Exercise walk-through

2.4.5
Trig Graphs

AQA · GCSE · Mathematics

Quadratic and other graphs

- Cubic, $y = x^3$: an S-shape.
- **Reciprocal** graph, $y = \frac{1}{x}$: two separate curves; $x = 0$ is not allowed.
- (H) Exponential, $y = 2^x$: grows faster and faster and never touches the x-axis.
- (H) $y = \sin x$, $y = \cos x$ and $y = \tan x$: waves that repeat every $360°$ ($180°$ for $\tan$).

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

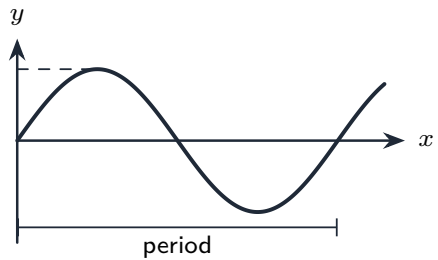
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

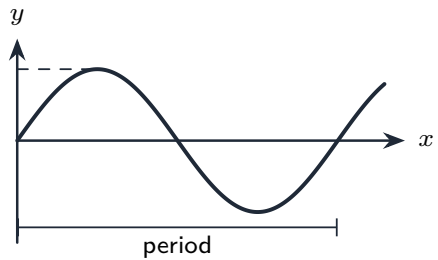
Exercise 2.1

2.1 Write down the value of (a) $\sin 90°$ (b) $\cos 180°$ (c) $\tan 45°$ [3]



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2.1

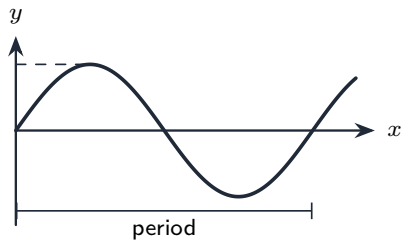
(a) **1**

(b) $-$**1**

(c) **1**

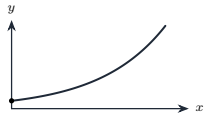
Setup for exercise 2.2

2.2 Write down the coordinates where $y = \cos x$ crosses the x-axis for $0° \leq x \leq 360°$. [2]

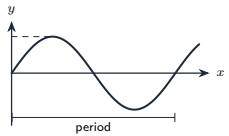


Exercise 2.2

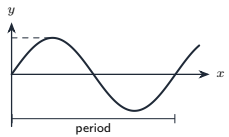
2.3 $y = 3^x$. Work out y when (a) $x = 2$ (b) $x = 0$ (c) $x = -2$. [2]



2.4 $\cos 60° = 0.5$. Find another angle between $0°$ and $360°$ whose cosine is 0.5. [1]

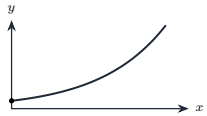


2.5 For which values of x between $0°$ and $360°$ is $\tan x$ not defined? [1]

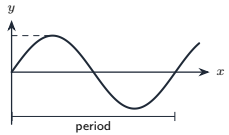


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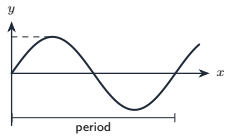
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2.2

- (**90°**, **0**) and (**270°**, **0**)

2.3

Full solution on next slide

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Full solution 2.2

2.2

- $(\mathbf{90}^\circ, \mathbf{0})$ and $(\mathbf{270}^\circ, \mathbf{0})$

2.3

- (a) $3^2 = \mathbf{9}$; (b) $3^0 = \mathbf{1}$; (c) $3^{-2} = \dfrac{\mathbf{1}}{\mathbf{9}}$

2.4

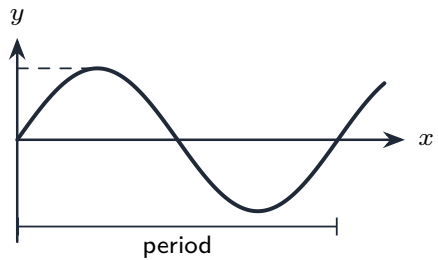
- $\cos x = \cos(360^\circ - x)$: $360^\circ - 60^\circ = \mathbf{300}^\circ$

2.5

- $\mathbf{90}^\circ$ and $\mathbf{270}^\circ$

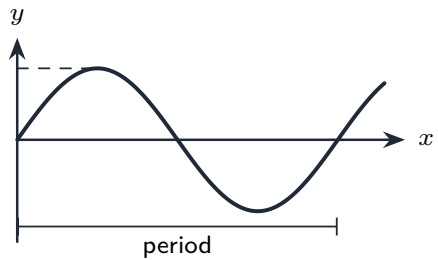
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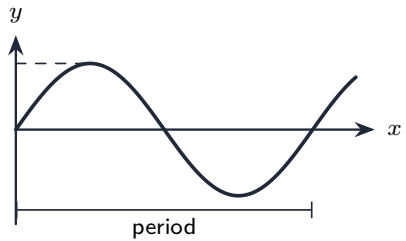


3.1

- the sine graph lies between -1 and 1, so it never reaches 2

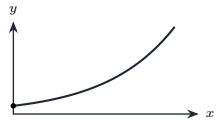
Setup for exercise 3.2

3.2 $\sin 40° = 0.643$ (3 d.p.). Find two angles between $180°$ and $360°$ for which $\sin x = -0.643$. [2]



Exercise 3.2

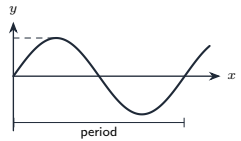
3.3 The curve $y = ab^x$ passes through $(0, 5)$ and $(2, 45)$. Find a and b, given $b > 0$. [3]



3.4 A car worth ¥120 000 loses 15% of its value each year. Its value after n years is $V = 120\,000 \times 0.85^n$. Work out its value after 3 years. [2]

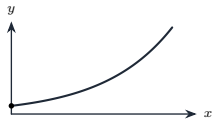


3.5 Bacteria double every hour, starting with 500. After how many whole hours does the number first exceed 30 000? [3]



Exercise 3.2

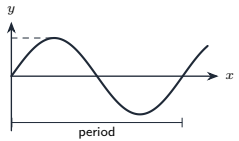
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3.2

- by symmetry, $\sin(180° + 40°) = \sin(360° - 40°) = -0.643$
- **220**° and **320**°

Full solution on next slide

Full solution 3.2

3.2

- by symmetry, $\sin(180° + 40°) = \sin(360° - 40°) = -0.643$
- $\mathbf{220}°$ and $\mathbf{320}°$

3.3

- at $(0, 5)$: $5 = ab^0 = a$, so $\boldsymbol{a = 5}$
- at $(2, 45)$: $45 = 5b^2$, so $b^2 = 9$
- $b > 0$, so $\boldsymbol{b = 3}$

3.4

- $V = 120\,000 \times 0.85^3 = 120\,000 \times 0.614125$
- $V = \mathbf{¥73\ 695}$

3.5

- number after h hours: 500×2^h; we need $2^h > 60$
- $2^5 = 32$ and $2^6 = 64$
- **6 hours**: $500 \times 64 = 32\,000$