Exercise walk-through

2.4.4 Quadratic, cubic and reciprocal graphs

AQA · GCSE · Mathematics

Quadratic and other graphs

- Cubic, $y = x^3$: an S-shape.
- **Reciprocal** graph, $y = \frac{1}{x}$: two separate curves; $x = 0$ is not allowed.
- (H) Exponential, $y = 2^x$: grows faster and faster and never touches the x-axis.
- (H) $y = \sin x$, $y = \cos x$ and $y = \tan x$: waves that repeat every $360°$ ($180°$ for $\tan$).

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

Exercise 2.1

2.1 Complete the table of values for $y = x^2 - 3$. [2]

x	-2	-1	0	1	2
y					



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2.1

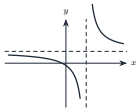
- $y = (-2)^2 - 3 = 1$, and so on: $\mathbf{1}, \mathbf{-2}, \mathbf{-3}, \mathbf{-2}, \mathbf{1}$

Exercise 2.2

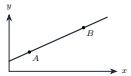
2.2 Work out y for $y = x^3 - 2$ when $x = -2$. [1]



2.3 Work out y for $y = \dfrac{6}{x}$ when (a) $x = 3$ (b) $x = -0.5$ [2]



2.4 Match each equation to its shape: $y = 2x + 1$, $y = x^2 - 4$, $y = \dfrac{1}{x}$, $y = -x^3$. Shapes: straight line, U-shaped, two branches, S-shaped falling. [2]



2.5 The graph of $y = x^2 - 6x + 5$ crosses the x-axis at 1 and 5. Write down the equation of its line of symmetry. [1]

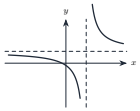


Exercise 2.2

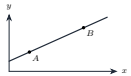
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2.2

- $y = (-2)^3 - 2 = -8 - 2 = \mathbf{-10}$

2.3

(a) $\dfrac{6}{3} = \mathbf{2}$

Full solution on next slide

Full solution 2.2

2.2

- $y = (-2)^3 - 2 = -8 - 2 = \mathbf{-10}$

2.3

(a) $\dfrac{6}{3} = \mathbf{2}$

(b) $\dfrac{6}{-0.5} = \mathbf{-12}$

2.4

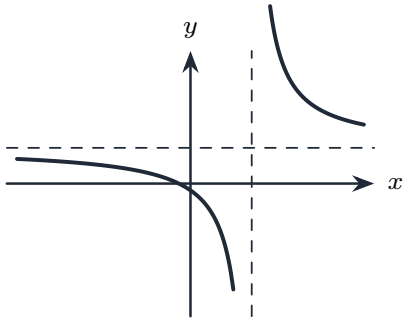
- $y = 2x + 1$: straight line; $y = x^2 - 4$: U-shaped; $y = \dfrac{1}{x}$: two branches; $y = -x^3$: S-shaped falling

2.5

- halfway between the roots: $\dfrac{1+5}{2} = 3$, so $\boldsymbol{x = 3}$

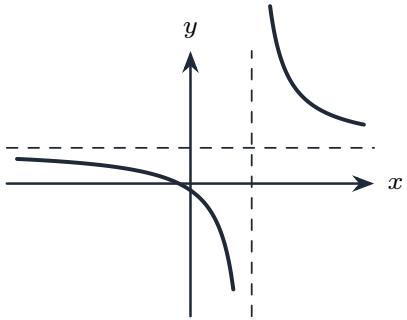
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3.1 Sam says the graph of $y = \dfrac{1}{x}$ crosses the y-axis at $(0, 1)$. Explain why he is wrong. [1]



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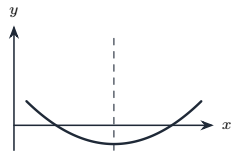


3.1

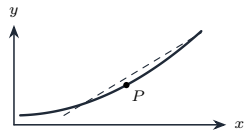
- at $x = 0$, $\frac{1}{0}$ has no value, so the graph never meets the y-axis

Setup for exercise 3.2

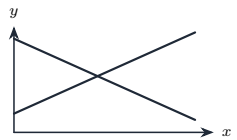
3.2 $y = x^2 + 2x - 8$. Find the coordinates of the points where the graph crosses the axes. [3]



3.3 Find the coordinates of the turning point of $y = x^2 + 2x - 8$. [2]



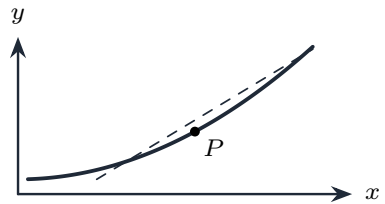
3.4 The graphs of $y = x^2$ and $y = 2x+3$ are drawn on the same axes. Find the coordinates of the points where they meet. [3]



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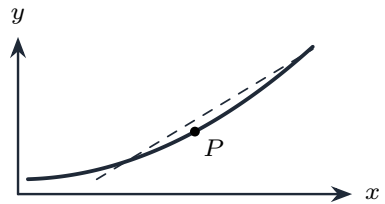
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3.5 A quadratic graph crosses the x-axis at $(-2, 0)$ and $(4, 0)$ and the y-axis at $(0, -8)$. Find its equation in the form $y = x^2 + bx + c$ and its turning point. [3]



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3.2

- y-axis: $x = 0$ gives $y = -8$, so $(\mathbf{0}, \mathbf{-8})$
- x-axis: $x^2 + 2x - 8 = 0$, so $(x + 4)(x - 2) = 0$
- $(\mathbf{-4}, \mathbf{0})$ and $(\mathbf{2}, \mathbf{0})$

3.3

Full solution on next slide

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Full solution 3.2

3.2

- y-axis: $x = 0$ gives $y = -8$, so $(\mathbf{0}, \mathbf{-8})$
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- $(\mathbf{-4}, \mathbf{0})$ and $(\mathbf{2}, \mathbf{0})$

3.3

- line of symmetry halfway between the roots: $x = \dfrac{-4 + 2}{2} = -1$
- $y = (-1)^2 + 2 \times (-1) - 8 = -9$, so $(\mathbf{-1}, \mathbf{-9})$

3.4

- where they meet, $x^2 = 2x + 3$, so $x^2 - 2x - 3 = 0$
- $(x - 3)(x + 1) = 0$, so $x = 3$ or $x = -1$
- $(\mathbf{3}, \mathbf{9})$ and $(\mathbf{-1}, \mathbf{1})$

3.5

- roots -2 and 4: $y = (x + 2)(x - 4) = x^2 - 2x - 8$; at $x = 0$ this gives -8 ✓
- line of symmetry $x = 1$; $y = 1 - 2 - 8 = -9$: turning point $(\mathbf{1}, \mathbf{-9})$