Exercise walk-through

2.3.2
Sequence Patterns

AQA · GCSE · Mathematics

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inequalities

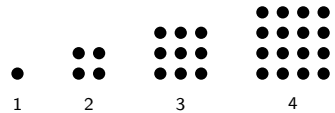
- Solve a linear inequality like an equation, but if you multiply or divide by a negative number, turn the sign round: from $-2x < 6$ you get $x > -3$.
- On a number line, an open circle means the end value is not included ($<$ or $>$) and a filled circle means it is ($\leq$ or $\geq$).
- **Worked example 15.** Find the integers n with $-3 < 2n + 1 \leq 9$.
- Subtract 1: $-4 < 2n \leq 8$.

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

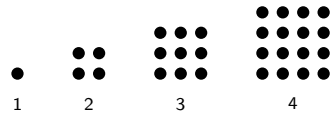
Exercise 2.1

2.1 Write down the next two terms of each sequence. (a) $1, 4, 9, 16, \ldots$ (b) $1, 8, 27, \ldots$ [2]



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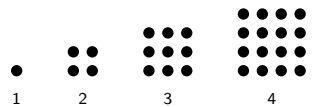
2.1

(a) square numbers: **25**, **36**

(b) cube numbers: **64**, **125**

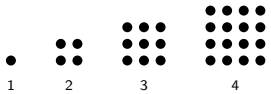
Setup for exercise 2.2

2.2 Find the 7th triangular number. [1]

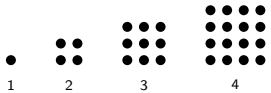


Exercise 2.2

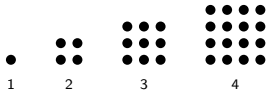
2.3 Write down the next two terms of the Fibonacci-type sequence $2, 3, 5, 8, \ldots$ [1]



2.4 A geometric sequence is $3, 12, 48, \ldots$ Find the common ratio and the next term. [2]

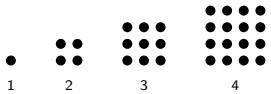


2.5 A geometric sequence is $80, 40, 20, \ldots$ Find the 6th term. [2]

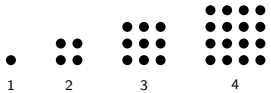


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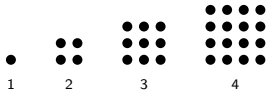
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2.2

- $1 + 2 + 3 + 4 + 5 + 6 + 7 = \mathbf{28}$

Full solution on next slide

Full solution 2.2

2.2

- $1 + 2 + 3 + 4 + 5 + 6 + 7 = \mathbf{28}$

2.3

- $5 + 8 = 13$ and $8 + 13 = 21$: $\mathbf{13}, \mathbf{21}$

2.4

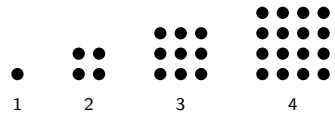
- $r = 12 \div 3 = \mathbf{4}$
- next term $48 \times 4 = \mathbf{192}$

2.5

- $r = 40 \div 80 = \frac{1}{2}$
- $20 \to 10 \to 5 \to 2.5$: the 6th term is $\mathbf{2.5}$

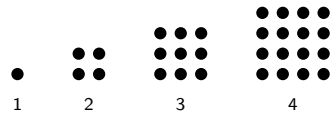
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3.1 Jo says, "The sequence $1, 4, 9, 16, \ldots$ is arithmetic because it is always increasing." Explain why she is wrong. [1]



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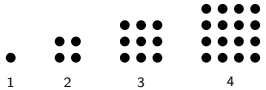
- an arithmetic sequence needs a **constant** difference; here the differences are $3, 5, 7,$ which change

Setup for exercise 3.2

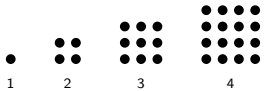
3.2 Here are three numbers: 6, 16, 27. For each one, say whether it is a square number, a cube number or a triangular number. [2]



3.3 The first two terms of a Fibonacci-type sequence are 4 and x. The 5th term is 32. Find x. [3]

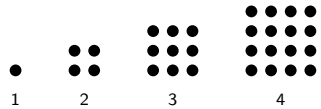


3.4 A ball is dropped from 162 cm. After each bounce it reaches $\frac{2}{3}$ of its previous height. Work out the height it reaches after the 4th bounce. [2]



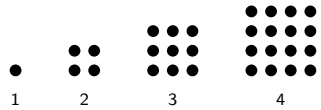
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3.2

- 6 is **triangular** $(1+2+3)$; 16 is **square** (4^2); 27 is **cube** (3^3)

3.3

- terms: $4,\ x,\ 4+x,\ 4+2x,\ 8+3x$
- $8+3x=32$, so $3x=24$
- $\boldsymbol{x=8}$; check: $4, 8, 12, 20, 32$

3.4

Full solution on next slide

Full solution 3.2

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- heights after each bounce: $162\times\frac{2}{3}=108$, then 72, then 48
- after the 4th bounce: $48\times\frac{2}{3}=\mathbf{32}$ cm

3.5

- common ratio: $\dfrac{5k}{5}=\dfrac{45}{5k}$, so $k=\dfrac{9}{k}$
- $k^2=9$
- $\boldsymbol{k=3}$ or $\boldsymbol{k=-3}$