
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3.3

- sand is 2 shares $= 60$ kg, so one share $= 30$ kg
- total $7 \times 30 =$ **210** kg

3.4

- make b the same: $2 : 3 = 8 : 12$ and $4 : 5 = 12 : 15$
- $a : b : c = 8 : 12 : 15$
- $a : c =$ **8** : **15**

3.5

- jar A: 4 shares of 15: 45 red, 15 green; jar B: 5 shares of 12: 24 red, 36 green
- red $45 + 24 = 69$; green $15 + 36 = 51$
- $69 : 51 =$ **23** : **17**