Exercise walk-through

2.2.3 Inequality Regions

AQA · GCSE · Mathematics

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inequalities

- Solve a linear inequality like an equation, but if you multiply or divide by a negative number, turn the sign round: from $-2x < 6$ you get $x > -3$.
- On a number line, an open circle means the end value is not included ($<$ or $>$) and a filled circle means it is ($\leq$ or $\geq$).
- **Worked example 15.** Find the integers n with $-3 < 2n + 1 \leq 9$.
- Subtract 1: $-4 < 2n \leq 8$.

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

Exercise 2.1

2.1 Solve $(x-3)(x-7) \leq 0$. [2]

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2.1

- roots $x = 3$ and $x = 7$; ≤ 0 means between the roots
- $\mathbf{3 \leq x \leq 7}$

Setup for exercise 2.2

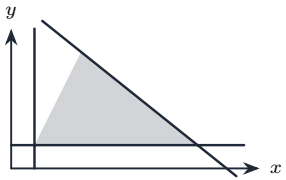
2.2 Solve $x^2 - 16 > 0$. [2]



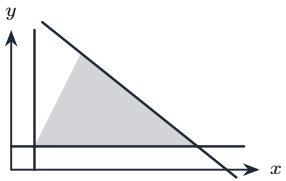
2.3 Solve $x^2 + 2x - 15 < 0$. [3]

Exercise 2.2

2.4 Does the point $(2, 4)$ satisfy both $y > 2x$ and $x + y \leq 6$? Show how you decide. [2]

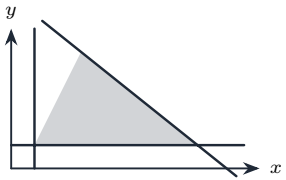


2.5 A region is bounded by the lines $x = 1$ (solid), $y = 3$ (dashed) and $y = x$ (solid). The point $(2, 2.5)$ lies inside it. Write down the three inequalities that describe the region. [3]

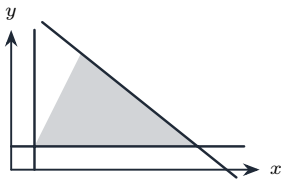


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2.2

- $(x - 4)(x + 4) > 0$: roots ± 4; > 0 means outside the roots

Full solution on next slide

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Full solution 2.2

2.2

- $(x-4)(x+4) > 0$: roots ± 4; > 0 means outside the roots
- $\boldsymbol{x < -4}$ or $\boldsymbol{x > 4}$

2.3

- factorise: $(x+5)(x-3) < 0$
- roots $x = -5$ and $x = 3$; < 0 means between them
- $\boldsymbol{-5 < x < 3}$

2.4

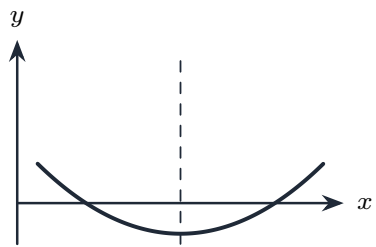
- $y > 2x$: $4 > 2 \times 2 = 4$ is **false**
- so the point does **not** satisfy both; it lies on the dashed line $y = 2x$

2.5

- $x = 1$ solid and $(2, 2.5)$ is to the right: $\boldsymbol{x \geq 1}$
- $y = 3$ dashed and $2.5 < 3$: $\boldsymbol{y < 3}$
- $y = x$ solid and $2.5 > 2$: $\boldsymbol{y \geq x}$

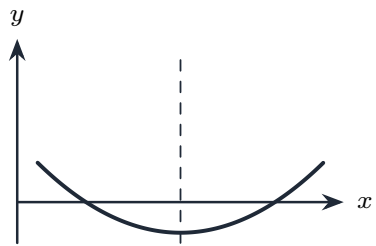
Exercise 3.1

3.1 Amir says that $x^2 > 9$ means $x > 3$. Explain why his answer is incomplete and give the full solution. [2]



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3.1

- $x^2 > 9$ is also true for negative numbers such as $x = -4$, since $(-4)^2 = 16$
- $\boldsymbol{x < -3}$ or $\boldsymbol{x > 3}$

Setup for exercise 3.2

3.2 Solve $2x^2 - 5x - 3 \geq 0$. [3]

Exercise 3.2

3.3 Sketch, on axes, the region where $x \geq 0$, $y \geq 0$ and $2x + y < 6$. Use solid and dashed lines correctly and label the region R. [3]

3.4 n is an integer with $n^2 < 20$. Find all possible values of n. [2]



3.5 Find the integer points (x, y) with $x > 0$, $y > 0$ and $x + 2y \leq 6$. [3]

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3.2

- factorise: $(2x + 1)(x - 3) \geq 0$
- roots $x = -\frac{1}{2}$ and $x = 3$; ≥ 0 means outside, roots included

Full solution on next slide

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Full solution 3.2

3.2

- factorise: $(2x+1)(x-3) \geq 0$
- roots $x = -\frac{1}{2}$ and $x = 3$; ≥ 0 means outside, roots included
- $\boldsymbol{x \leq -\frac{1}{2}}$ or $\boldsymbol{x \geq 3}$

3.3

- solid lines along the axes $x = 0$ and $y = 0$
- dashed line $2x + y = 6$ through $(3, 0)$ and $(0, 6)$
- R is the triangle between the axes and the dashed line; test $(0, 0)$: $0 < 6$ is true

3.4

- $n^2 < 20$ means $-\sqrt{20} < n < \sqrt{20}$, and $\sqrt{20} \approx 4.47$
- $\boldsymbol{-4, -3, -2, -1, 0, 1, 2, 3, 4}$

3.5

- $y = 1$: $x \leq 4$, so $x = 1, 2, 3, 4$; $y = 2$: $x \leq 2$, so $x = 1, 2$; $y = 3$: $x \leq 0$, none
- $\boldsymbol{(1,1), (2,1), (3,1), (4,1), (1,2), (2,2)}$