Exercise walk-through

2.2
Equations, inequalities, formulae and rearranging (A5–A7)

AQA · GCSE · Mathematics

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inequalities

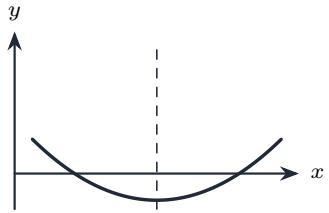
- Solve a linear inequality like an equation, but if you multiply or divide by a negative number, turn the sign round: from $-2x < 6$ you get $x > -3$.
- On a number line, an open circle means the end value is not included ($<$ or $>$) and a filled circle means it is ($\leq$ or $\geq$).
- **Worked example 15.** Find the integers n with $-3 < 2n + 1 \leq 9$.
- Subtract 1: $-4 < 2n \leq 8$.

Sequences

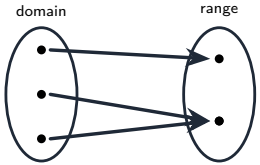
- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

Exercises 2.1·2.2

2.1 For $f(x) = 3x - 5$ and $g(x) = x^2$, find fg(2) [2]

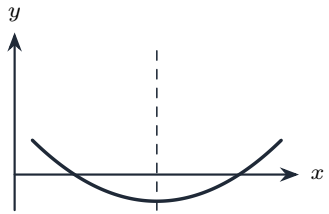


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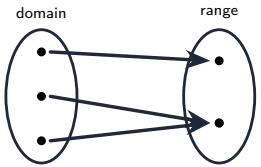
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2.1 $g(2) = 4$; $f(4) = 7$.

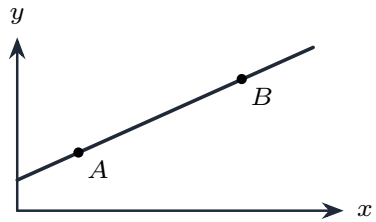
2.2 Find $f^{-1}(x)$ for $f(x) = 3x - 5$ [2]



2.2 $y = 3x - 5 \Rightarrow x = \frac{y+5}{3}$, so $f^{-1}(x) = \frac{x+5}{3}$.

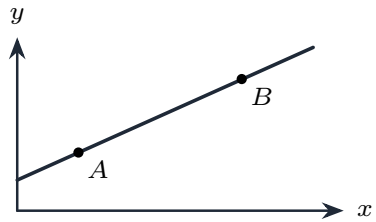
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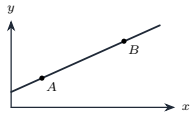
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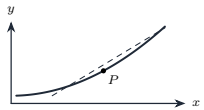
2.3 $m = 2$, so $y = 2x + 5$.

Exercise 2.4

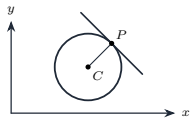
2.4 Write down the gradient of the line perpendicular to $m = \frac{2}{5}$ [1]



2.5 Write $y = x^2 - 6x + 5$ in the form $(x + a)^2 + b$ and state the minimum point [3]

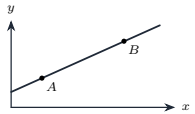


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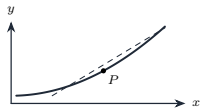


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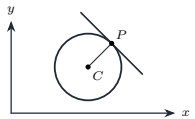
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2.5 $(x - 3)^2 - 4$; minimum $(3, -4)$.

2.6 Radius gradient $4/3$; tangent $-3/4$.

Full solution 2.4

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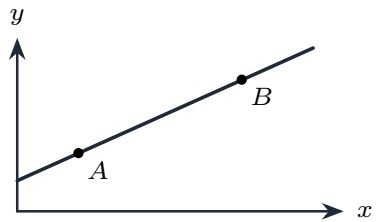
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Exercise 3.1

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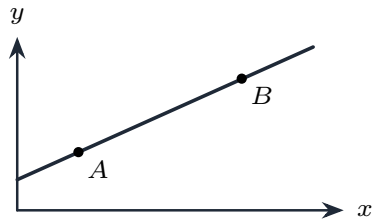
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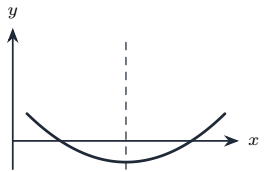
3.1 (a) $m = \frac{11-3}{3-(-1)} = 2$, so $y = 2x + 5$. (b) Midpoint $(1, 7)$; perpendicular gradient $-\frac{1}{2}$: $y - 7 = -\frac{1}{2}(x - 1)$, so $y = -\frac{1}{2}x + \frac{15}{2}$.

Exercise 3.2

3.2 *(Original.)* Sketch or describe $y = 2^x$, giving the y-intercept and the behaviour as x grows and as $x \to -\infty$. [3]

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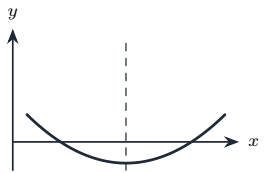


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3.2 Passes $(0, 1)$; rises ever faster as x grows; approaches the x-axis (never 0) as $x \to -\infty$.

3.3 (a) $(x + 3)^2 - 7$. (b) $(x + 3)^2 = 7$, so $x = -3 \pm \sqrt{7}$.

Local review guidance: gradient then point-slope form; exact roots; graph descriptions name intercept AND end behaviour. This is authored classroom guidance.