Exercise walk-through

2.5.2 Simultaneous

AQA · GCSE · Mathematics

Straight-line graphs

- Every straight line, except a vertical one, has an equation $y = mx + c$.
- The number m is the **gradient**, the steepness: how far y goes up when x increases by 1.
- The number c is the y-intercept, where the line crosses the y-axis.
- Between two points,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

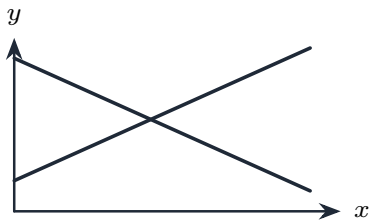
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

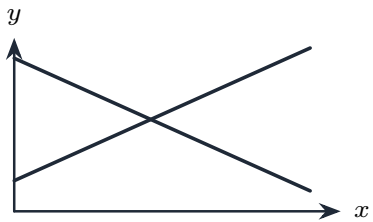
Exercise 2.1

2.1 Solve $x + y = 10$ and $x - y = 4$. [2]



Exercise 2.1

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2.1

- add: $2x = 14$, so $x = 7$
- $\boldsymbol{x = 7}$, $\boldsymbol{y = 3}$

Exercise 2.2

2.2 Solve $2x + y = 11$ and $x + y = 7$. [2]



2.3 Solve $3x + 2y = 16$ and $x - 2y = 0$. [2]



2.4 Solve $y = 2x - 1$ and $3x + y = 14$ by substitution. [2]



2.5 Solve $4x + 3y = 25$ and $3x - 2y = 6$. [3]



Exercise 2.2

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2.2

- subtract: $(2x + y) - (x + y) = 11 - 7$, so $x = 4$
- $\boldsymbol{x = 4}$, $\boldsymbol{y = 3}$

2.3

Full solution on next slide

Full solution 2.2

2.2

- subtract: $(2x + y) - (x + y) = 11 - 7$, so $x = 4$
- $\boldsymbol{x = 4}$, $\boldsymbol{y = 3}$

2.3

- add: $4x = 16$, so $x = 4$
- $x - 2y = 0$ gives $y = 2$: $\boldsymbol{x = 4}$, $\boldsymbol{y = 2}$

2.4

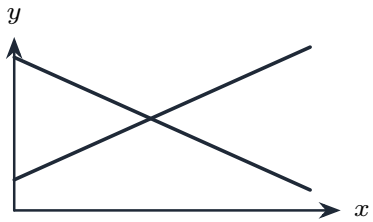
- $3x + (2x - 1) = 14$, so $5x = 15$ and $x = 3$
- $y = 2 \times 3 - 1 = 5$: $\boldsymbol{x = 3}$, $\boldsymbol{y = 5}$

2.5

- make the y-coefficients equal: $\times 2$ gives $8x + 6y = 50$ and $\times 3$ gives $9x - 6y = 18$
- add: $17x = 68$, so $x = 4$
- $4 \times 4 + 3y = 25$, so $3y = 9$: $\boldsymbol{x = 4}$, $\boldsymbol{y = 3}$

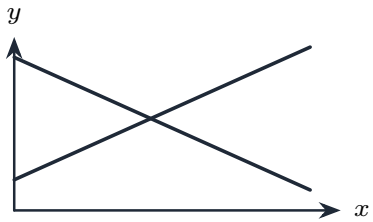
Exercise 3.1

3.1 Mia solves $2x + 3y = 13$ and $2x - y = 1$ by adding them and gets $4x + 2y = 14$. Explain why this does not help, and solve the equations. [3]



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3.1

- adding keeps both letters, so neither is eliminated; subtract instead, because the $2x$ terms match
- $(2x + 3y) - (2x - y) = 13 - 1$, so $4y = 12$ and $y = 3$
- $2x - 3 = 1$, so $\boldsymbol{x = 2}$, $\boldsymbol{y = 3}$

Exercise 3.2

3.2 Three pens and two rulers cost ¥23. One pen and four rulers cost ¥21. Find the cost of one pen and one ruler. [3]



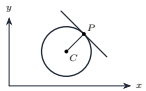
3.3 The lines $y = 3x - 4$ and $y = 8 - x$ cross at P. Find the coordinates of P. [2]



3.4 *(H)* Solve $y = 2x + 3$ and $x^2 + y = 6$. [4]



3.5 *(H)* Solve $x + y = 7$ and $x^2 + y^2 = 25$. [4]



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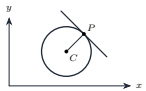
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3.2

- pen p, ruler r: $3p + 2r = 23$ and $p + 4r = 21$
- double the first: $6p + 4r = 46$; subtract the second: $5p = 25$, so $p = 5$
- $5 + 4r = 21$, so $r = 4$: pen **¥5**, ruler **¥4**

3.3

Full solution on next slide

Full solution 3.2 (1/2)

3.2

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3.3

- $3x - 4 = 8 - x$, so $4x = 12$ and $x = 3$
- $y = 8 - 3 = 5$: $\mathbf{P(3, 5)}$

3.4

Full solution 3.2 (2/2)

- substitute: $x^2 + 2x + 3 = 6$, so $x^2 + 2x - 3 = 0$
- $(x + 3)(x - 1) = 0$, so $x = -3$ or $x = 1$
- $x = 1$: $y = 5$; $x = -3$: $y = -3$
- $(\mathbf{1}, \mathbf{5})$ and $(-\mathbf{3}, -\mathbf{3})$

3.5

- $y = 7 - x$, so $x^2 + (7 - x)^2 = 25$
- $2x^2 - 14x + 49 = 25$, so $x^2 - 7x + 12 = 0$
- $(x - 3)(x - 4) = 0$, so $x = 3$ or $x = 4$
- $(\mathbf{3}, \mathbf{4})$ and $(\mathbf{4}, \mathbf{3})$