Exercise walk-through

2.5.3
Equation Iteration

AQA · GCSE · Mathematics

Straight-line graphs

- Every straight line, except a vertical one, has an equation $y = mx + c$.
- The number m is the **gradient**, the steepness: how far y goes up when x increases by 1.
- The number c is the y-intercept, where the line crosses the y-axis.
- Between two points,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

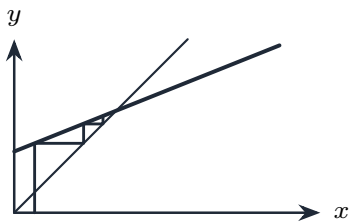
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

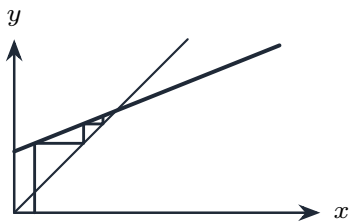
Exercise 2.1

2.1 Use $x_{n+1} = \sqrt{x_n + 2}$ with $x_0 = 1$ to find x_1. [1]



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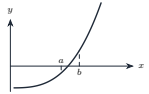


2.1

- $x_1 = \sqrt{1+2} = \sqrt{3} = \mathbf{1.73205}$

Exercise 2.2

2.2 Show that $x^3 + x - 4 = 0$ has a root between 1 and 2. [2]



2.3 Show that $x^2 - 5x + 2 = 0$ can be rearranged to $x = \dfrac{x^2 + 2}{5}$. [1]



2.4 Use $x_{n+1} = \dfrac{x_n^2 + 2}{5}$ with $x_0 = 0$ to find x_1, x_2 and x_3. Give x_3 to 4 decimal places. [2]

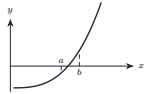


2.5 Use $x_{n+1} = 3 - \dfrac{1}{x_n}$ with $x_0 = 3$ to find x_1 and x_2. [2]



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2.2

- $x = 1$: $1 + 1 - 4 = -2$; $x = 2$: $8 + 2 - 4 = 6$
- the sign changes from negative to positive, so there is a root between 1 and 2

2.3

- $x^2 + 2 = 5x$, then divide by 5: $x = \dfrac{x^2 + 2}{5}$

Full solution on next slide

Full solution 2.2

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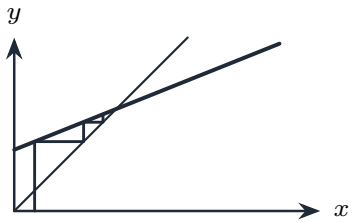
- $x_1 = \dfrac{0 + 2}{5} = 0.4, \quad x_2 = \dfrac{0.16 + 2}{5} = 0.432$
- $x_3 = \dfrac{0.432^2 + 2}{5} = \mathbf{0.4373}$

2.5

- $x_1 = 3 - \frac{1}{3} = 2.66667$
- $x_2 = 3 - \dfrac{1}{2.66667} = \mathbf{2.625}$

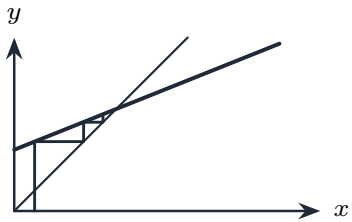
Exercise 3.1

3.1 Ella iterates $x_{n+1} = \sqrt{x_n + 2}$ and writes $x_1 = \sqrt{3}$, $x_2 = \sqrt{3}$, $x_3 = \sqrt{3}$. Explain her mistake. [1]



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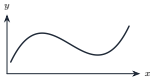


3.1

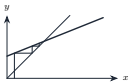
- she used x_0 every time; each step must use the **previous** iterate: $x_2 = \sqrt{x_1 + 2} = \sqrt{3.73205}$

Exercise 3.2

3.2 Show that $x^3 - 3x - 5 = 0$ can be rearranged to $x = \sqrt[3]{3x + 5}$. [1]



3.3 Use $x_{n+1} = \sqrt[3]{3x_n + 5}$ with $x_0 = 2$ to find x_3 to 4 decimal places. [3]



3.4 Show that $x = 2.28$ is a solution of $x^3 - 3x - 5 = 0$ correct to 2 decimal places. [2]

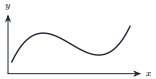


3.5 Show that $x^3 + 4x = 1$ has a solution between 0 and 1, and use $x_{n+1} = \dfrac{1 - x_n^3}{4}$ with $x_0 = 0.5$ to find the solution to 3 decimal places. [4]

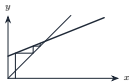


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3.2

- $x^3 = 3x + 5$, then take the cube root: $x = \sqrt[3]{3x + 5}$

3.3

- $x_1 = \sqrt[3]{11} = 2.22398$
- $x_2 = \sqrt[3]{11.67194} = 2.26837$

Full solution on next slide

Full solution 3.2 (1/2)

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- $x_1 = \sqrt[3]{11} = 2.22398$
- $x_2 = \sqrt[3]{11.67194} = 2.26837$
- $x_3 = \sqrt[3]{11.80512} = \mathbf{2.2770}$

3.4

Full solution 3.2 (2/2)

- $x = 2.275$: $2.275^3 - 3 \times 2.275 - 5 = -0.0505\ldots$; $x = 2.285$: $2.285^3 - 3 \times 2.285 - 5 = 0.0756\ldots$
- the sign changes between 2.275 and 2.285, so the root rounds to **2.28**

3.5

- $f(x) = x^3 + 4x - 1$: $f(0) = -1$ and $f(1) = 4$, a change of sign
- $x_1 = \dfrac{1 - 0.125}{4} = 0.21875$, $x_2 = \dfrac{1 - 0.21875^3}{4} = 0.24738$
- $x_3 = 0.24622$, $x_4 = 0.24627$, $x_5 = 0.24627$

- $\mathbf{x = 0.246}$ (3 d.p.)