Exercise walk-through

2.5
Quadratics and simultaneous equations - HT (A17–A23)

AQA · GCSE · Mathematics

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inequalities

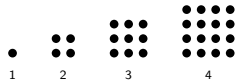
- Solve a linear inequality like an equation, but if you multiply or divide by a negative number, turn the sign round: from $-2x < 6$ you get $x > -3$.
- On a number line, an open circle means the end value is not included ($<$ or $>$) and a filled circle means it is ($\leq$ or $\geq$).
- **Worked example 15.** Find the integers n with $-3 < 2n + 1 \leq 9$.
- Subtract 1: $-4 < 2n \leq 8$.

Sequences

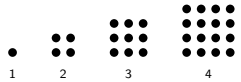
- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

Exercises 2.1·2.3

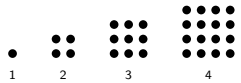
2.1 Write the next two terms of $5, 9, 17, 29, 45, \ldots$ [2]



2.2 Find the nth term of $4, 7, 12, 19, 28, \ldots$ [3]

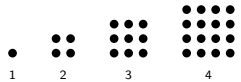


2.3 Find the nth term of $6, 11, 18, 27, 38, \ldots$ [3]



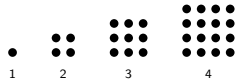
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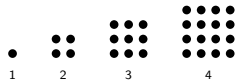
2.1 Differences $20, 24$ follow the $+4$ pattern: terms $65, 89$.

2.2 Find the nth term of $4, 7, 12, 19, 28, \ldots$ [3]



2.2 Differences $3, 5, 7, 9$; second difference 2, so $a = 1$: n^2 leaves $3, 3, 3, 3$, giving $n^2 + 3$.

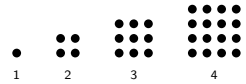
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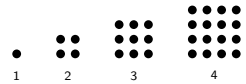
2.3 Differences $5, 7, 9, 11$; second difference 2, so $a = 1$: subtracting n^2 (1, 4, 9, 16) leaves $5, 7, 9, 11 = 2n + 3$: $n^2 + 2n + 3$.

Exercises 2.4·3.1

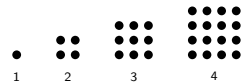
2.4 A Fibonacci-type sequence starts $2, 5, \ldots$: write the next three terms. [2]



2.5 Which term of the sequence with nth term $3n + 2$ equals 62? [2]

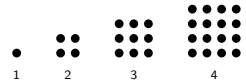


3.1 *(Original.)* A sequence has nth term $n^2 + 2n$. (a) Write down the first four terms. (b) Which term equals 168? [4]

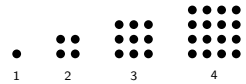


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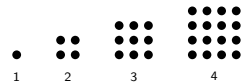
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2.4 $7, 12, 19$.

2.5 $3n + 2 = 62$ gives $n = 20$.

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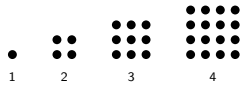
3.1 (a) $3, 8, 15, 24$. (b) $n^2 + 2n = 168$ gives $(n + 1)^2 = 169$, so $n = 12$.

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Exercise 3.2

3.2 *(Original.)* The first differences of a sequence are $5, 9, 13, 17, \ldots$.

(a) Find the next difference. (b) The first term is 3: write the first four terms. (c) Find the nth term. [5]



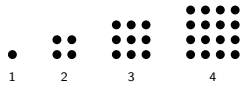
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3.2 (a) Differences increase by 4: next is 21. (b) $3, 8, 17, 30$. (c) Second difference of the terms is 4, so $a = 2$; subtracting $2n^2$ (which gives $2, 8, 18, 32$) leaves $1, 0, -1, -2 = 2 - n$: so the nth term is $2n^2 - n + 2$ – check $n = 4$: $32 - 4 + 2 = 30$. ✓

Full solution on next slide

Full solution 3.2

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3.3 (a) $y = kx^2$: $48 = 16k$ gives $k = 3$. (b) $y = 3 \times 49 = 147$.

Local review guidance: second differences before guessing a; nth terms checked on two positions; sequence types named. This is authored classroom guidance.