Exercise walk-through

2.4.2
Function Machine

AQA · GCSE · Mathematics

Straight-line graphs

- Every straight line, except a vertical one, has an equation $y = mx + c$.
- The number m is the **gradient**, the steepness: how far y goes up when x increases by 1.
- The number c is the y-intercept, where the line crosses the y-axis.
- Between two points,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

Exercise 2.1

2.1 The machine is "$\times 5$, then -2". Find the output when the input is (a) 3 (b) -1. [2]



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2.1

(a) $3 \times 5 - 2 = \mathbf{13}$

(b) $-1 \times 5 - 2 = \mathbf{-7}$

Setup for exercise 2.2

2.2 The machine is "÷4, then +6". The output is 10. Find the input. [2]



Exercise 2.2

2.3 The machine is "-3, then $\times 4$". Write an expression for the output when the input is n. [1]



2.4 The machine is "$\times 2$, then $+3$". The output is -5. Find the input. [2]



2.5 The machine is "square, then -1". The output is 48. Find the two possible inputs. [2]



Exercise 2.2

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2.2

- undo in reverse order: $10 - 6 = 4$

Full solution on next slide

Full solution 2.2

2.2

- undo in reverse order: $10 - 6 = 4$
- $4 \times 4 = \mathbf{16}$

2.3

- subtract first, so a bracket: $\mathbf{4(n - 3)}$

2.4

- $-5 - 3 = -8$
- $-8 \div 2 = \mathbf{-4}$

2.5

- $48 + 1 = 49$
- square root: $\mathbf{7}$ or $\mathbf{-7}$, because $(-7)^2 = 49$ too

Exercise 3.1

3.1 The machine is "+4, then ×3". Ella says the output for input x is $x + 12$. Explain why she is wrong and give the correct expression. [2]



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3.1

- the 4 is added **before** multiplying, so all of $x + 4$ is multiplied by 3
- $\mathbf{3(x + 4) = 3x + 12}$

Setup for exercise 3.2

3.2 A phone shop converts a price in yuan to a sale price with the machine "×0.8, then −10". A phone's sale price is ¥390. Work out its original price. [2]



3.3 The machine is "×3, then +7". For which input are the input and the output equal? [3]



Exercise 3.2

3.4 Machine A is "$\times 4$, then -5". Machine B is "$\times 2$, then $+9$". The same number is put into both machines and they give the same output. Find the number and the output. [3]



3.5 The machine is "$\times a$, then $+b$". An input of 2 gives 13 and an input of 5 gives 25. Find a and b. [3]



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3.2

- undo -10: $390 + 10 = 400$

Full solution on next slide

Full solution 3.2

3.2

- undo -10: $390 + 10 = 400$
- undo $\times 0.8$: $400 \div 0.8 = \mathbf{¥500}$

3.3

- output is $3x + 7$; set it equal to the input: $3x + 7 = x$
- $2x = -7$
- $\boldsymbol{x = -3.5}$

3.4

- A gives $4x - 5$, B gives $2x + 9$; $4x - 5 = 2x + 9$
- $2x = 14$, so $x = \mathbf{7}$
- output $4 \times 7 - 5 = \mathbf{23}$

3.5

- $2a + b = 13$ and $5a + b = 25$
- subtract: $3a = 12$, so $\boldsymbol{a = 4}$
- $b = 13 - 8 = \mathbf{5}$