Exercise walk-through

2.1.3 Manipulation

AQA · GCSE · Mathematics

Solving equations

- **Linear equations.** Collect the letters on one side and the numbers on the other. **Worked example 8.** $7x - 22 = 4x + 29$.
- Subtract $4x$: $3x - 22 = 29$.
- Add 22: $3x = 51$, so $x = 17$.
- **Quadratic equations by factorising.** Make one side zero and factorise.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inequalities

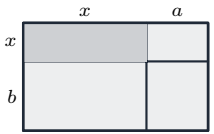
- Solve a linear inequality like an equation, but if you multiply or divide by a negative number, turn the sign round: from $-2x < 6$ you get $x > -3$.
- On a number line, an open circle means the end value is not included ($<$ or $>$) and a filled circle means it is ($\leq$ or $\geq$).
- **Worked example 15.** Find the integers n with $-3 < 2n + 1 \leq 9$.
- Subtract 1: $-4 < 2n \leq 8$.

Sequences

- A sequence is a list of numbers that follows a rule.
- A term-to-term rule says how to get the next term; a position-to-term rule, the **nth term** , gives any term from its position n.
- You should know the square numbers (1, 4, 9, 16, ...), cube numbers (1, 8, 27, ...), triangular numbers (1, 3, 6, 10, ...) and Fibonacci-type sequences, where each term is the sum of the two before.
- **Arithmetic sequences.** An **arithmetic progression** adds the same number each time.

Exercises 2.1·2.2

2.1 Simplify $5a + 3b - 2a + b$. [2]

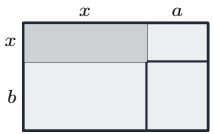


2.2 Simplify $3a \times 4ab$. [1]



Exercises 2.1·2.2

2.1 Simplify $5a + 3b - 2a + b$. [2]



2.1

- collect the a-terms and the b-terms: $5a - 2a = 3a$ and $3b + b = 4b$
- $\mathbf{3a + 4b}$

2.2 Simplify $3a \times 4ab$. [1]

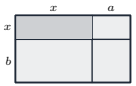


2.2

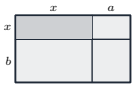
- numbers: $3 \times 4 = 12$; letters: $a \times a \times b = a^2b$, so $\mathbf{12a^2b}$

Exercise 2.3

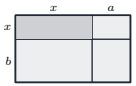
2.3 Expand and simplify $3(x+4)-2(x-1)$. [2]



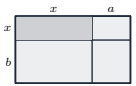
2.4 Factorise fully $8x^2-12x$. [2]



2.5 Expand and simplify $(x+5)(x-2)$. [2]

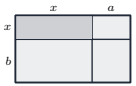


2.6 Factorise (a) $x^2+7x+12$ (b) y^2-49 [3]

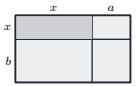


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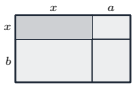
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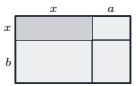
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2.3

- $3x+12-2x+2$: the -2 multiplies -1 to give $+2$
- $\boldsymbol{x+14}$

Full solution on next slide

Full solution 2.3

2.3

- $3x + 12 - 2x + 2$: the -2 multiplies -1 to give $+2$
- $\boldsymbol{x + 14}$

2.4

- HCF of $8x^2$ and $12x$ is $4x$
- $\boldsymbol{4x(2x - 3)}$

2.5

- $x^2 - 2x + 5x - 10$
- $\boldsymbol{x^2 + 3x - 10}$

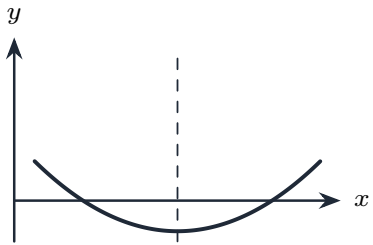
2.6

(a) $3 \times 4 = 12$ and $3 + 4 = 7$: $\boldsymbol{(x + 3)(x + 4)}$

(b) difference of two squares, $49 = 7^2$: $\boldsymbol{(y - 7)(y + 7)}$

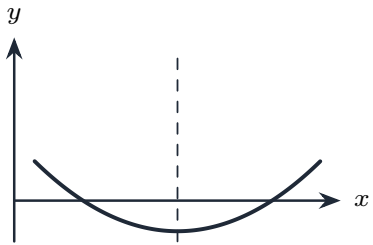
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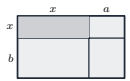


3.1

- **No**: x^2 and x are unlike terms, so they cannot be collected; for example at $x = 2$, $x^2 + x = 6$ but $x^3 = 8$

Exercise 3.2

3.2 Expand and simplify $(2x - 3)^2$. [2]



3.3 A rectangle has length $(x + 6)$ cm and width $(x - 1)$ cm. A square has side x cm. Show that the rectangle's area is $(5x - 6)$ cm^2 more than the square's. [3]



3.4 *(H)* Factorise $3x^2 + 10x - 8$. [2]

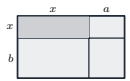


3.5 *(H)* Simplify fully $\dfrac{x^2 - 9}{x^2 + x - 6}$. [3]



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3.2

- $(2x-3)(2x-3) = 4x^2-6x-6x+9$
- $\mathbf{4x^2-12x+9}$

3.3

Full solution on next slide

Full solution 3.2 (1/2)

3.2

- $(2x-3)(2x-3) = 4x^2 - 6x - 6x + 9$
- $\mathbf{4x^2 - 12x + 9}$

3.3

- rectangle: $(x+6)(x-1) = x^2 - x + 6x - 6 = x^2 + 5x - 6$
- square: x^2
- difference: $(x^2 + 5x - 6) - x^2 = 5x - 6$

Full solution 3.2 (2/2)

3.4

- $a \times c = 3 \times (-8) = -24$; two numbers multiplying to -24 and adding to 10: 12 and -2
- $3x^2 + 12x - 2x - 8 = 3x(x+4) - 2(x+4) = \mathbf{(3x-2)(x+4)}$

3.5

- top: $x^2 - 9 = (x-3)(x+3)$
- bottom: $x^2 + x - 6 = (x+3)(x-2)$
- cancel the common factor $(x+3)$: $\dfrac{\boldsymbol{x-3}}{\boldsymbol{x-2}}$