Exercise walk-through

2.3.3 Quadratic Sequences

AQA · GCSE · Mathematics

Straight-line graphs

- Every straight line, except a vertical one, has an equation $y = mx + c$.
- The number m is the **gradient**, the steepness: how far y goes up when x increases by 1.
- The number c is the y-intercept, where the line crosses the y-axis.
- Between two points,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

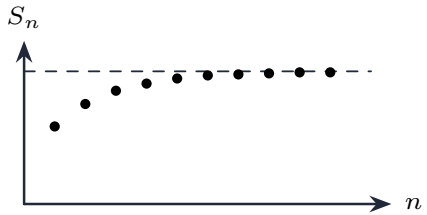
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

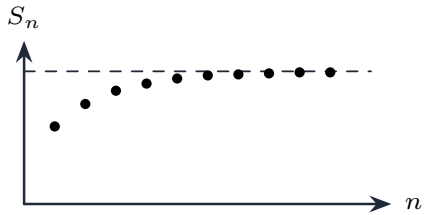
Exercise 2.1

2.1 Work out the first four terms of the sequence with nth term $n^2 - 3$. [2]



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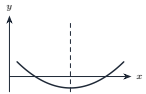


2.1

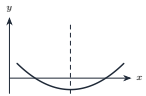
- $1 - 3, 4 - 3, 9 - 3, 16 - 3$
- $\mathbf{-2}, \mathbf{1}, \mathbf{6}, \mathbf{13}$

Exercise 2.2

2.2 Show that $2, 6, 12, 20, 30, \ldots$ is a quadratic sequence. [2]



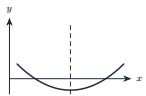
2.3 A quadratic sequence has a second difference of 6. Write down the coefficient of n^2 in its nth term. [1]



2.4 Find the nth term of $2, 8, 18, 32, 50, \ldots$ [2]

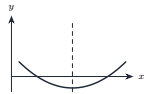


2.5 Find the nth term of $3, 6, 11, 18, 27, \ldots$ [2]



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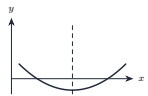
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2.2

- first differences $4, 6, 8, 10$
- second differences $2, 2, 2$: constant, so the sequence is quadratic

2.3

Full solution on next slide

Full solution 2.2

2.2

- first differences $4, 6, 8, 10$
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2.3

- half the second difference: $\mathbf{3}$, so the nth term starts $3n^2$

2.4

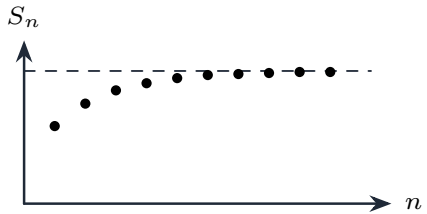
- second difference 4, so $a = 2$; $2n^2$ gives $2, 8, 18, 32, 50$, matching every term
- $\boldsymbol{2n^2}$

2.5

- second difference 2, so n^2; subtract $1, 4, 9, 16, 25$ to leave $2, 2, 2, 2, 2$
- $\boldsymbol{n^2 + 2}$

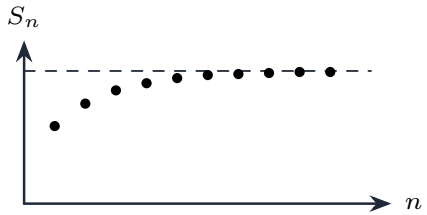
Exercise 3.1

3.1 Ali says the coefficient of n^2 is always equal to the second difference. Use the sequence $1, 4, 9, 16, \ldots$ to show that he is wrong. [1]



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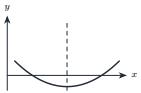


3.1

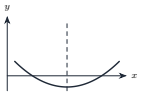
- first differences $3, 5, 7$; second difference 2, but the nth term is n^2, with coefficient **1**: the coefficient is half the second difference

Exercise 3.2

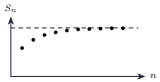
3.2 Find the nth term of $5, 12, 23, 38, 57, \ldots$ [3]



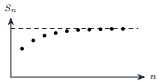
3.3 Find the nth term of $0, 3, 10, 21, 36, \ldots$ [3]



3.4 The nth term of a sequence is $n^2 + 4n$. Show that 140 is a term of the sequence, and state its position. [3]

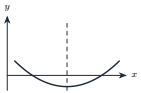


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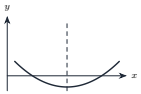


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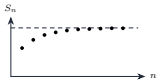
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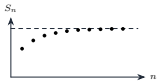
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3.2

- first differences $7, 11, 15, 19$; second difference 4, so $a = 2$
- subtract $2n^2$: $5-2, 12-8, 23-18, 38-32 = 3, 4, 5, 6$, which is $n+2$
- $\mathbf{2n^2 + n + 2}$

3.3

Full solution on next slide

Full solution 3.2 (1/2)

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- first differences $3, 7, 11, 15$; second difference 4, so $a = 2$
- subtract $2n^2$: $0 - 2, 3 - 8, 10 - 18, 21 - 32 = -2, -5, -8, -11$, which is $1 - 3n$
- $\boldsymbol{2n^2 - 3n + 1}$

Full solution 3.2 (2/2)

3.4

- $n^2 + 4n = 140$, so $n^2 + 4n - 140 = 0$
- $(n + 14)(n - 10) = 0$, so $n = 10$ or $n = -14$
- $n = 10$ is a positive whole number, so 140 is the **10th** term

3.5

- common ratio $r = \dfrac{\sqrt{3}}{1} = \sqrt{3}$; the nth term is $(\sqrt{3})^{n-1}$
- 7th term: $(\sqrt{3})^6 = 3^3 = 27$
- 6th term: $(\sqrt{3})^5 = 3^2 \times \sqrt{3} = \mathbf{9\sqrt{3}}$