Exercise walk-through

2.4.3 Functions

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Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inequalities

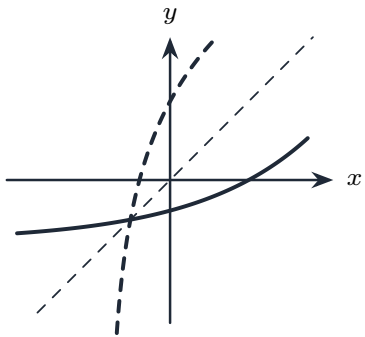
- Solve a linear inequality like an equation, but if you multiply or divide by a negative number, turn the sign round: from $-2x < 6$ you get $x > -3$.
- On a number line, an open circle means the end value is not included ($<$ or $>$) and a filled circle means it is ($\leq$ or $\geq$).
- **Worked example 15.** Find the integers n with $-3 < 2n + 1 \leq 9$.
- Subtract 1: $-4 < 2n \leq 8$.

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

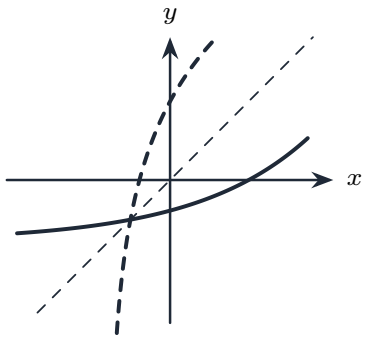
Exercise 2.1

2.1 $f(x) = 2x + 3$. Find (a) $f(4)$ (b) $f(-5)$ [2]



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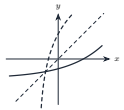
2.1

(a) $f(4) = 2 \times 4 + 3 = \mathbf{11}$

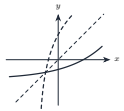
(b) $f(-5) = 2 \times (-5) + 3 = \mathbf{-7}$

Exercise 2.2

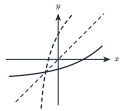
2.2 $f(x) = 2x + 3$. Solve $f(x) = 11$. [2]



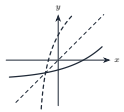
2.3 $f(x) = 2x + 3$ and $g(x) = x^2$. Find $fg(4)$ and $gf(4)$. [2]



2.4 $f(x) = 2x + 3$. Find $f^{-1}(x)$. [2]

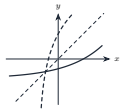


2.5 $h(x) = \sqrt{x - 2}$. Find $h(27)$. [1]

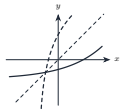


Exercise 2.2

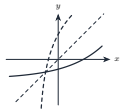
2.2 $f(x) = 2x + 3$. Solve $f(x) = 11$. [2]



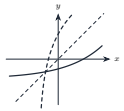
2.3 $f(x) = 2x + 3$ and $g(x) = x^2$. Find $fg(4)$ and $gf(4)$. [2]



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2.5 $h(x) = \sqrt{x - 2}$. Find $h(27)$. [1]



2.2

- $2x + 3 = 11$, so $2x = 8$
- $\boldsymbol{x = 4}$

2.3

- $fg(4)$: $g(4) = 16$, then $f(16) = 35$, so **35**

Full solution on next slide

Full solution 2.2

2.2

- $2x + 3 = 11$, so $2x = 8$
- $\boldsymbol{x = 4}$

2.3

- $fg(4)$: $g(4) = 16$, then $f(16) = 35$, so $\mathbf{35}$
- $gf(4)$: $f(4) = 11$, then $g(11) = 121$, so $\mathbf{121}$

2.4

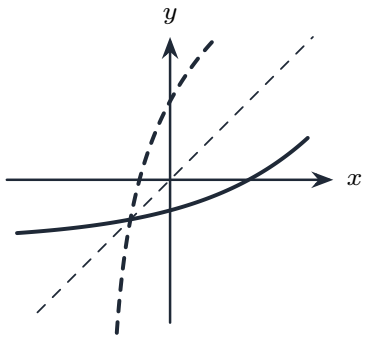
- $y = 2x + 3$, so $x = \dfrac{y - 3}{2}$
- $\boldsymbol{f^{-1}(x) = \dfrac{x - 3}{2}}$

2.5

- $h(27) = \sqrt{27 - 2} = \sqrt{25} = \mathbf{5}$

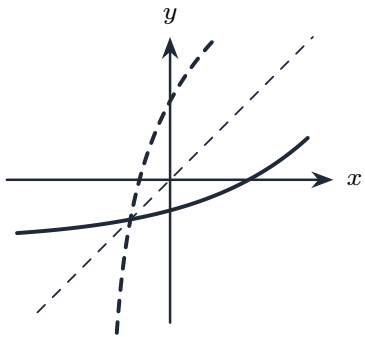
Exercise 3.1

3.1 $f(x) = x + 5$ and $g(x) = 3x$. Lily says $fg(x) = 3x + 15$. Is she correct? Give a reason. [2]



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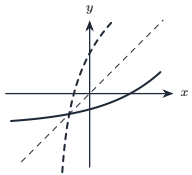


3.1

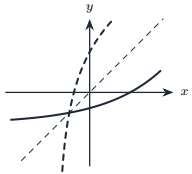
- $fg(x) = f(3x) = 3x + 5$
- **No**: g is applied first, then 5 is added; $3x + 15$ is $gf(x)$

Setup for exercise 3.2

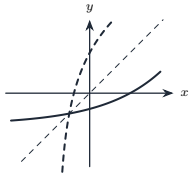
3.2 $f(x) = 4x - 1$ and $g(x) = x^2 + 2$. Find $gf(x)$ in the form $ax^2 + bx + c$. [3]



3.3 $f(x) = \dfrac{x+7}{2}$. Find $f^{-1}(x)$ and work out $f^{-1}(6)$. [3]

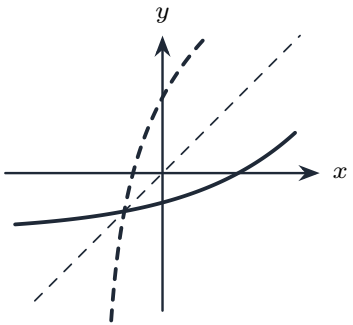


3.4 $f(x) = 3x - 2$ and $g(x) = \dfrac{1}{x}$. Solve $gf(x) = \dfrac{1}{4}$. [3]



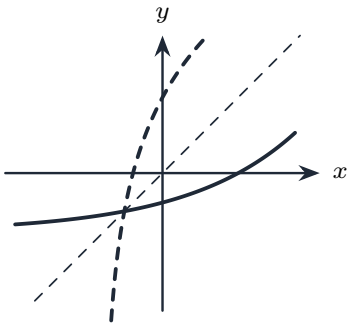
Exercise 3.2

3.5 $f(x) = x + 2$ and $g(x) = x^2$. Solve $fg(x) = gf(x)$. [3]



Exercise 3.2

3.5 $f(x) = x + 2$ and $g(x) = x^2$. Solve $fg(x) = gf(x)$. [3]



3.2

- $gf(x) = g(4x - 1) = (4x - 1)^2 + 2$
- $(4x - 1)^2 = 16x^2 - 8x + 1$
- $\mathbf{16x^2 - 8x + 3}$

3.3

Full solution on next slide

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Full solution 3.2 (1/2)

3.2

- $gf(x) = g(4x-1) = (4x-1)^2 + 2$
- $(4x-1)^2 = 16x^2 - 8x + 1$
- $\mathbf{16x^2 - 8x + 3}$

3.3

- $y = \dfrac{x+7}{2}$, so $2y = x + 7$ and $x = 2y - 7$
- $\boldsymbol{f^{-1}(x) = 2x - 7}$
- $f^{-1}(6) = 2 \times 6 - 7 = \mathbf{5}$

Full solution 3.2 (2/2)

3.4

- $gf(x) = \dfrac{1}{3x-2} = \dfrac{1}{4}$
- $3x - 2 = 4$
- $\boldsymbol{x = 2}$

3.5

- $fg(x) = f(x^2) = x^2 + 2$ and $gf(x) = g(x+2) = (x+2)^2 = x^2 + 4x + 4$
- $x^2 + 2 = x^2 + 4x + 4$: the x^2 terms cancel, leaving $4x = -2$
- $\boldsymbol{x = -\frac{1}{2}}$