Exercise walk-through

2.4.9
Circle Coordinates

AQA · GCSE · Mathematics

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inequalities

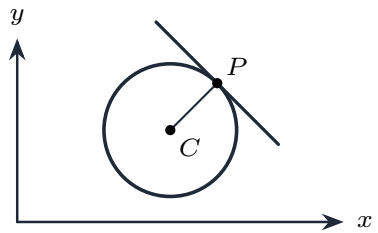
- Solve a linear inequality like an equation, but if you multiply or divide by a negative number, turn the sign round: from $-2x < 6$ you get $x > -3$.
- On a number line, an open circle means the end value is not included ($<$ or $>$) and a filled circle means it is ($\leq$ or $\geq$).
- **Worked example 15.** Find the integers n with $-3 < 2n + 1 \leq 9$.
- Subtract 1: $-4 < 2n \leq 8$.

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

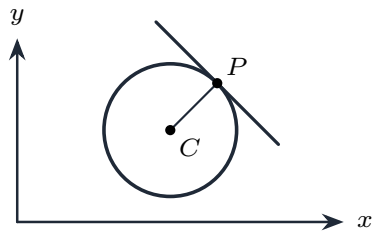
Exercise 2.1

2.1 Write down the centre and radius of the circle $x^2 + y^2 = 49$. [2]



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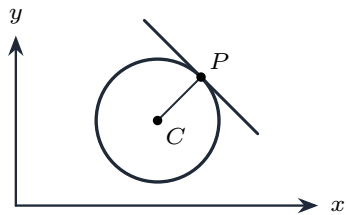


2.1

- centre $(\mathbf{0}, \mathbf{0})$; radius $\sqrt{49} = \mathbf{7}$

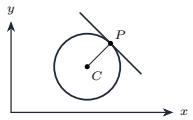
Setup for exercise 2.2

2.2 Write down the equation of the circle with centre $(0, 0)$ and radius 6. [1]

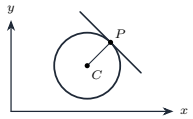


Exercise 2.2

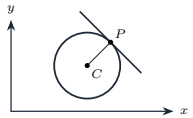
2.3 Does the point $(5, 2)$ lie inside, on or outside the circle $x^2 + y^2 = 30$? Show how you decide. [2]



2.4 $P(-6, 8)$ lies on a circle centred at the origin. Find the radius of the circle. [2]

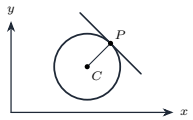


2.5 The point $(8, 6)$ lies on $x^2 + y^2 = 100$. Find the gradient of the tangent at $(8, 6)$. [2]

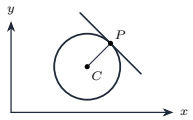


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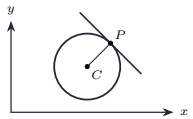
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2.2

- $r^2 = 36$: $\boldsymbol{x^2 + y^2 = 36}$

2.3

Full solution on next slide

Full solution 2.2

2.2

- $r^2 = 36$: $\boldsymbol{x^2 + y^2 = 36}$

2.3

- $5^2 + 2^2 = 25 + 4 = 29$
- $29 < 30$, so the point is **inside**

2.4

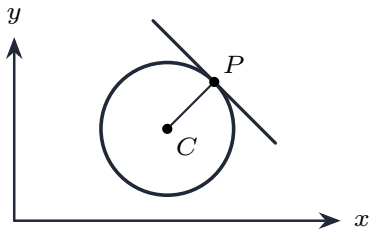
- $r^2 = (-6)^2 + 8^2 = 36 + 64 = 100$
- $r = \mathbf{10}$

2.5

- radius gradient $\dfrac{6}{8} = \dfrac{3}{4}$
- tangent gradient $-\dfrac{\mathbf{4}}{\mathbf{3}}$

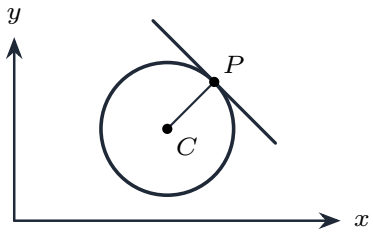
Exercise 3.1

3.1 Leo says the tangent at $(3, 4)$ to $x^2 + y^2 = 25$ has gradient $\frac{4}{3}$. Explain his mistake. [1]



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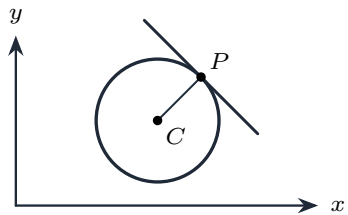


3.1

- $\frac{4}{3}$ is the gradient of the **radius**; the tangent is perpendicular to it, so its gradient is $-\frac{3}{4}$

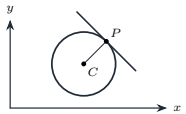
Setup for exercise 3.2

3.2 The point $(k, 12)$ lies on the circle $x^2 + y^2 = 169$. Find the two possible values of k. [2]

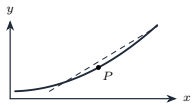


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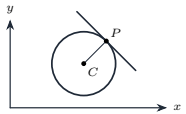
3.3 Find the equation of the tangent to $x^2+y^2=20$ at the point $(2,4)$. Give your answer in the form $y=mx+c$. [3]



3.4 The tangent in **3.3** crosses the x-axis at A. Find the coordinates of A. [1]

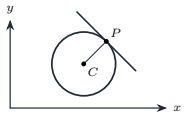


3.5 Find the coordinates of the points where the line $y=x+1$ meets the circle $x^2+y^2=13$. [4]

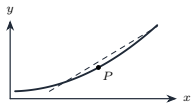


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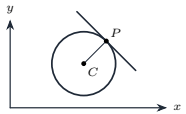
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3.2

- $k^2+144=169$, so $k^2=25$
- $\boldsymbol{k=5}$ or $\boldsymbol{k=-5}$

Full solution on next slide

Full solution 3.2

3.2

- $k^2 + 144 = 169$, so $k^2 = 25$
- $\boldsymbol{k = 5}$ or $\boldsymbol{k = -5}$

3.3

- radius gradient $\dfrac{4}{2} = 2$, so the tangent gradient is $-\frac{1}{2}$
- substitute $(2, 4)$: $4 = -\frac{1}{2} \times 2 + c$, so $c = 5$
- $\boldsymbol{y = -\frac{1}{2}x + 5}$

3.4

- $y = 0$: $\frac{1}{2}x = 5$, so $x = 10$: $(\mathbf{10}, \mathbf{0})$

3.5

- substitute: $x^2 + (x + 1)^2 = 13$
- $2x^2 + 2x + 1 = 13$, so $x^2 + x - 6 = 0$
- $(x + 3)(x - 2) = 0$, so $x = -3$ or $x = 2$
- $(\mathbf{2}, \mathbf{3})$ and $(\mathbf{-3}, \mathbf{-2})$