Exercise walk-through

2.4.8
Graph Rates

AQA · GCSE · Mathematics

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inequalities

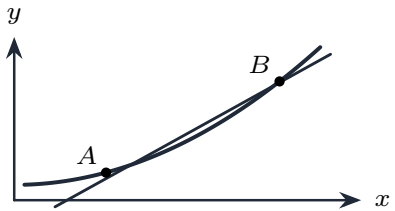
- Solve a linear inequality like an equation, but if you multiply or divide by a negative number, turn the sign round: from $-2x < 6$ you get $x > -3$.
- On a number line, an open circle means the end value is not included ($<$ or $>$) and a filled circle means it is ($\leq$ or $\geq$).
- **Worked example 15.** Find the integers n with $-3 < 2n + 1 \leq 9$.
- Subtract 1: $-4 < 2n \leq 8$.

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

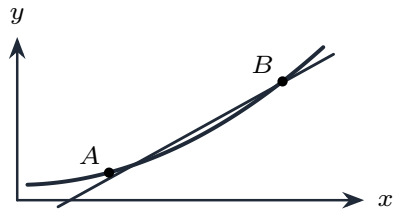
Exercise 2.1

2.1 The points $(1, 1)$ and $(3, 9)$ lie on $y = x^2$. Find the gradient of the chord joining them. [2]



Exercise 2.1

2.1 The points $(1, 1)$ and $(3, 9)$ lie on $y = x^2$. Find the gradient of the chord joining them. [2]

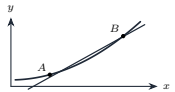


2.1

- $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{9 - 1}{3 - 1} = \dfrac{8}{2} = \mathbf{4}$

Exercise 2.2

2.2 A tangent drawn to a curve passes through $(0, -2)$ and $(4, 10)$. Find its gradient. [1]



2.3 A velocity–time graph is a straight line from $(0, 4)$ to $(5, 14)$, in seconds and m/s. Work out the acceleration. [2]



2.4 Using the graph in **2.3**, work out the distance travelled in the 5 seconds. [2]

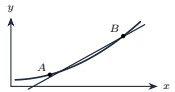


2.5 On a velocity–time graph, what does a horizontal line show? [1]



Exercise 2.2

2.2 A tangent drawn to a curve passes through $(0, -2)$ and $(4, 10)$. Find its gradient. [1]



2.3 A velocity–time graph is a straight line from $(0, 4)$ to $(5, 14)$, in seconds and m/s. Work out the acceleration. [2]



2.4 Using the graph in **2.3**, work out the distance travelled in the 5 seconds. [2]



2.5 On a velocity–time graph, what does a horizontal line show? [1]



2.2

- $\dfrac{10 - (-2)}{4 - 0} = \dfrac{12}{4} = \mathbf{3}$

2.3

Full solution on next slide

Full solution 2.2

2.2

- $\dfrac{10-(-2)}{4-0}=\dfrac{12}{4}=\mathbf{3}$

2.3

- $\text{acceleration}=\dfrac{\text{change in velocity}}{\text{time}}=\dfrac{(14-4)\ \text{m/s}}{5\ \text{s}}=\mathbf{2}\ \text{m/s}^2$

2.4

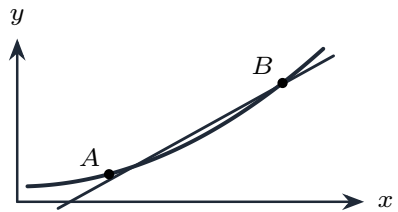
- area of the trapezium $=\frac{1}{2}(a+b)h=\frac{1}{2}\times(4+14)\times 5=\mathbf{45}\ \text{m}$

2.5

- **constant velocity**: zero acceleration

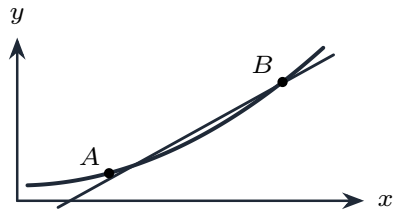
Exercise 3.1

3.1 Ella says the gradient of a chord gives the speed at an exact moment. Explain why she is wrong. [1]



Exercise 3.1

3.1 Ella says the gradient of a chord gives the speed at an exact moment. Explain why she is wrong. [1]



3.1

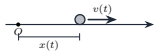
- a chord joins two points, so its gradient is the **average** speed over that time; the speed at an instant needs a **tangent**

Exercise 3.2

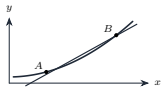
3.2 A car accelerates from rest to 20 m/s in 8 s, travels at 20 m/s for 12 s, then decelerates steadily to rest in 5 s. Work out the total distance travelled. [3]



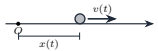
3.3 The velocity of a cyclist at $t = 0, 2, 4, 6$ s is $0, 3, 7, 12$ m/s. Use three strips of equal width to estimate the distance travelled in the 6 seconds. [3]



3.4 The depth of water in a bath is $d = 2t^2$ cm after t minutes, for $0 \le t \le 4$. Find the average rate of change of depth between $t = 1$ and $t = 3$. [2]



3.5 Is your estimate in **3.3** an overestimate or an underestimate? The velocity graph bends upwards. Give a reason. [1]

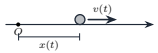


Exercise 3.2

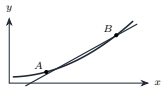
3.2 A car accelerates from rest to 20 m/s in 8 s, travels at 20 m/s for 12 s, then decelerates steadily to rest in 5 s. Work out the total distance travelled. [3]



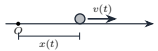
3.3 The velocity of a cyclist at $t = 0, 2, 4, 6$ s is $0, 3, 7, 12$ m/s. Use three strips of equal width to estimate the distance travelled in the 6 seconds. [3]



3.4 The depth of water in a bath is $d = 2t^2$ cm after t minutes, for $0 \le t \le 4$. Find the average rate of change of depth between $t = 1$ and $t = 3$. [2]



3.5 Is your estimate in **3.3** an overestimate or an underestimate? The velocity graph bends upwards. Give a reason. [1]



3.2

- area under the graph: $\frac{1}{2} \times 8 \times 20 = 80$ m and $12 \times 20 = 240$ m
- $\frac{1}{2} \times 5 \times 20 = 50$ m
- total $80 + 240 + 50 = \mathbf{370}$ m

Full solution on next slide

Full solution 3.2

3.2

- area under the graph: $\frac{1}{2} \times 8 \times 20 = 80$ m and $12 \times 20 = 240$ m
- $\frac{1}{2} \times 5 \times 20 = 50$ m
- total $80 + 240 + 50 = \mathbf{370}$ m

3.3

- strips of width 2: $\frac{1}{2}(0 + 3) \times 2 = 3$, $\frac{1}{2}(3 + 7) \times 2 = 10$, $\frac{1}{2}(7 + 12) \times 2 = 19$
- distance $\approx 3 + 10 + 19 = \mathbf{32}$ m

3.4

- $d(1) = 2$ cm and $d(3) = 18$ cm
- average rate $= \dfrac{(18 - 2) \text{ cm}}{(3 - 1) \text{ min}} = \mathbf{8}$ cm/min

3.5

- an **overestimate**: each straight trapezium top lies above a curve that bends upwards, so the trapezia include extra area