Exercise walk-through

# 2.1.4
# Algebra Argument

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# Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

# Inequalities

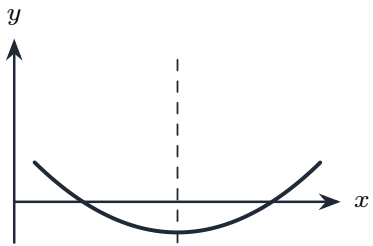
- Solve a linear inequality like an equation, but if you multiply or divide by a negative number, turn the sign round: from $-2x < 6$ you get $x > -3$.
- On a number line, an open circle means the end value is not included ($<$ or $>$) and a filled circle means it is ($\leq$ or $\geq$).
- **Worked example 15.** Find the integers $n$ with $-3 < 2n + 1 \leq 9$.
- Subtract 1: $-4 < 2n \leq 8$.

# Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position $n$.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

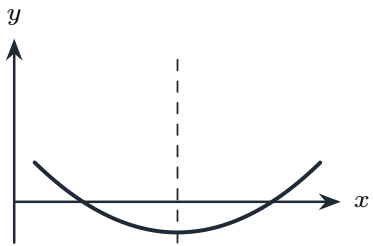
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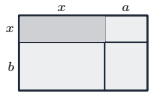



**2.1**

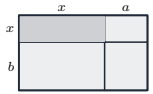
- an **equation**: it is true only for $x = 0$ and $x = 1$; at $x = 2$, $4 \neq 2$

# Exercise 2.2

**2.2** Show that $5(x+2)-3x \equiv 2(x+5)$. [2]




**2.3** Show that $(x+3)^2-9 \equiv x(x+6)$. [2]




**2.4** Write down an expression for (a) any even number (b) the next even number after it. [2]

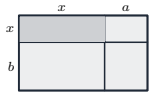
assume → deduce → conclude

**2.5** Find a value of $n$ to show that this statement is false: "$n^2+1$ is always even." [1]

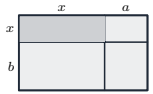
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**2.2**

- left: $5x + 10 - 3x = 2x + 10$
- right: $2(x+5) = 2x + 10$, so the two sides are the same for every $x$

Full solution on next slide

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# Full solution 2.2

**2.2**

- left: $5x + 10 - 3x = 2x + 10$
- right: $2(x + 5) = 2x + 10$, so the two sides are the same for every $x$

**2.3**

- $(x + 3)^2 = x^2 + 6x + 9$, so $(x + 3)^2 - 9 = x^2 + 6x$
- factorise: $x^2 + 6x = x(x + 6)$

**2.4**

(a) $\boldsymbol{2n}$

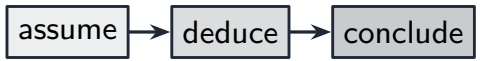
(b) $\boldsymbol{2n + 2}$

**2.5**

- e.g. $n = 2$: $2^2 + 1 = 5$, which is odd

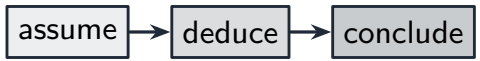
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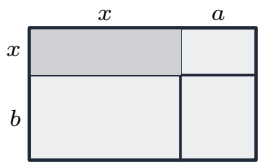



**3.1**

- e.g. $n = 4$: $2 \times 4 + 1 = 9 = 3 \times 3$, which is not prime

# Setup for exercise 3.2

**3.2** Find the value of $a$ such that $4(x-3)+2(x+a) \equiv 6x-2$. [3]



# Exercise 3.2

**3.3** *(H)* Prove that the sum of three consecutive integers is a multiple of 3. [3]

assume → deduce → conclude

**3.4** *(H)* Prove that the sum of any two odd numbers is even. [3]

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**3.5** *(H)* Prove that $(n+5)^2 - (n+3)^2$ is a multiple of 4 for every integer $n$. [3]

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**3.2**

- expand the left: $4x - 12 + 2x + 2a = 6x + (2a - 12)$

Full solution on next slide

# Full solution 3.2 (1/2)

**3.2**

- expand the left: $4x - 12 + 2x + 2a = 6x + (2a - 12)$
- match the constants on both sides: $2a - 12 = -2$
- $2a = 10$, so $\boldsymbol{a = 5}$

**3.3**

- let the integers be $n$, $n + 1$ and $n + 2$
- sum: $n + (n + 1) + (n + 2) = 3n + 3$
- $3n + 3 = 3(n + 1)$, and $n + 1$ is an integer, so the sum is a multiple of 3

# Full solution 3.2 (2/2)

**3.4**

- let the odd numbers be $2m+1$ and $2n+1$, where $m$ and $n$ are integers: two **different** letters, so the numbers need not be equal
- sum: $(2m+1)+(2n+1) = 2m+2n+2 = 2(m+n+1)$, which is $2 \times$ (an integer), so it is even

**3.5**

- $(n+5)^2 = n^2+10n+25$ and $(n+3)^2 = n^2+6n+9$
- difference: $(n^2+10n+25)-(n^2+6n+9) = 4n+16$
- $4n+16 = 4(n+4)$, and $n+4$ is an integer, so it is a multiple of 4