Exercise walk-through

2.5.1 Quadratic

AQA · GCSE · Mathematics

Straight-line graphs

- Every straight line, except a vertical one, has an equation $y = mx + c$.
- The number m is the **gradient**, the steepness: how far y goes up when x increases by 1.
- The number c is the y-intercept, where the line crosses the y-axis.
- Between two points,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

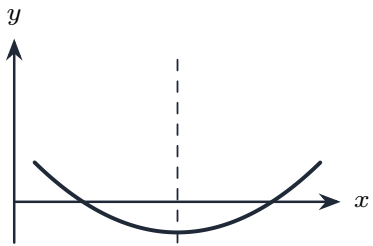
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

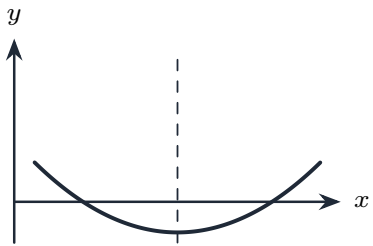
Exercise 2.1

2.1 Solve $x^2 = 49$. [1]



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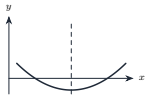


2.1

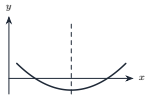
- $\boldsymbol{x = 7}$ or $\boldsymbol{x = -7}$

Exercise 2.2

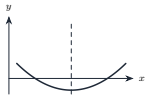
2.2 Solve $x^2 + 7x + 10 = 0$. [2]



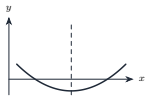
2.3 Solve $x^2 - 5x = 0$. [2]



2.4 Solve $x^2 - x - 12 = 0$. [2]

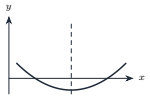


2.5 *(H)* Solve $3x^2 - 10x - 8 = 0$ by factorising. [3]

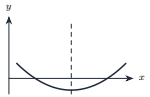


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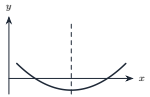
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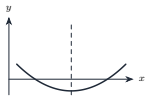
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2.2

- $(x + 2)(x + 5) = 0$
- $\boldsymbol{x = -2}$ or $\boldsymbol{x = -5}$

2.3

Full solution on next slide

Full solution 2.2

2.2

- $(x+2)(x+5)=0$
- $\boldsymbol{x=-2}$ or $\boldsymbol{x=-5}$

2.3

- common factor: $x(x-5)=0$
- $\boldsymbol{x=0}$ or $\boldsymbol{x=5}$

2.4

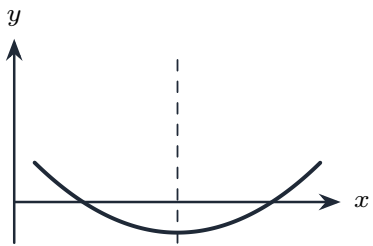
- $(x-4)(x+3)=0$
- $\boldsymbol{x=4}$ or $\boldsymbol{x=-3}$

2.5

- $ac=-24$; numbers -12 and 2: $3x^2-12x+2x-8=3x(x-4)+2(x-4)$
- $(3x+2)(x-4)=0$
- $\boldsymbol{x=-\frac{2}{3}}$ or $\boldsymbol{x=4}$

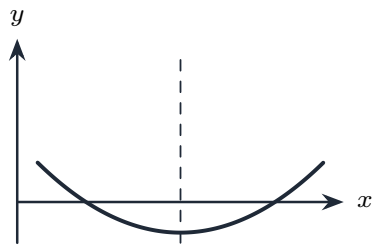
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3.1 Ravi solves $x^2 = 6x$ by dividing both sides by x, and writes $x = 6$. Explain what he has missed. [2]



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3.1

- dividing by x loses the solution $x = 0$
- $x^2 - 6x = 0$, so $x(x - 6) = 0$: $\boldsymbol{x = 0}$ or $\boldsymbol{x = 6}$

Exercise 3.2

3.2 A rectangle has width x cm and length $(x + 5)$ cm. Its area is 84 cm^2. Form and solve an equation to find the width. [4]



3.3 *(H)* Solve $x^2 + 4x - 7 = 0$. Give your answers to 2 decimal places. [3]



3.4 *(H)* Write $x^2 - 8x + 3$ in the form $(x + p)^2 + q$ and hence solve $x^2 - 8x + 3 = 0$, giving exact answers. [3]



3.5 *(H)* Solve $\frac{4}{x} + \frac{3}{x+1} = 2$. [4]



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3.2

- area: $x(x + 5) = 84$, so $x^2 + 5x - 84 = 0$
- $(x + 12)(x - 7) = 0$
- $x = 7$ or $x = -12$
- a width cannot be negative, so the width is **7 cm**

Full solution on next slide

Full solution 3.2 (1/2)

3.2

- area: $x(x+5) = 84$, so $x^2 + 5x - 84 = 0$
- $(x+12)(x-7) = 0$
- $x = 7$ or $x = -12$
- a width cannot be negative, so the width is **7 cm**

3.3

- $a = 1$, $b = 4$, $c = -7$: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-4 \pm \sqrt{16 + 28}}{2} = \dfrac{-4 \pm \sqrt{44}}{2}$
- $\boldsymbol{x = 1.32}$ or $\boldsymbol{x = -5.32}$

Full solution 3.2 (2/2)

3.4

- $x^2 - 8x + 3 = (x-4)^2 - 16 + 3 = (x-4)^2 - 13$
- $(x-4)^2 = 13$, so $\boldsymbol{x = 4 \pm \sqrt{13}}$

3.5

- multiply every term by $x(x+1)$: $4(x+1) + 3x = 2x(x+1)$
- $7x + 4 = 2x^2 + 2x$, so $2x^2 - 5x - 4 = 0$
- $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{5 \pm \sqrt{25 + 32}}{4} = \dfrac{5 \pm \sqrt{57}}{4}$
- $\boldsymbol{x = 3.14}$ or $\boldsymbol{x = -0.64}$ (2 d.p.)