Exercise walk-through

2.4.6
Graph Transformations

AQA · GCSE · Mathematics

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inequalities

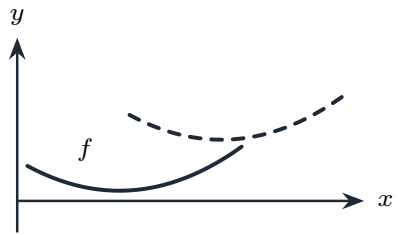
- Solve a linear inequality like an equation, but if you multiply or divide by a negative number, turn the sign round: from $-2x < 6$ you get $x > -3$.
- On a number line, an open circle means the end value is not included ($<$ or $>$) and a filled circle means it is ($\leq$ or $\geq$).
- **Worked example 15.** Find the integers n with $-3 < 2n + 1 \leq 9$.
- Subtract 1: $-4 < 2n \leq 8$.

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

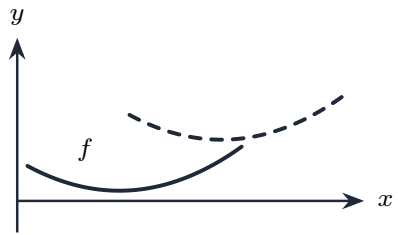
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2.1 Describe the transformation that maps $y = f(x)$ onto $y = f(x) - 4$. [1]



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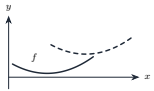


2.1

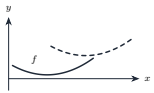
- translation $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$: 4 down

Exercise 2.2

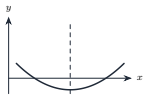
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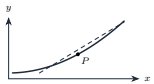
2.3 The point $(4, 3)$ lies on $y = f(x)$. Write down its image on (a) $y = -f(x)$ (b) $y = f(-x)$. [2]



2.4 Write $x^2 + 8x + 5$ in the form $(x + p)^2 + q$. [2]

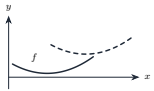


2.5 Write down the turning point of $y = (x + 1)^2 - 7$. [1]

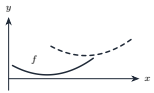


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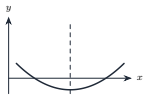
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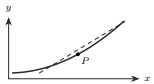
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2.2

- translation $\binom{-2}{0}$: 2 left

2.3

(a) reflect in the x-axis: $(\mathbf{4}, \mathbf{-3})$

Full solution on next slide

Full solution 2.2

2.2

- translation $\binom{-2}{0}$: 2 left

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(a) reflect in the x-axis: $(\mathbf{4}, \mathbf{-3})$

(b) reflect in the y-axis: $(\mathbf{-4}, \mathbf{3})$

2.4

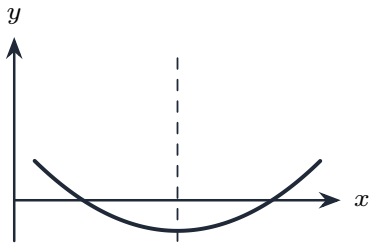
- half of 8 is 4: $(x+4)^2 = x^2 + 8x + 16$
- $x^2 + 8x + 5 = (x+4)^2 - 16 + 5 = \boldsymbol{(x+4)^2 - 11}$

2.5

- $(\mathbf{-1}, \mathbf{-7})$

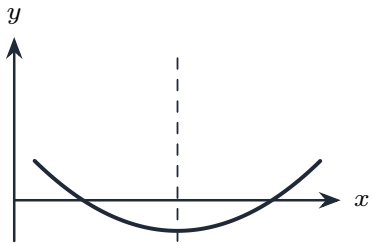
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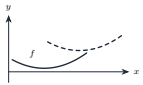


3.1

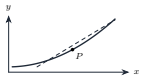
- **No**: $+5$ inside the bracket moves the graph 5 to the **left**; the vertex is at $(-5, 0)$

Exercise 3.2

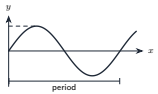
3.2 The point $(2, -6)$ is the minimum of $y = f(x)$. Write down the minimum point of $y = f(x-1) + 4$. [2]



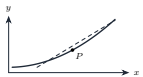
3.3 Write $x^2 - 10x + 18$ in the form $(x+p)^2 + q$ and hence write down the turning point of $y = x^2 - 10x + 18$. [3]



3.4 $y = \sin x$ is reflected in the x-axis. Write down the equation of the new graph and its value at $x = 90°$. [2]

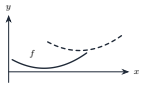


3.5 Write $2x^2 + 12x + 7$ in the form $a(x+p)^2 + q$ and find the minimum value of $2x^2 + 12x + 7$. [3]

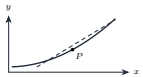


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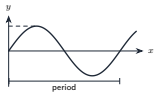
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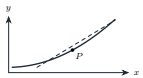
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3.2

- $f(x-1)$: 1 right; $+4$: 4 up
- $(2+1, -6+4) = (\mathbf{3}, \mathbf{-2})$

3.3

- half of -10 is -5: $(x-5)^2 = x^2 - 10x + 25$

Full solution on next slide

Full solution 3.2

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- half of -10 is -5: $(x-5)^2 = x^2 - 10x + 25$
- $x^2 - 10x + 18 = \boldsymbol{(x-5)^2 - 7}$
- turning point $(\mathbf{5}, \mathbf{-7})$

3.4

- $\boldsymbol{y = -\sin x}$
- at $90°$: $-\sin 90° = \mathbf{-1}$

3.5

- take out 2 from the x-terms: $2(x^2+6x) + 7 = 2[(x+3)^2 - 9] + 7 = \boldsymbol{2(x+3)^2 - 11}$
- $(x+3)^2 \geq 0$, so the minimum value is $\mathbf{-11}$, at $x = -3$

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