Exercise walk-through

2.3.1 Sequences

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Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inequalities

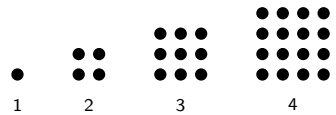
- Solve a linear inequality like an equation, but if you multiply or divide by a negative number, turn the sign round: from $-2x < 6$ you get $x > -3$.
- On a number line, an open circle means the end value is not included ($<$ or $>$) and a filled circle means it is ($\leq$ or $\geq$).
- **Worked example 15.** Find the integers n with $-3 < 2n + 1 \leq 9$.
- Subtract 1: $-4 < 2n \leq 8$.

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

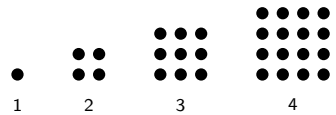
Exercise 2.1

2.1 The first term of a sequence is 4. The term-to-term rule is "multiply by 2, then subtract 1". Write down the next three terms. [2]



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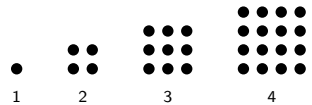


2.1

- $4 \times 2 - 1 = 7$, then $7 \times 2 - 1 = 13$, then $13 \times 2 - 1 = 25$
- $\mathbf{7}, \mathbf{13}, \mathbf{25}$

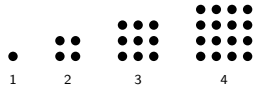
Setup for exercise 2.2

2.2 The nth term of a sequence is $5n - 3$. Work out the first three terms and the 10th term. [2]

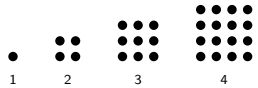


Exercise 2.2

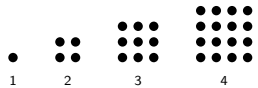
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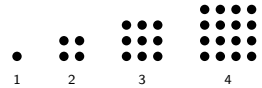


2.5 The nth term of a sequence is $4n + 1$. Is 81 a term of the sequence? Show how you decide. [2]

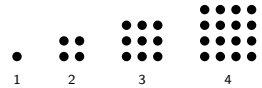


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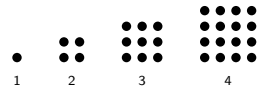
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2.2

- $n = 1, 2, 3$: $5 - 3 = 2$, $10 - 3 = 7$, $15 - 3 = 12$, so $\mathbf{2}, \mathbf{7}, \mathbf{12}$

Full solution on next slide

Full solution 2.2

2.2

- $n = 1, 2, 3$: $5 - 3 = 2$, $10 - 3 = 7$, $15 - 3 = 12$, so $\mathbf{2}, \mathbf{7}, \mathbf{12}$
- $n = 10$: $5 \times 10 - 3 = \mathbf{47}$

2.3

- difference 4, so $4n$; zero term $7 - 4 = 3$
- $\boldsymbol{4n + 3}$

2.4

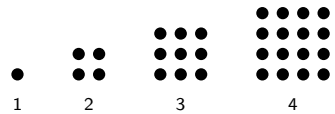
- difference -3, so $-3n$; zero term $20 + 3 = 23$
- $\boldsymbol{23 - 3n}$

2.5

- $4n + 1 = 81$ gives $4n = 80$
- $n = 20$, a whole number, so **yes**: 81 is the 20th term

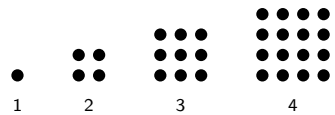
Exercise 3.1

3.1 Here are the first four terms of a sequence: $3, 9, 15, 21$. Zoe says the nth term is $n+6$. Explain why she is wrong and give the correct nth term. [2]



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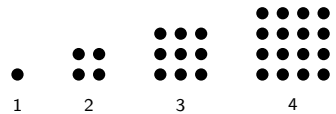
3.1

- $n + 6$ gives $7, 8, 9, \ldots$; the difference is 6, so the nth term must start $6n$
- zero term $3 - 6 = -3$: $\boldsymbol{6n - 3}$

Exercise 3.2 (a)

3.2 Patterns are made from sticks. Pattern 1 uses 4 sticks, pattern 2 uses 7 sticks and pattern 3 uses 10 sticks.

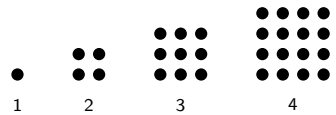
(a) Find an expression for the number of sticks in pattern n. [2]



Exercise 3.2 (a)

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(a) Find an expression for the number of sticks in pattern n. [2]



(a)

- difference 3, so $3n$; zero term $4 - 3 = 1$
- $\boldsymbol{3n + 1}$ sticks

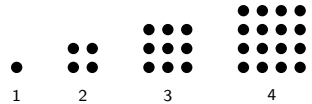
Setup for exercise 3.2 (1/2)

3.2 Patterns are made from sticks. Pattern 1 uses 4 sticks, pattern 2 uses 7 sticks and pattern 3 uses 10 sticks.

Setup for exercise 3.2 (2/2)

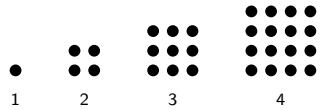
(b) Sam has 150 sticks. What is the largest pattern he can make? [2]

3.3 The nth term of a sequence is $6n - 4$. Find the first term that is greater than 100. [2]

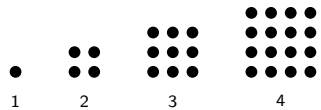


Exercise 3.2 (b)

3.4 Two sequences have nth terms $3n+10$ and $5n-2$. Which term number has the same value in both sequences, and what is that value? [3]

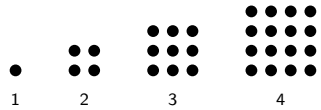


3.5 An arithmetic sequence has 3rd term 17 and 7th term 37. Find its nth term. [3]

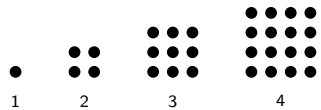


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(b)

- $3n + 1 \leq 150$ gives $3n \leq 149$, so $n \leq 49.6\ldots$

Full solution on next slide

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Full solution 3.2 (b)

(b)

- $3n + 1 \leq 150$ gives $3n \leq 149$, so $n \leq 49.6\ldots$
- the largest pattern is **pattern 49**, using 148 sticks

3.3

- $6n - 4 > 100$ gives $6n > 104$, so $n > 17.3\ldots$, so $n = 18$
- $6 \times 18 - 4 =$ **104**

3.4

- set them equal: $3n + 10 = 5n - 2$
- $12 = 2n$, so $n = 6$
- both give $3 \times 6 + 10 = 28$: the **6th** term, value **28**

3.5

- from the 3rd to the 7th term is 4 differences: $4d = 37 - 17 = 20$, so $d = 5$
- 3rd term $= 3 \times 5 + c = 17$, so $c = 2$
- $\boldsymbol{5n + 2}$