Exercise walk-through

2.1.5
Algebraic Fractions

AQA · GCSE · Mathematics

Straight-line graphs

- Every straight line, except a vertical one, has an equation $y = mx + c$.
- The number m is the **gradient**, the steepness: how far y goes up when x increases by 1.
- The number c is the y-intercept, where the line crosses the y-axis.
- Between two points,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

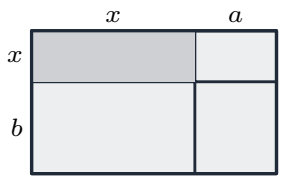
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

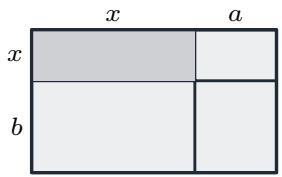
Exercise 2.1

2.1 Simplify $\dfrac{6x^2}{9x}$. [1]



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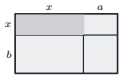
- cancel the common factor $3x$: $\dfrac{6x^2}{9x} = \dfrac{\mathbf{2x}}{\mathbf{3}}$

Exercise 2.2

2.2 Simplify $\dfrac{4x+12}{x^2+3x}$. [2]



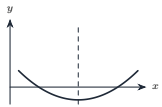
2.3 Simplify $\dfrac{x^2+5x+6}{x^2-4}$. [3]



2.4 Write $\dfrac{x}{3}+\dfrac{x+1}{4}$ as a single fraction. [2]



2.5 Work out $\dfrac{x+2}{5}\times\dfrac{10}{x^2+2x}$. Give your answer in its simplest form. [2]

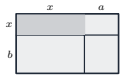


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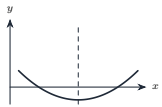
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2.2

- factorise: $\dfrac{4(x+3)}{x(x+3)}$
- cancel $(x+3)$: $\boldsymbol{\dfrac{4}{x}}$

Full solution on next slide

Full solution 2.2 (1/2)

2.2

- factorise: $\dfrac{4(x+3)}{x(x+3)}$
- cancel $(x+3)$: $\boldsymbol{\dfrac{4}{x}}$

2.3

- top: $x^2+5x+6=(x+2)(x+3)$
- bottom: $x^2-4=(x-2)(x+2)$

Full solution 2.2 (2/2)

- cancel $(x+2)$: $\boldsymbol{\dfrac{x+3}{x-2}}$

2.4

- common denominator 12: $\dfrac{4x}{12} + \dfrac{3(x+1)}{12}$
- $\dfrac{4x+3x+3}{12} = \boldsymbol{\dfrac{7x+3}{12}}$

2.5

- factorise: $\dfrac{x+2}{5} \times \dfrac{10}{x(x+2)}$
- cancel $(x+2)$ and 5: $\boldsymbol{\dfrac{2}{x}}$

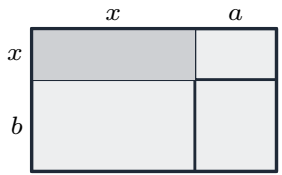
Exercise 3.1

3.1 Ravi simplifies $\dfrac{x+6}{x+2}$ to $\dfrac{6}{2} = 3$. Explain why he is wrong. [1]



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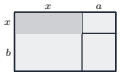
- x and 6 are **terms** added together, not factors: only a factor of the whole top and whole bottom can cancel; at $x=1$ the fraction is $\frac{7}{3}$, not 3

Exercise 3.2

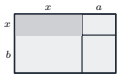
3.2 Write $\dfrac{3}{x+2} - \dfrac{2}{x-1}$ as a single fraction in its simplest form. [3]



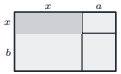
3.3 Simplify $\dfrac{x^2-1}{x^2+3x+2} \div \dfrac{x-1}{x+2}$. [3]



3.4 Simplify fully $\dfrac{2x^2-5x-3}{4x^2-1}$. [3]



3.5 Solve $\dfrac{3}{x-2} + \dfrac{1}{x} = 2$. [4]

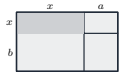


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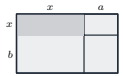
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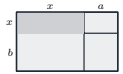
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3.2

- common denominator $(x+2)(x-1)$: $\dfrac{3(x-1)-2(x+2)}{(x+2)(x-1)}$
- top: $3x-3-2x-4 = x-7$

Full solution on next slide

Full solution 3.2 (1/2)

3.2

- common denominator $(x+2)(x-1)$: $\dfrac{3(x-1)-2(x+2)}{(x+2)(x-1)}$
- top: $3x-3-2x-4=x-7$
- single fraction: $\boldsymbol{\dfrac{x-7}{(x+2)(x-1)}}$

3.3

- multiply by the reciprocal: $\dfrac{x^2-1}{x^2+3x+2}\times\dfrac{x+2}{x-1}$
- factorise: $\dfrac{(x-1)(x+1)}{(x+1)(x+2)}\times\dfrac{x+2}{x-1}$
- every factor cancels: $\mathbf{1}$ (for $x\neq 1,-1,-2$)

Full solution 3.2 (2/2)

3.4

- top: $2x^2 - 5x - 3 = (2x + 1)(x - 3)$
- bottom: $4x^2 - 1 = (2x - 1)(2x + 1)$
- cancel $(2x + 1)$: $\boldsymbol{\dfrac{x - 3}{2x - 1}}$

3.5

- multiply every term by $x(x - 2)$: $3x + (x - 2) = 2x(x - 2)$
- $4x - 2 = 2x^2 - 4x$, so $2x^2 - 8x + 2 = 0$, that is $x^2 - 4x + 1 = 0$
- quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$
- $\boldsymbol{x = 2 + \sqrt{3}}$ or $\boldsymbol{x = 2 - \sqrt{3}}$; neither is 0 or 2, so both are allowed