

Course notes

AQA · GCSE · Mathematics

8300: course teaching notes

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These are original teaching notes, and each lesson is labelled with its tier.

3.1 · Exact arithmetic and estimation · Foundation

Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor**: A prime number that divides the integer exactly.

The method

- Prime factors reveal shared structure.
- Use the smallest common prime powers for the HCF and the largest for the LCM.
- Estimate before calculating; use brackets to preserve the order of operations.

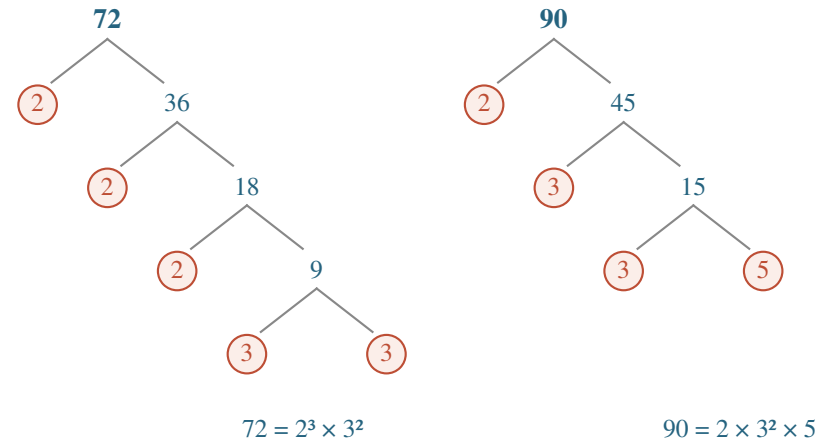
A worked case

$$72 = 2^3 \times 3^2, \quad 90 = 2 \times 3^2 \times 5, \quad \text{HCF} = 18$$

■ Worked example

$72 = 2^3 \times 3^2$ and $90 = 2 \times 3^2 \times 5$. Their HCF is $2 \times 9 = 18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5 = 360$. A prime integer is greater than 1 and has exactly two positive factors. The number 1 is not prime; a composite positive integer greater than 1 has more than two positive factors. Every integer greater than 1 has a unique product of prime factors apart from their order. For example, $60 = 2^2 \times 3 \times 5$. A factor divides a number; a multiple is the result of multiplying it by an integer. HCF uses common smallest powers, while LCM uses largest powers.

Factor trees: $\text{HCF}(72, 90) = 2 \times 3^2 = 18$



Where it goes wrong

- The HCF divides both numbers; the LCM is a multiple of both.
- They answer different questions.
- A decimal estimate is not an exact fraction.

■ Watch out

Not true: The HCF of two positive integers is larger than either integer. The HCF divides both numbers, so it can never be larger than the smaller one: the HCF of 72 and 90 is 18.

Applying it

- For a non-calculator paper, keep fractions exact and show cancellation.
- For a calculator paper, enter the full expression and compare with your estimate.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.1.

■ Key idea

- **Prime factor:** A prime number that divides the integer exactly.
- Prime factors reveal shared structure.
- Watch for: the HCF divides both numbers; the LCM is a multiple of both.

3.1 · Exact arithmetic and estimation · Higher

Can we pack without leftovers?

- A supplier packs 72 pencils and 90 pens into identical gift bags. How can we avoid leftovers?
- This lesson studies **prime factor**: A prime number that divides the integer exactly.

The method

- Prime factors reveal shared structure.
- Use the smallest common prime powers for the HCF and the largest for the LCM.
- Estimate before calculating; use brackets to preserve the order of operations.

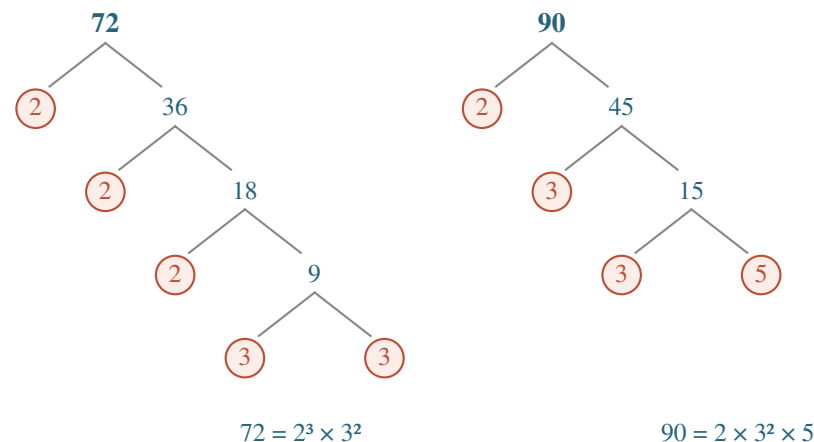
A worked case

$$a = \prod p_i^{\alpha_i}, \quad b = \prod p_i^{\beta_i}, \quad \text{HCF}(a, b) = \prod p_i^{\min(\alpha_i, \beta_i)}$$

■ Worked example

$72=2^3 \times 3^2$ and $90=2 \times 3^2 \times 5$. Their HCF is $2 \times 9=18$. Make 18 bags with 4 pencils and 5 pens each. Their LCM is $2^3 \times 3^2 \times 5=360$. A prime integer is greater than 1 and has exactly two positive factors. The number 1 is not prime; a composite positive integer greater than 1 has more than two positive factors. Every integer greater than 1 has a unique product of prime factors apart from their order. For example, $60=2^2 \times 3 \times 5$. A factor divides a number; a multiple is the result of multiplying it by an integer. HCF uses common smallest powers, while LCM uses largest powers.

Factor trees: $\text{HCF}(72, 90) = 2 \times 3^2 = 18$



Where it goes wrong

- The HCF divides both numbers; the LCM is a multiple of both.
- They answer different questions.
- A decimal estimate is not an exact fraction.

■ Watch out

Not true: The HCF of two positive integers is larger than either integer. The HCF divides both numbers, so it can never be larger than the smaller one: the HCF of 72 and 90 is 18.

Applying it

- For a non-calculator paper, keep fractions exact and show cancellation.
- For a calculator paper, enter the full expression and compare with your estimate.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.1.

■ Key idea

- **Prime factor:** A prime number that divides the integer exactly.
- Prime factors reveal shared structure.
- Watch for: the HCF divides both numbers; the LCM is a multiple of both.

3.1 · Integer indices and standard form · Foundation

How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
- This lesson studies **index** : The power to which a base is raised.

The method

- Use integer powers, square and cube roots and standard form with $1 \leq a < 10$.
- In multiplying powers with the same base, add indices; in division, subtract them.

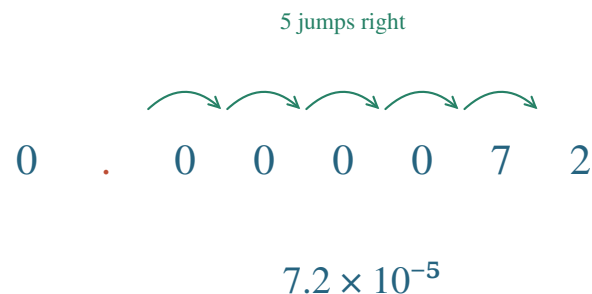
A worked case

$$a^m a^n = a^{m+n}$$

■ Worked example

$0.000072 = 7.2 \times 10^{-5}$. Also $2^3 \times 2^4 = 2^7 = 128$. The square root of 81 is 9. Check a standard-form answer by writing it out as a decimal. Useful powers include $3^3 = 27$, $4^3 = 64$, $5^3 = 125$, $15^2 = 225$ and $10^6 = 1,000,000$. Integer laws give $2^0 = 1$, $2^{-3} = 1/8$ and $(2^3)^2 = 2^6 = 64$. For standard-form multiplication, $(3 \times 10^5)(4 \times 10^{-3}) = 12 \times 10^2 = 1.2 \times 10^3$. Division gives $(6 \times 10^5)/(2 \times 10^2) = 3 \times 10^3$. For addition, first align exponents: $3 \times 10^4 + 2 \times 10^3 = 3.2 \times 10^4$.

$0.000072 = 7.2 \times 10^{-5}$: the point moves 5 places



coefficient 7.2 is between 1 and 10; the power is -5

Where it goes wrong

- Index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 .
- Do not round a surd when an exact answer is requested.

■ Watch out

Not true: For every positive a , $a^2 + a^3$ equals a^5 . Index laws only add powers when the terms are multiplied: $2^2 + 2^3 = 12$, but $2^5 = 32$.

Applying it

- This Foundation/Core lesson excludes fractional powers and surd rationalisation.
- Estimate a result before using a calculator and retain the required precision.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.1.

■ Key idea

- **Index:** The power to which a base is raised.
- Use integer powers, square and cube roots and standard form with $1 \leq a < 10$.
- Watch for: index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 .

3.1 · Indices, surds and standard form · Higher

How small is a microscopic length?

- A microscope records a length of 0.000072 metres. A compact representation must keep its size correct.
- This lesson studies **index** : The power to which a base is raised.

The method

- For the same positive base, multiplication adds indices and division subtracts them.
- A negative index means reciprocal; a fractional index represents a root.
- Standard form has $1 \leq a < 10$.

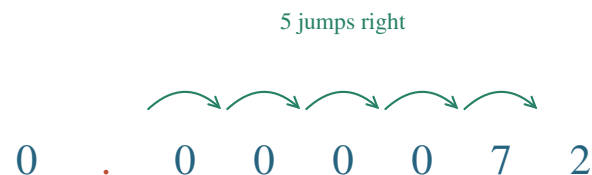
A worked case

$$a^m a^n = a^{m+n}, \quad a^{-n} = \frac{1}{a^n}, \quad a^{m/n} = (\sqrt[n]{a})^m$$

■ Worked example

$0.000072 = 7.2 \times 10^{-5}$. Also $16^{3/4} = (16^{1/4})^3 = 2^3 = 8$. Simplify $\sqrt{72} = 6\sqrt{2}$, then rationalise $1/\sqrt{2} = \sqrt{2}/2$. Useful powers include $3^3 = 27$, $4^3 = 64$, $5^3 = 125$, $15^2 = 225$ and $10^6 = 1,000,000$. Integer laws give $2^0 = 1$, $2^{-3} = 1/8$ and $(2^3)^2 = 2^6 = 64$. For standard-form multiplication, $(3 \times 10^5)(4 \times 10^{-3}) = 12 \times 10^2 = 1.2 \times 10^3$. Division gives $(6 \times 10^5)/(2 \times 10^2) = 3 \times 10^3$. For addition, first align exponents: $3 \times 10^4 + 2 \times 10^3 = 3.2 \times 10^4$. For a positive base, $a^{(m/n)} = (\text{the } n\text{th root of } a)^m$. Thus $27^{2/3} = 3^2 = 9$ and $16^{(-1/2)} = 1/4$. To estimate $\sqrt{20}$ without a calculator, $4^2 < 20 < 5^2$ gives $4 < \sqrt{20} < 5$; testing $4.5^2 = 20.25$ shows $\sqrt{20}$ is just below 4.5. Do not use a rough estimate as an exact surd answer.

$0.000072 = 7.2 \times 10^{-5}$: the point moves 5 places



$$7.2 \times 10^{-5}$$

coefficient 7.2 is between 1 and 10; the power is -5

Where it goes wrong

- Index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 .
- Do not round a surd when an exact answer is requested.

■ Watch out

Not true: For every positive a , $a^2 + a^3$ equals a^5 . Index laws only add powers when the terms are multiplied: $2^2 + 2^3 = 12$, but $2^5 = 32$.

Applying it

- Check powers of ten against the original quantity.
- Use surds for exact geometry, and round only the final length when the question asks for a decimal.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.1.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Index:** The power to which a base is raised.
- For the same positive base, multiplication adds indices and division subtracts them.
- Watch for: index laws do not turn a sum into a single power: $2^3 + 2^4 = 24$, not 2^7 .

3.1 · Accuracy, bounds and compound measures · Higher

Is the printed measurement exact?

- A rectangular panel is labelled 8.0 cm by 5.0 cm, each to the nearest 0.1 cm. Its true area is not fixed at 40 cm^2 .
- This lesson studies **lower bound**: The smallest possible value consistent with a stated rounding rule.

The method

- A value rounded to the nearest unit u lies from stated value $-u/2$ up to, but usually not including, stated value $+u/2$.
- For positive quantities, combine extremes according to the operation.

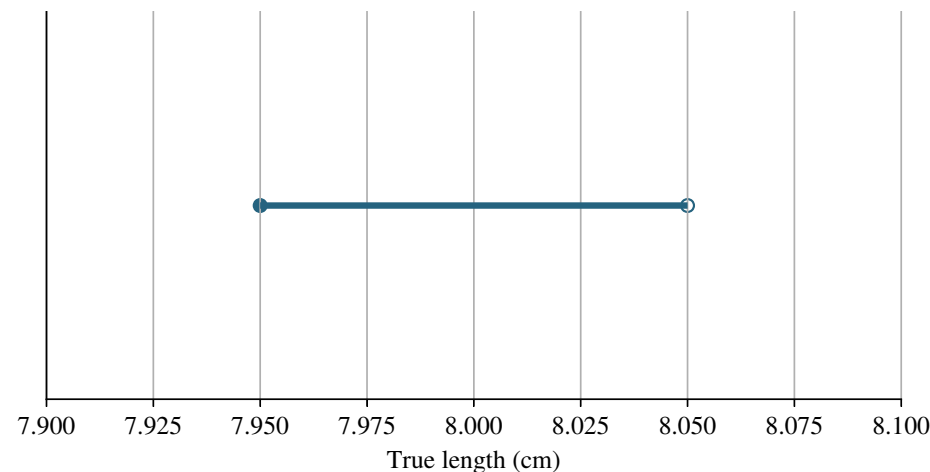
A worked case

$$A = LW, \quad v = \frac{d}{t}$$

■ Worked example

The lengths satisfy $7.95 \leq L < 8.05$ and $4.95 \leq W < 5.05$. Since $A = LW$, $39.3525 \leq A < 40.6525$. For speed d/t , the largest speed uses the largest distance and smallest positive time.

8.0 cm to nearest 0.1: $7.95 \leq L < 8.05$



Where it goes wrong

- An upper bound is not automatically achieved.
- Dividing upper distance by upper time does not give the largest speed.
- Keep enough digits in intermediate calculations.

■ Watch out

Not true: The greatest value of positive d/t uses the greatest d and greatest t . To make d/t as large as possible, use the greatest d and the least t ; a larger t makes the quotient smaller.

Applying it

- Distinguish measurement uncertainty from arithmetic rounding.
- A sensible reported precision cannot be finer than the measurements justify.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.1.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Lower bound:** The smallest possible value consistent with a stated rounding rule.
- A value rounded to the nearest unit u lies from stated value $-u/2$ up to, but usually not including, stated value $+u/2$.
- Watch for: an upper bound is not automatically achieved.

3.1 · Signed numbers, place value and operation order · Foundation

Which way does the balance move?

- An account starts 12 yuan overdrawn. A payment of 35 yuan arrives, then a 9-yuan charge is taken. Which direction does each change move the balance?
- This lesson studies **reciprocal**: A number that multiplies a nonzero original number to give 1.

The method

- Larger numbers lie farther right on the number line.
- Adding a negative moves left; subtracting a negative moves right.
- Multiply or divide signs first, then magnitudes.
- Evaluate brackets, powers and roots before multiplication/division, then addition/subtraction, working left to right within equal priority.

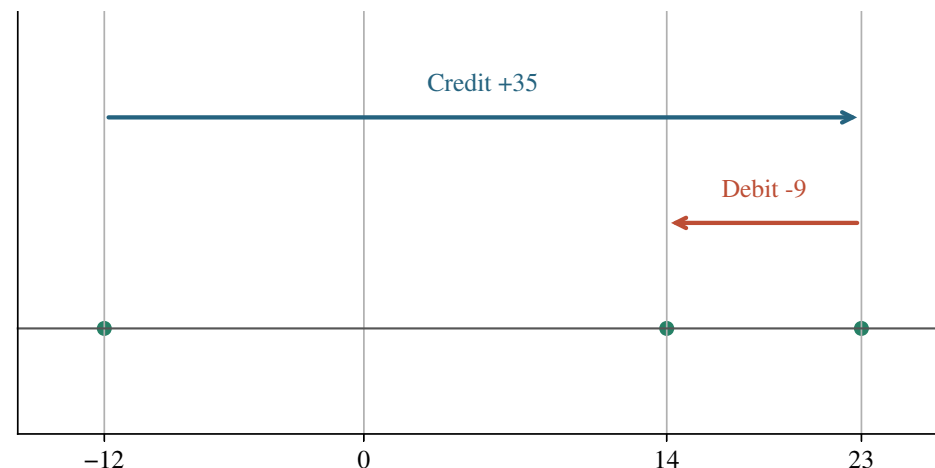
A worked case

$$-12 + 35 - 9 = 14, \quad 18 \div 3 \times 2 = 12$$

■ Worked example

The balance is $-12 + 35 - 9 = 14$ yuan. Also $-4(-7) = 3$ and $(-6) \times (-3) = 18$. The reciprocal of -4 is $-1/4$, since their product is 1. For $18 \div 3 \times 2$, work left to right to obtain 12. In 3.047, the 4 means four hundredths and the 7 means seven thousandths. For decimal multiplication, first calculate 24×35 : $24 \times 30 + 24 \times 5 = 720 + 120 = 840$. The original 2.4×0.35 has three decimal places in total, giving 0.840. For $5.04 \div 0.12$ multiply both numbers by 100 to get $504 \div 12$; $12 \times 40 = 480$ leaves 24, so the answer is 42. To order negative fractions, $-3/4 = -0.75$ and $-4/5 = -0.8$, hence $-4/5 < -3/4$. The symbols $\leq$ and $\geq$ include equality; $\neq$ means unequal.

The balance moves from -12 to +14



Where it goes wrong

- Subtraction is not commutative: $3-8$ and $8-3$ differ.
- A negative sign outside a square is not inside its base: $-3^2 = -9$, but $(-3)^2 = 9$.
- Zero has no reciprocal.

■ Watch out

Not true: Multiplication must be performed before division, even when division is farther left. Multiplication and division have equal priority and are done from left to right: $12 \div 3 \times 2 = 8$.

Applying it

- Use a signed starting balance, a positive credit and a negative debit.
- A shop buying for 48 yuan and selling for 60 makes 12 yuan profit; reversing the prices gives a 12-yuan loss.
- Estimate the sign before calculating.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.1.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Reciprocal:** A number that multiplies a nonzero original number to give 1.
- Larger numbers lie farther right on the number line.
- Watch for: subtraction is not commutative: 3-8 and 8-3 differ.

3.1 · Signed numbers, place value and operation order · Higher

Which way does the balance move?

- An account starts 12 yuan overdrawn. A payment of 35 yuan arrives, then a 9-yuan charge is taken. Which direction does each change move the balance?
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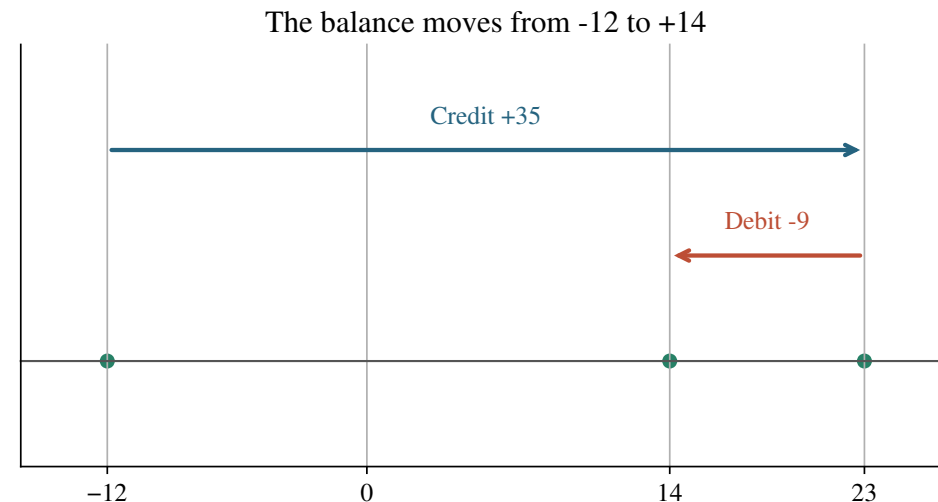
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$$-12 + 35 - 9 = 14, \quad 18 \div 3 \times 2 = 12$$

■ Worked example

The balance is $-12+35-9=14$ yuan. Also $-4(-7)=3$ and $(-6)\times(-3)=18$. The reciprocal of -4 is $-1/4$, since their product is 1. For $18\div3\times2$, work left to right to obtain 12. In 3.047, the 4 means four hundredths and the 7 means seven thousandths. For decimal multiplication, first calculate 24×35 : $24\times30+24\times5=720+120=840$. The original 2.4×0.35 has three decimal places in total, giving 0.840. For $5.04\div0.12$ multiply both numbers by 100 to get $504\div12$; $12\times40=480$ leaves 24, so the answer is 42. To order

negative fractions, $-3/4=-0.75$ and $-4/5=-0.8$, hence $-4/5<-3/4$. The symbols $\leq$ and $\geq$ include equality; $\neq$ means unequal.



Where it goes wrong

- Subtraction is not commutative: 3-8 and 8-3 differ.
- A negative sign outside a square is not inside its base: $-3^2=-9$, but $(-3)^2=9$.
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Not true: Multiplication must be performed before division, even when division is farther left. Multiplication and division have equal priority and are done from left to right: $12\div3\times2=8$.

Applying it

- Use a signed starting balance, a positive credit and a negative debit.
- A shop buying for 48 yuan and selling for 60 makes 12 yuan profit; reversing the prices gives a 12-yuan loss.
- Estimate the sign before calculating.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.1.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Reciprocal:** A number that multiplies a nonzero original number to give 1.
- Larger numbers lie farther right on the number line.
- Watch for: subtraction is not commutative: $3-8$ and $8-3$ differ.

3.1 · Exact fractions and mixed-number operations · Foundation

Can we keep the recipe exact?

- A recipe uses $1\frac{1}{2}$ cups for one batch. How much remains after using $\frac{3}{4}$ of a cup, and why should the answer stay exact?
- This lesson studies **improper fraction**: A fraction whose numerator is at least as large as its positive denominator.

The method

- Convert mixed numbers to improper fractions.
- Add or subtract using a common denominator; multiply numerators and denominators; divide by a nonzero fraction by multiplying its reciprocal.
- Cancel common factors, not added terms.
- These rules also apply to negative fractions.

A worked case

$$\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \times \frac{8}{5} = \frac{6}{5}$$

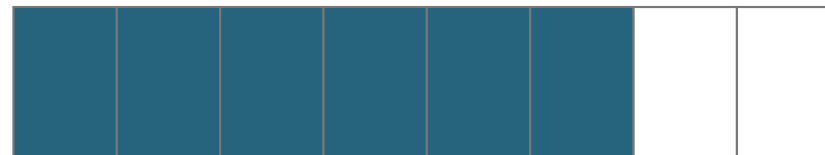
■ Worked example

$1\frac{1}{2} - \frac{3}{4} = 6/4 - 3/4 = 3/4$. Also $(-2/3) \times (9/4) = -18/12 = -3/2$ and $(3/4) \div (5/8) = (3/4) \times (8/5) = 6/5$. A common denominator gives $5/6 + 3/4 = 10/12 + 9/12 = 19/12$. Check division by multiplying $6/5$ by $5/8$ to recover $3/4$. Exact multiples of π follow ordinary arithmetic: $3\pi + 2\pi = 5\pi$ and $6\pi/3 = 2\pi$. Leave an answer such as 5π exact when requested; $\pi \approx 3.14$ would introduce

approximation. For subtraction, $5/6 - 3/4 = 10/12 - 9/12 = 1/12$; for a negative mixed number, $-1\frac{1}{2}$ means $-(1 + 1/2) = -3/2$.

Six eighths contain $6/5$ groups of five eighths

$$3/4 = 6/8$$



One group of $5/8$

Where it goes wrong

- Adding denominators does not preserve the unit size: $1/2 + 1/3$ is not $2/5$.
- Cancel only factors of a whole numerator and denominator.
- A division by zero is undefined.

■ Watch out

Not true: To add two fractions, add their numerators and add their denominators. Use a common denominator first: $1/2 + 1/3 = 5/6$, not $2/5$.

Applying it

- For a non-calculator question show the common denominator or reciprocal step.
- Convert $19/12$ to $1\frac{7}{12}$ if a mixed number is requested; round only when the question explicitly needs a decimal.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.1.

■ Key idea

- **Improper fraction:** A fraction whose numerator is at least as large as its positive denominator.
- Convert mixed numbers to improper fractions.
- Watch for: adding denominators does not preserve the unit size: $1/2 + 1/3$ is not $2/5$.

3.1 · Exact fractions and mixed-number operations · Higher

Can we keep the recipe exact?

- A recipe uses $1\frac{1}{2}$ cups for one batch. How much remains after using $\frac{3}{4}$ of a cup, and why should the answer stay exact?
- This lesson studies **improper fraction**: A fraction whose numerator is at least as large as its positive denominator.

The method

- Convert mixed numbers to improper fractions.
- Add or subtract using a common denominator; multiply numerators and denominators; divide by a nonzero fraction by multiplying its reciprocal.
- Cancel common factors, not added terms.
- These rules also apply to negative fractions.

A worked case

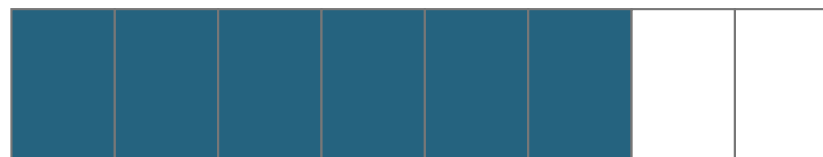
$$\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \times \frac{8}{5} = \frac{6}{5}$$

■ Worked example

$1\frac{1}{2} - \frac{3}{4} = 6/4 - 3/4 = 3/4$. Also $(-2/3) \times (9/4) = -18/12 = -3/2$ and $(3/4) \div (5/8) = (3/4) \times (8/5) = 6/5$. A common denominator gives $5/6 + 3/4 = 10/12 + 9/12 = 19/12$. Check division by multiplying $6/5$ by $5/8$ to recover $3/4$. Exact multiples of π follow ordinary arithmetic: $3\pi + 2\pi = 5\pi$ and $6\pi/3 = 2\pi$. Leave an answer such as 5π exact when requested; $\pi \approx 3.14$ would introduce approximation. For subtraction, $5/6 - 3/4 = 10/12 - 9/12 = 1/12$; for a negative mixed number, $-1\frac{1}{2}$ means $-(1 + 1/2) = -3/2$.

Six eighths contain $6/5$ groups of five eighths

$$3/4 = 6/8$$



One group of $5/8$

Where it goes wrong

- Adding denominators does not preserve the unit size: $1/2 + 1/3$ is not $2/5$.
- Cancel only factors of a whole numerator and denominator.
- A division by zero is undefined.

■ Watch out

Not true: To add two fractions, add their numerators and add their denominators. Use a common denominator first: $1/2 + 1/3 = 5/6$, not $2/5$.

Applying it

- For a non-calculator question show the common denominator or reciprocal step.
- Convert $19/12$ to $1\frac{7}{12}$ if a mixed number is requested; round only when the question explicitly needs a decimal.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.1.

■ Key idea

- **Improper fraction:** A fraction whose numerator is at least as large as its positive denominator.
- Convert mixed numbers to improper fractions.
- Watch for: adding denominators does not preserve the unit size: $1/2 + 1/3$ is not $2/5$.

3.1 · Systematic lists and possibility grids · Foundation

Have we listed every choice?

- A cafe offers tea or juice with apple, banana or melon. How can a list prove that no lunch choice was missed?
- This lesson studies **systematic list** : A list made in a fixed order so every allowed outcome appears exactly once.

The method

- Fix one first choice and list every allowed second choice before moving to the next first choice.
- Use a table to check that no outcome is missing or duplicated.
- If combinations are forbidden, remove those entries from the list.

A worked case

$$\{TA, TB, TM, JA, JB, JM\}$$

■ Worked example

List tea-apple, tea-banana, tea-melon, juice-apple, juice-banana, juice-melon: six choices. Removing tea-melon leaves five. The codes using 1,2,3 once each are 123,132,213,231,312,321: six. These answers follow from complete lists.

A complete grid lists six choices once each

Juice + Apple

Juice + Banana

Juice + Melon

Tea + Apple

Tea + Banana

Tea + Melon

Where it goes wrong

- State whether order matters and whether repetition is allowed.
- Choosing A then B can be different from B then A for a code, but not for an unordered two-person team.
- A restriction can make a simple product invalid.

■ Watch out

Not true: Three available first choices and four second choices give twelve outcomes even if some combinations are forbidden. The product rule assumes every pair is allowed; forbidden combinations must be removed from the 12.

Applying it

- Foundation should justify answers with a complete ordered list or grid.
- Higher may compress a verified structure using the product rule, but must still check restrictions.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.1.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Systematic list:** A list made in a fixed order so every allowed outcome appears exactly once.
- Fix one first choice and list every allowed second choice before moving to the next first choice.
- Watch for: state whether order matters and whether repetition is allowed.

3.1 · Systematic lists and the product rule · Higher

Have we listed every choice?

- A cafe offers tea or juice with apple, banana or melon. How can a list prove that no lunch choice was missed?
- This lesson studies **systematic list** : A list made in a fixed order so every allowed outcome appears exactly once.

The method

- Fix one first choice and list every permitted second choice before moving to the next first choice.
- A table gives the same structure.
- When every one of m first choices allows n second choices, the Higher product rule gives mn outcomes.
- If restrictions change the options, count the allowed rows separately.

A worked case

$$\{TA, TB, TM, JA, JB, JM\}$$

■ Worked example

The list is tea-apple, tea-banana, tea-melon, juice-apple, juice-banana, juice-melon: six outcomes. If tea-melon is unavailable, five remain. For a three-digit code with digits 1,2,3 and no repeats, choose 3 then 2 then 1 possibilities, giving six codes: 123,132,213,231,312,321.

A complete grid lists six choices once each

Juice + Apple	Juice + Banana	Juice + Melon
Tea + Apple	Tea + Banana	Tea + Melon

Where it goes wrong

- State whether order matters and whether repetition is allowed.
- Choosing A then B can be different from B then A for a code, but not for an unordered two-person team.
- A restriction can make a simple product invalid.

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Not true: Three available first choices and four second choices give twelve outcomes even if some combinations are forbidden. The product rule assumes every pair is allowed; forbidden combinations must be removed from the 12.

Applying it

- Foundation should justify answers with a complete ordered list or grid.
- Higher may compress a verified structure using the product rule, but must still check restrictions.

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■ Key idea

- **Systematic list:** A list made in a fixed order so every allowed outcome appears exactly once.
- Fix one first choice and list every permitted second choice before moving to the next first choice.
- Watch for: state whether order matters and whether repetition is allowed.

3.1 · Exact surds and rationalising denominators · Higher

Can a root stay exact?

- A square has area 12 cm^2 . Can its side be written exactly without a long calculator decimal?
- This lesson studies **surd**: An irrational root kept in exact form rather than replaced by a rounded decimal.

The method

- Extract square factors: $\sqrt{a^2b} = a\sqrt{b}$ for $a \geq 0, b \geq 0$.
- Add only matching root parts.
- Multiply roots with nonnegative radicands.
- To rationalise a denominator, multiply numerator and denominator by the same suitable root or conjugate.

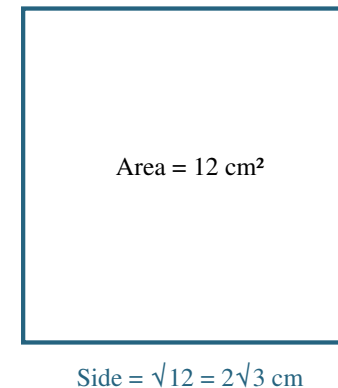
A worked case

$$\sqrt{12} + \sqrt{27} = 5\sqrt{3}, \quad \frac{6}{\sqrt{3}} = 2\sqrt{3}$$

■ Worked example

The side is $\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3} \text{ cm}$. Thus $\sqrt{12} + \sqrt{27} = 2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$. Also $6/\sqrt{3} = 6\sqrt{3}/3 = 2\sqrt{3}$. For $1/(2+\sqrt{3})$, multiply by $(2-\sqrt{3})/(2-\sqrt{3})$: the denominator becomes $4-3=1$, giving $2-\sqrt{3}$. A circle of radius 3 has exact area 9π ; a decimal is an approximation.

An exact root describes the square



Where it goes wrong

- $\sqrt{a+b}$ generally differs from $\sqrt{a} + \sqrt{b}$.
- Match the radicand before collecting terms.
- Rationalising changes the form, not the value; multiplying only the denominator changes the value.

■ Watch out

Not true: For every pair of positive numbers, $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$. $\sqrt{9+16} = 5$, but $\sqrt{9} + \sqrt{16} = 7$; only products split: $\sqrt{ab} = \sqrt{a}\sqrt{b}$.

Applying it

- AQA N8 Higher requires exact surds and rationalisation.
- Foundation retains exact fractions and multiples of π ; do not assign this surd lesson to Foundation.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.1.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Surd:** An irrational root kept in exact form rather than replaced by a rounded decimal.
- Extract square factors: $\sqrt{a^2b} = a\sqrt{b}$ for $a \geq 0, b \geq 0$.
- Watch for: $\sqrt{a+b}$ generally differs from $\sqrt{a} + \sqrt{b}$.

3.1 · Terminating decimals and fractions · Foundation

Does the display tell the whole number?

- A calculator shows 0.333333. Is the display the exact value of one third, or only the digits that fit on the screen?
- This lesson studies **terminating decimal**: A decimal that ends after a finite number of digits.

The method

- Use the place value of the last digit to write a power-of-ten denominator, then simplify.
- Compare numbers using matching decimal places or a common denominator.

A worked case

$$0.375 = \frac{375}{1000} = \frac{3}{8}$$

■ Worked example

$0.375 = 375/1000 = 3/8$. Also $3.5 = 35/10 = 7/2$ and $0.06 = 6/100 = 3/50$. To order $3/8$ and 0.4 , write 0.375 and 0.400 ; therefore $3/8 < 0.4$.

Different exact forms, the same shaded fraction

$$3/8 = 0.375 = 375/1000$$



Where it goes wrong

- Keep the place value: 0.06 is six hundredths, not six tenths.
- A fraction may exceed 1.
- Simplify the entire numerator and denominator by the same common factor.

■ Watch out

Not true: The finite decimal 0.375 equals $375/1000$. 0.333333 is a finite decimal equal to $333333/1000000$; one third is 0.333... repeating forever.

Applying it

- AQA N10 Foundation covers terminating decimals and fractions, including ordering.
- Recurring-decimal conversion is reserved for the Higher lesson.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.1.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Terminating decimal:** A decimal that ends after a finite number of digits.
- Use the place value of the last digit to write a power-of-ten denominator, then simplify.
- Watch for: keep the place value: 0.06 is six hundredths, not six tenths.

3.1 · Terminating and recurring decimal conversions · Higher

Does the display tell the whole number?

- A calculator shows 0.333333. Is the display the exact value of one third, or only the digits that fit on the screen?
- This lesson studies **recurring decimal** : A decimal with a digit or block of digits that repeats forever.

The method

- A finite decimal uses a power-of-ten denominator before simplifying.
- For a recurring decimal, multiply by powers of ten so the repeated tails align, then subtract.
- Use matching decimal places to compare numbers.
- A displayed rounded decimal need not be the exact fraction.

A worked case

$$100x - x = 27, \quad x = \frac{27}{99} = \frac{3}{11}$$

■ Worked example

$0.375 = 375/1000 = 3/8$. For $x = 0.272727\dots$, $100x = 27.272727\dots$, so $99x = 27$ and $x = 27/99 = 3/11$. For $y = 0.16666\dots$, $100y - 10y = 16.666\dots - 1.666\dots = 15$, so $y = 15/90 = 1/6$. In a reduced fraction, a denominator containing only factors 2 and 5 gives a terminating decimal.

Different exact forms, the same shaded fraction

$$3/8 = 0.375 = 375/1000$$



Where it goes wrong

- Align the recurring tails before subtracting. 0.333333 is finite and differs from 0.333333... .
- A non-recurring prefix needs a second power of ten; blindly dividing every digit block by 99 fails.

■ Watch out

Not true: The finite decimal 0.333333 is exactly one third. 0.333333 is a finite decimal equal to 333333/1000000; one third is 0.333... repeating forever.

Applying it

- AQA N10 Higher includes recurring conversion.
- Both tiers convert and order terminating decimals and fractions.
- Check a conversion by long division; 3 divided by 8 gives 0.375.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.1.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Recurring decimal:** A decimal with a digit or block of digits that repeats forever.
- A finite decimal uses a power-of-ten denominator before simplifying.
- Watch for: align the recurring tails before subtracting. 0.333333 is finite and differs from 0.333333... .

3.1 · Metric and compound-unit conversions · Foundation

Why must the factor be squared?

- A floor tile measures 0.4 m by 0.4 m. Why is its area not 0.16×100 square centimetres?
- This lesson studies **conversion factor** : The multiplier that expresses the same quantity in another unit.

The method

- Convert each dimension.
- Since 1 m=100 cm, $1 \text{ m}^2=10,000 \text{ cm}^2$ and $1 \text{ m}^3=1,000,000 \text{ cm}^3$.
- Also 1 litre= 1000 cm^3 , 1 kg=1000 g and 1 hour=3600 seconds.
- For a compound unit convert numerator and denominator, keeping the physical quantity unchanged.

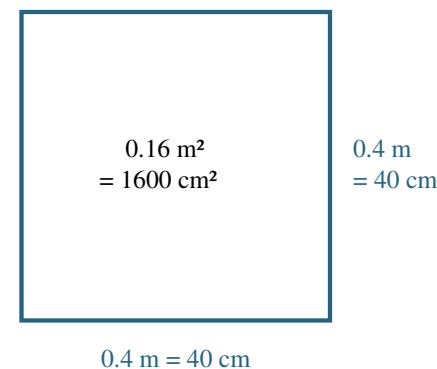
A worked case

$$1 \text{ m}^2 = 100^2 \text{ cm}^2, \quad 72 \frac{\text{km}}{\text{h}} = 20 \frac{\text{m}}{\text{s}}$$

■ Worked example

0.4 m=40 cm, so the tile area is $40 \times 40 = 1600 \text{ cm}^2$, agreeing with $0.16 \times 10,000$. A 0.002 m^3 container holds $2000 \text{ cm}^3 = 2$ litres. A speed of 72 km/h is $72,000/3600 = 20 \text{ m/s}$. Write the units at each stage so the conversion can be checked.

Convert both dimensions before multiplying



Where it goes wrong

- The area multiplier is the square of the length multiplier; the volume multiplier is its cube.
- An hour is 60 minutes, not 100.
- Converting only the numerator of a speed gives an inconsistent unit.

■ Watch out

Not true: To convert square metres to square centimetres, multiply by 100. $1 \text{ m}^2 = 100 \text{ cm} \times 100 \text{ cm} = 10\,000 \text{ cm}^2$, so multiply by 10 000.

Applying it

- AQA N13 and R1 include metric length, area, volume, capacity and compound measures.
- Use given conversion factors for unfamiliar imperial units; do not guess them.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.1.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Conversion factor:** The multiplier that expresses the same quantity in another unit.
- Convert each dimension.
- Watch for: the area multiplier is the square of the length multiplier; the volume multiplier is its cube.

3.1 · Metric and compound-unit conversions · Higher

Why must the factor be squared?

- A floor tile measures 0.4 m by 0.4 m. Why is its area not 0.16×100 square centimetres?
- This lesson studies **conversion factor** : The multiplier that expresses the same quantity in another unit.

The method

- Convert each dimension.
- Since $1 \text{ m} = 100 \text{ cm}$, $1 \text{ m}^2 = 10,000 \text{ cm}^2$ and $1 \text{ m}^3 = 1,000,000 \text{ cm}^3$.
- Also $1 \text{ litre} = 1000 \text{ cm}^3$, $1 \text{ kg} = 1000 \text{ g}$ and $1 \text{ hour} = 3600 \text{ seconds}$.
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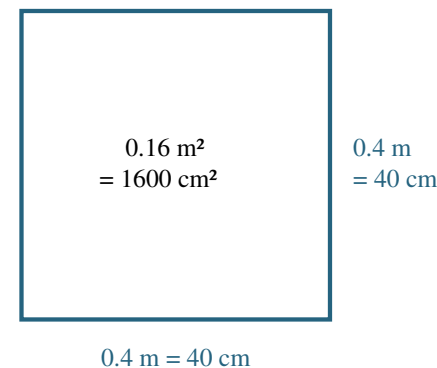
A worked case

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■ Worked example

$0.4 \text{ m} = 40 \text{ cm}$, so the tile area is $40 \times 40 = 1600 \text{ cm}^2$, agreeing with $0.16 \times 10,000$. A 0.002 m^3 container holds $2000 \text{ cm}^3 = 2 \text{ litres}$. A speed of 72 km/h is $72,000/3600 = 20 \text{ m/s}$. Write the units at each stage so the conversion can be checked.

Convert both dimensions before multiplying



Where it goes wrong

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Not true: To convert square metres to square centimetres, multiply by 100. $1 \text{ m}^2 = 100 \text{ cm} \times 100 \text{ cm} = 10\,000 \text{ cm}^2$, so multiply by 10 000.

Applying it

- AQA N13 and R1 include metric length, area, volume, capacity and compound measures.
- Use given conversion factors for unfamiliar imperial units; do not guess them.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.1.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Conversion factor:** The multiplier that expresses the same quantity in another unit.
- Convert each dimension.
- Watch for: the area multiplier is the square of the length multiplier; the volume multiplier is its cube.

3.1 · Estimation, rounding and simple error intervals · Foundation

Would the estimate catch a typo?

- A calculator reports $49.8 \times 19.7 \div 10.2$. Could 962 be a reasonable answer, or does a quick estimate expose an input error?
- This lesson studies **significant figure**: A digit counted from the first nonzero digit when reporting numerical precision.

The method

- For a rough check use easy nearby numbers: $50 \times 20 \div 10 = 100$.
- Decimal places count digits after the point; significant figures start at the first nonzero digit.
- Round only the final result.
- Nearest-unit rounding has an interval extending half a unit either way; positive truncation keeps values from the stated value up to the next unit.

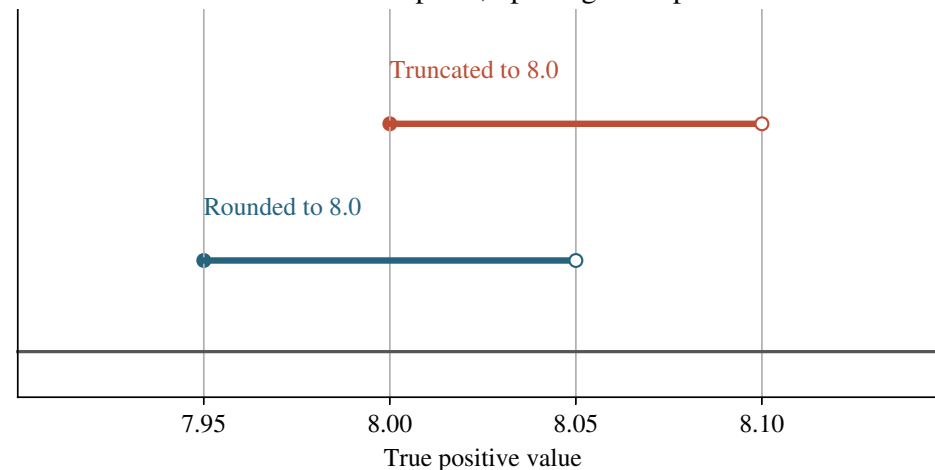
A worked case

$$7.95 \leq L < 8.05$$

■ Worked example

The calculation is about 96.18, consistent with the estimate 100. The number 0.004786 rounds to 0.0048 at 2 significant figures, but to 0.005 at 3 decimal places. If length L rounds to 8.0 cm at 1 decimal place, $7.95 \leq L < 8.05$. If a positive value is truncated to 8.0 at 1 decimal place, $8.0 \leq L < 8.1$ instead. Reported precision must fit the question. Seventeen items packed six per box need three whole boxes, since two boxes hold only twelve items. Rounding $17/6$ to two boxes would fail the physical requirement. For a nearest-0.1 reading of 8.0, each possible value differs from the report by at most 0.05; a claim of 8.08 lies outside the interval. Carry full calculator precision until a final money, length or accuracy requirement is applied.

Closed left endpoint; open right endpoint



Where it goes wrong

- Zeros before the first nonzero digit do not count as significant figures.
- Rounding and truncation give different intervals.
- Do not turn an approximate check into an exact answer, or round every intermediate result.

■ Watch out

Not true: Rounding a positive value and truncating it to the same decimal place give the same error interval. Rounding gives an interval on both sides of the value; truncation gives an interval on one side only.

Applying it

- AQA N14–N16 require accuracy interpretation at both tiers.
- Foundation uses simple intervals and limits of accuracy.
- Higher combines upper and lower bounds for calculated quantities in the separate bounds lesson.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.1.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Significant figure:** A digit counted from the first nonzero digit when reporting numerical precision.
- For a rough check use easy nearby numbers: $50 \times 20 \div 10 = 100$.
- Watch for: zeros before the first nonzero digit do not count as significant figures.

3.1 · Estimation, rounding and simple error intervals · Higher

Would the estimate catch a typo?

- A calculator reports $49.8 \times 19.7 \div 10.2$. Could 962 be a reasonable answer, or does a quick estimate expose an input error?
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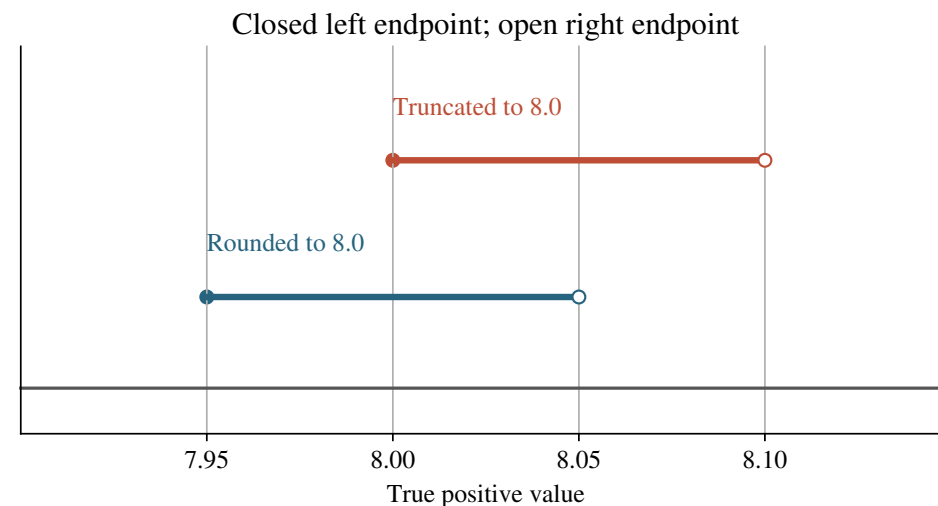
A worked case

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Seventeen items packed six per box need three whole boxes, since two boxes hold only twelve items. Rounding $17/6$ to two boxes would fail the physical requirement. For a nearest-0.1 reading of 8.0, each possible value differs from the report by at most 0.05; a claim of 8.08 lies outside the interval. Carry full calculator precision until a final money, length or accuracy requirement is applied.



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- Foundation uses simple intervals and limits of accuracy.
- Higher combines upper and lower bounds for calculated quantities in the separate bounds lesson.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.1.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Significant figure:** A digit counted from the first nonzero digit when reporting numerical precision.
- For a rough check use easy nearby numbers: $50 \times 20 \div 10 = 100$.
- Watch for: zeros before the first nonzero digit do not count as significant figures.

3.1 · Ratio shares and fraction operators · Foundation

Which fraction describes the whole?

- A club shares 84 yuan between two teams in the ratio 2:5. Does the first team receive two fifths of the total?
- This lesson studies **ratio**: A comparison that states the relative numbers of equal-sized parts.

The method

- Add the ratio parts to find the whole: 2:5 has 7 equal parts.
- Divide the total by 7, then multiply by the required part count.
- A fraction acts as a multiplier; a percentage p acts as $p/100$.
- Keep part-to-part and part-to-whole comparisons separate.

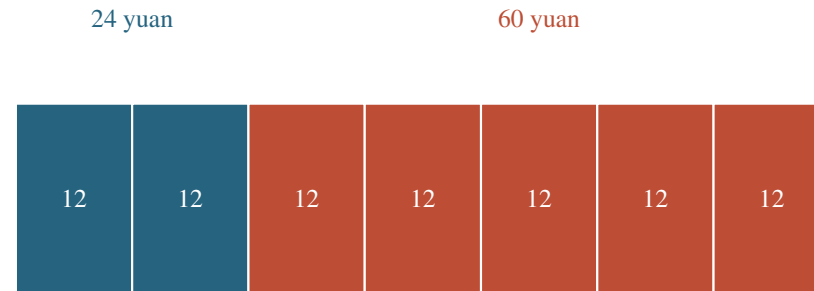
A worked case

$$84 \times \frac{2}{2+5} = 24$$

■ Worked example

Each part is $84/7=12$ yuan, giving shares 24 and 60. The first team has $2/7$ of the whole, while its share is $2/5$ of the other share. A $3/4$ portion of 28 is 21. A 120% amount of 35 is $1.2 \times 35 = 42$, so percentages may exceed 100.

Ratio 2:5 has seven total parts



Where it goes wrong

- A ratio 2:5 does not give the first share as $2/5$ of the total.
- The total has 7 parts.
- A multiplier over 1 increases a positive quantity; it does not automatically mean a probability.

■ Watch out

Not true: For a 2:5 division, the first share is $2/5$ of the whole. A 2:5 ratio has 7 parts, so the first share is $2/7$ of the whole.

Applying it

- AQA N11/N12 and R3–R8 link ratio parts to fractions and operators.
- Check the shares add to the stated total and simplify a ratio by dividing every part by the same positive factor.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.1.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Ratio:** A comparison that states the relative numbers of equal-sized parts.
- Add the ratio parts to find the whole: 2:5 has 7 equal parts.
- Watch for: A ratio 2:5 does not give the first share as $\frac{2}{5}$ of the total.

3.1 · Ratio shares and fraction operators · Higher

Which fraction describes the whole?

- A club shares 84 yuan between two teams in the ratio 2:5. Does the first team receive two fifths of the total?
- This lesson studies **ratio** : A comparison that states the relative numbers of equal-sized parts.

The method

- Add the ratio parts to find the whole: 2:5 has 7 equal parts.
- Divide the total by 7, then multiply by the required part count.
- A fraction acts as a multiplier; a percentage p acts as $p/100$.
- Keep part-to-part and part-to-whole comparisons separate.

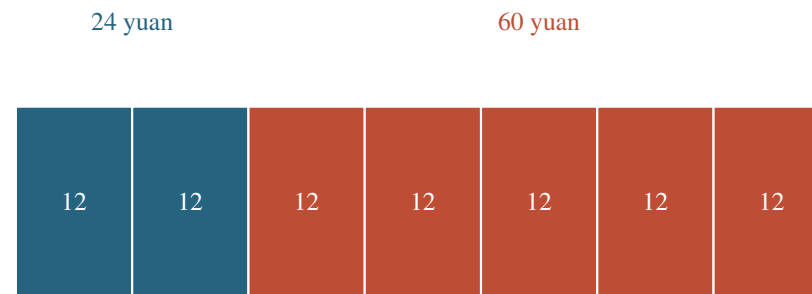
A worked case

$$84 \times \frac{2}{2+5} = 24$$

■ Worked example

Each part is $84/7=12$ yuan, giving shares 24 and 60. The first team has $\frac{2}{7}$ of the whole, while its share is $\frac{2}{5}$ of the other share. A $\frac{3}{4}$ portion of 28 is 21. A 120% amount of 35 is $1.2 \times 35 = 42$, so percentages may exceed 100.

Ratio 2:5 has seven total parts



Where it goes wrong

- A ratio 2:5 does not give the first share as $\frac{2}{5}$ of the total.
- The total has 7 parts.
- A multiplier over 1 increases a positive quantity; it does not automatically mean a probability.

■ Watch out

Not true: For a 2:5 division, the first share is $\frac{2}{7}$ of the whole. A 2:5 ratio has 7 parts, so the first share is $\frac{2}{7}$ of the whole.

Applying it

- AQA N11/N12 and R3–R8 link ratio parts to fractions and operators.
- Check the shares add to the stated total and simplify a ratio by dividing every part by the same positive factor.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.1.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Ratio:** A comparison that states the relative numbers of equal-sized parts.
- Add the ratio parts to find the whole: 2:5 has 7 equal parts.
- Watch for: A ratio 2:5 does not give the first share as $\frac{2}{5}$ of the total.

3.2 · Equations, identities and rearrangement · Foundation

When do two plans cost the same?

- Two mobile plans cost $20+3x$ and $44+x$ yuan for x GB. When do they cost the same?
- This lesson studies **identity**: An equality that holds for every allowed value of its variable.

The method

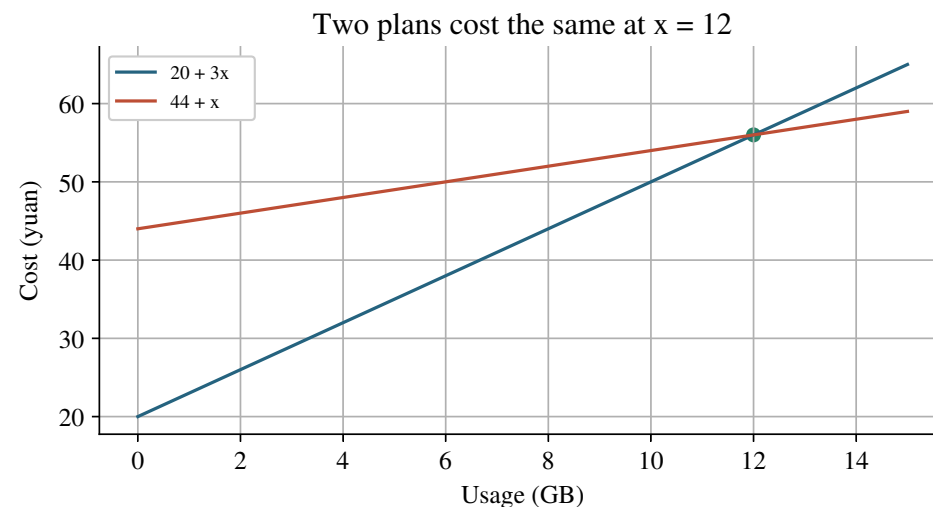
- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs.
- Preserve equality by applying the same operation to both sides.
- State restrictions before dividing by a variable.

A worked case

$$C_1 = 20 + 3x, \quad C_2 = 44 + x, \quad C_1 = C_2$$

■ Worked example

$20+3x=44+x$ gives $2x=24$ and $x=12$. Both plans then cost 56. In $A=\pi r^2$, divide by π and take the positive square root to obtain $r=\sqrt{(A/\pi)}$, because r is a length.



Where it goes wrong

- Cancelling a term is not the same as cancelling a factor.
- In $(x^2+2x)/x$, factor the numerator and retain $x \neq 0$.
- Check a rearrangement by substitution.

■ Watch out

Not true: Cancelling x from $(x+3)/x$ leaves 3 for every nonzero x . x is a term of the numerator, not a factor, so it cannot be cancelled: at $x=1$, $(1+3)/1=4$, not 3.

Applying it

- Define the unknown and set up a linear equation.
- Check the answer by substitution.
- Restrict this Foundation/Core lesson to simple expressions and equations.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Identity:** An equality that holds for every allowed value of its variable.
- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs.
- Watch for: cancelling a term is not the same as cancelling a factor.

3.2 · Equations, identities and rearrangement · Higher

When do two plans cost the same?

- Two mobile plans cost $20+3x$ and $44+x$ yuan for x GB. When do they cost the same?
- This lesson studies **identity**: An equality that holds for every allowed value of its variable.

The method

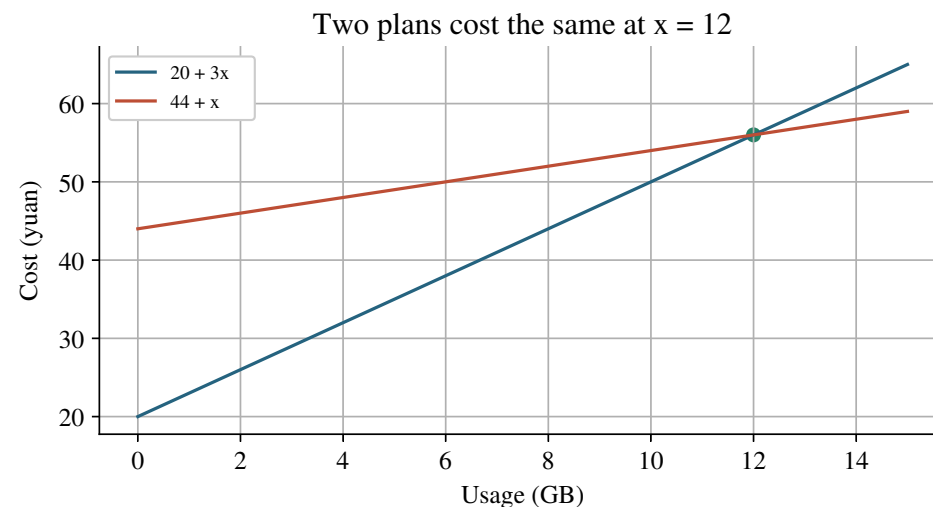
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- State restrictions before dividing by a variable.

A worked case

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■ Worked example

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- In $(x^2+2x)/x$, factor the numerator and retain $x \neq 0$.
- Check a rearrangement by substitution.

■ Watch out

Not true: Cancelling x from $(x+3)/x$ leaves 3 for every nonzero x . x is a term of the numerator, not a factor, so it cannot be cancelled: at $x=1$, $(1+3)/1=4$, not 3.

Applying it

- Set up the equation from units and the meaning of the unknown.
- A negative or fractional solution may be algebraically correct but impossible for a count.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Identity:** An equality that holds for every allowed value of its variable.
- An equation asks which inputs satisfy an equality; an identity holds for all allowed inputs.
- Watch for: cancelling a term is not the same as cancelling a factor.

3.2 · Factorising and solving simple quadratics · Foundation

Which widths make enough space?

- A rectangular enclosure has area $x(10-x)$. What widths give at least 21 square metres?
- This lesson studies **root** : An input for which the expression has value zero.

The method

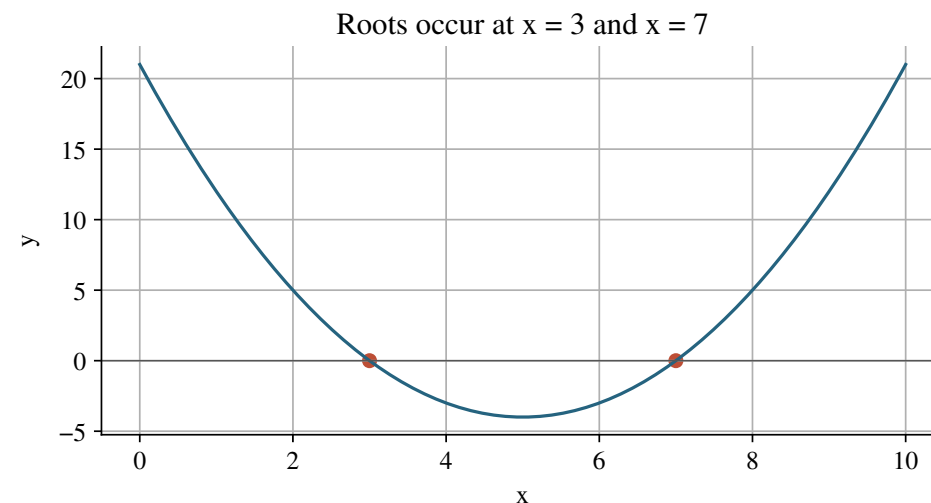
- Expand brackets and factorise simple quadratics.
- Solve by setting each factor equal to zero, and use a graph to interpret the roots.

A worked case

$$(x - 3)(x - 7) = 0$$

■ Worked example

$x^2 - 10x + 21 = (x - 3)(x - 7)$. Hence the equation $x^2 - 10x + 21 = 0$ has roots 3 and 7. Check each root by substitution and mark both intercepts on the graph.



Where it goes wrong

- Multiplying an inequality by a negative number reverses its direction.
- A sketch must show which side of each root satisfies the inequality.
- Geometry may restrict x further.

■ Watch out

Not true: A positive discriminant means that a quadratic has no real roots. A positive discriminant means two distinct real roots; a negative discriminant means no real roots.

Applying it

- This Foundation/Core lesson uses factorisation and graphical roots; the discriminant, quadratic formula and quadratic inequalities are reserved for the advanced tier.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Root:** An input for which the expression has value zero.
- Expand brackets and factorise simple quadratics.
- Watch for: multiplying an inequality by a negative number reverses its direction.

3.2 · Quadratics and inequalities · Higher

Which widths make enough space?

- A rectangular enclosure has area $x(10-x)$. What widths give at least 21 square metres?
- This lesson studies **discriminant** : The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.

The method

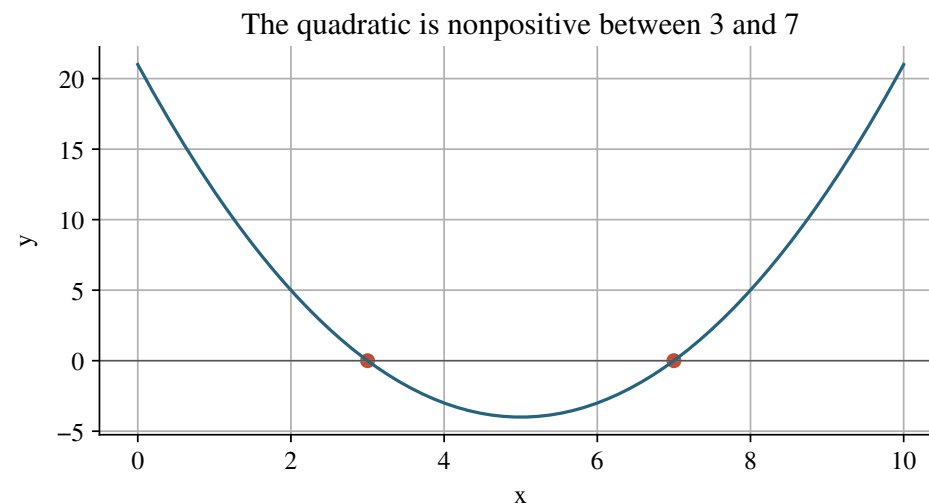
- Factor where possible; otherwise complete the square or use the quadratic formula.
- A quadratic inequality needs the sign on intervals, not only the roots.
- The discriminant identifies repeated or missing real roots.

A worked case

$$ax^2 + bx + c = 0, \quad \Delta = b^2 - 4ac$$

■ Worked example

The equation $x^2-10x+21=0$ factorises as $(x-3)(x-7)=0$, giving roots 3 and 7. Complete the square: $x^2-10x+21=(x-5)^2-4$, giving turning point (5,-4) and symmetry line $x=5$. The formula $x=[-b\pm\sqrt{(b^2-4ac)}]/(2a)$ also gives $(10\pm4)/2=3,7$. For $2x^2+5x-3=0$, the discriminant is 49 and roots are $(-5\pm7)/4=1/2,-3$. A graph gives approximate roots when exact factorisation is inconvenient. For an enclosure with area $A=x(10-x)$, complete the square to get $A=25-(x-5)^2$. The greatest area is 25 at $x=5$, within $0<x<10$.



Where it goes wrong

- Multiplying an inequality by a negative number reverses its direction.
- A sketch must show which side of each root satisfies the inequality.
- Geometry may restrict x further.

■ Watch out

Not true: A positive discriminant means that a quadratic has no real roots. A positive discriminant means two distinct real roots; a negative discriminant means no real roots.

Applying it

- AQA A11/A18 Higher interprets roots, intercepts and turning points, completes the square and uses the quadratic formula, including equations needing rearrangement.
- Factorisation and substitution check each result.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Discriminant:** The quantity b^2-4ac that determines the real roots of $ax^2+bx+c=0$.
- Factor where possible; otherwise complete the square or use the quadratic formula.
- Watch for: multiplying an inequality by a negative number reverses its direction.

3.2 · Two linear simultaneous equations · Foundation

Can one equation determine two counts?

- Two ticket types raise a total amount. A single equation cannot identify both unknown counts.
- This lesson studies **elimination** : Combining equations to remove one variable while retaining the same solutions.

The method

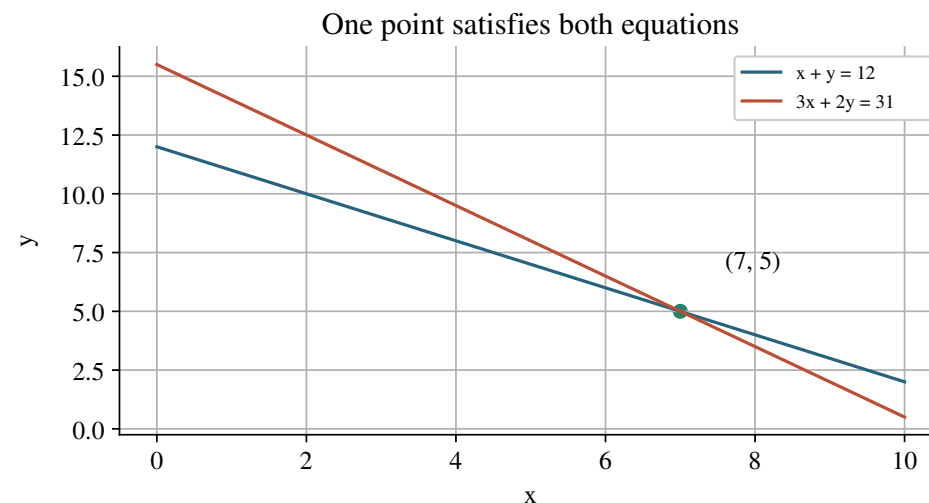
- Multiply equations to make a variable cancel, or substitute an expression from one equation into the other.
- Solve the remaining linear equation and recover the second variable.
- The intersection is a point satisfying both original equations.

A worked case

$$x + y = 12, \quad 3x + 2y = 31$$

■ Worked example

For $x+y=12$ and $3x+2y=31$, subtract twice the first equation from the second: $x=7$. Then $y=5$. Check both $7+5=12$ and $3 \times 7 + 2 \times 5 = 31$. The two straight-line graphs meet at $(7,5)$.



Where it goes wrong

- Check the ordered pair in both original equations.
- Multiplying an equation means multiplying every term, including the constant.
- Parallel distinct lines have no common solution.

■ Watch out

Not true: A pair of simultaneous equations is solved by checking just one of them. A solution must satisfy both equations; check it in each one.

Applying it

- AQA A19 Foundation solves two linear equations by elimination or substitution and interprets the graph intersection.
- Linear/quadratic systems belong to Higher.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Elimination:** Combining equations to remove one variable while retaining the same solutions.
- Multiply equations to make a variable cancel, or substitute an expression from one equation into the other.
- Watch for: check the ordered pair in both original equations.

3.2 · Linear and linear/quadratic simultaneous equations · Higher

Can one equation determine two counts?

- Two ticket types raise a total amount. A single equation cannot identify both unknown counts.
- This lesson studies **elimination** : Combining equations to remove one variable while retaining the same solutions.

The method

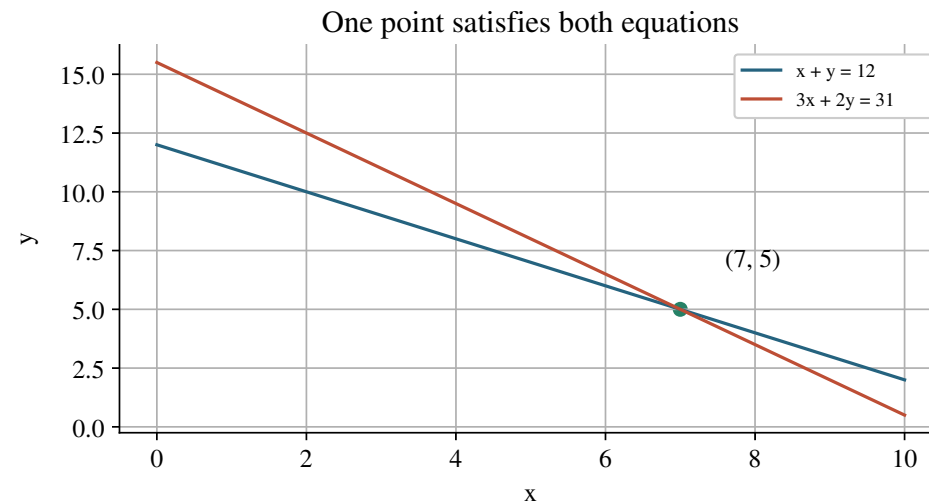
- For two linear equations, use elimination or substitution and check both equations.
- For a line and a quadratic, substitute the linear relation first; then solve the resulting quadratic.
- For inequalities, shade the region satisfying every condition.

A worked case

$$x + y = 12, \quad 3x + 2y = 31$$

■ Worked example

For $x+y=12$ and $3x+2y=31$, subtract twice the first equation to obtain $x=7, y=5$. For $y=x+2$ and $y=x^2$, equate outputs: $x^2-x-2=0$, hence $x=2$ or -1 . The intersections are $(2,4)$ and $(-1,1)$, and both satisfy the line and parabola.



Where it goes wrong

- One equation checked is not enough.
- A line can meet a quadratic twice, so retain both solutions unless the context removes one.
- Inequality boundaries may be included or excluded according to the sign.

■ Watch out

Not true: A pair of simultaneous equations is solved by checking just one of them. A solution must satisfy both equations; check it in each one.

Applying it

- AQA A19 Higher includes two linear equations and linear/quadratic systems.
- Elimination suits linear pairs; substitution reduces a line/curve pair to a quadratic.
- Retain every solution and check both original equations.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Elimination:** Combining equations to remove one variable while retaining the same solutions.
- For two linear equations, use elimination or substitution and check both equations.
- Watch for: one equation checked is not enough.

3.2 · Straight lines and gradients · Foundation

How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
- This lesson studies **gradient** : The change in y divided by the corresponding change in x .

The method

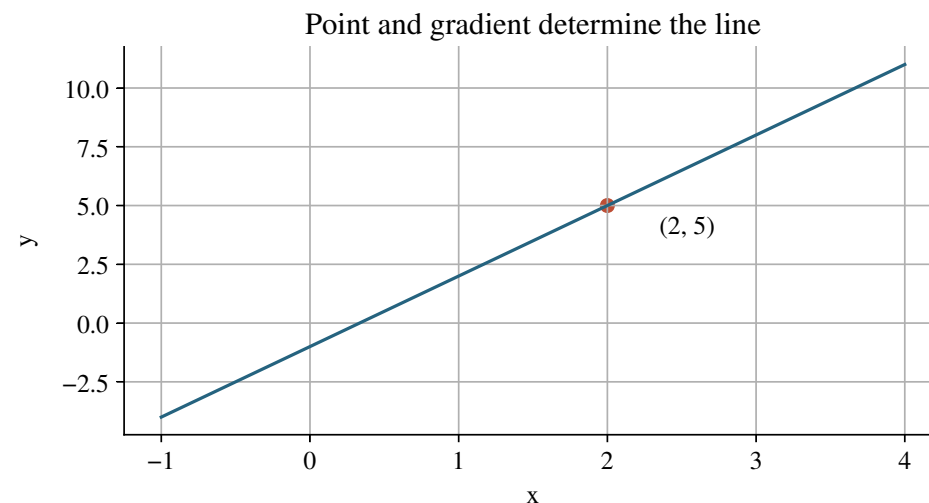
- Gradient is change in y divided by change in x .
- A straight line has $y=mx+c$, where c is its y -intercept.
- Parallel lines have equal gradients.

A worked case

$$y = mx + c$$

■ Worked example

Through $(2,5)$ with gradient 3, substitute to get $5=3\times 2+c$, so $c=-1$ and $y=3x-1$. Points $(1,2)$ and $(4,8)$ give gradient $(8-2)/(4-1)=2$.



Where it goes wrong

- A vertical line has no finite gradient; do not force it into $y=mx+c$.
- Read the signs of a circle's centre carefully.
- The radius to a tangent is perpendicular to the tangent.

■ Watch out

Not true: Perpendicular nonvertical lines have equal gradients. Perpendicular gradients multiply to -1 : a gradient of 2 needs $-1/2$. Parallel lines have equal gradients.

Applying it

- Plot a straight line using two checked points and label its intercept.
- Perpendicular-gradient formulae and circle equations are not part of this Foundation/Core lesson.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Gradient:** The change in y divided by the corresponding change in x .
- Gradient is change in y divided by change in x .
- Watch for: A vertical line has no finite gradient; do not force it into $y=mx+c$.

3.2 · Coordinate geometry and tangents · Higher

How does a path's slope become an equation?

- A path rises 6 metres over a horizontal distance of 3 metres. Its gradient connects a diagram to an equation.
- This lesson studies **gradient**: The change in y divided by the corresponding change in x .

The method

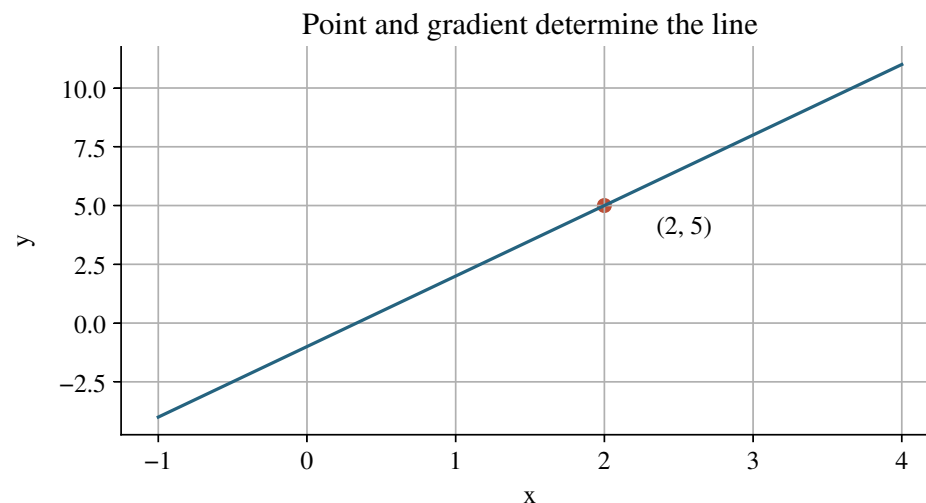
- A line through (x_1, y_1) with gradient m has $y - y_1 = m(x - x_1)$.
- Parallel lines have equal gradients.
- Finite perpendicular gradients multiply to -1 .
- A circle has $(x-a)^2 + (y-b)^2 = r^2$.

A worked case

$$y - y_1 = m(x - x_1), \quad (x - a)^2 + (y - b)^2 = r^2$$

■ Worked example

Through $(2, 5)$ with gradient 3, $y - 5 = 3(x - 2)$, so $y = 3x - 1$. A perpendicular through the same point has $y - 5 = -(x - 2)/3$. The circle $(x - 2)^2 + (y + 1)^2 = 25$ has centre $(2, -1)$ and radius 5.



Where it goes wrong

- A vertical line has no finite gradient; do not force it into $y=mx+c$.
- Read the signs of a circle's centre carefully.
- The radius to a tangent is perpendicular to the tangent.

■ Watch out

Not true: Perpendicular nonvertical lines have equal gradients. Perpendicular gradients multiply to -1 : a gradient of 2 needs $-1/2$. Parallel lines have equal gradients.

Applying it

- Before solving a line-circle intersection, predict whether there are zero, one or two intersections.
- Substitution produces a quadratic whose discriminant checks the prediction.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Gradient:** The change in y divided by the corresponding change in x .
- A line through (x_1, y_1) with gradient m has $y - y_1 = m(x - x_1)$.
- Watch for: A vertical line has no finite gradient; do not force it into $y = mx + c$.

3.2 · Arithmetic sequences and n th terms · Foundation

Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common difference**: The constant added between consecutive terms of an arithmetic sequence.

The method

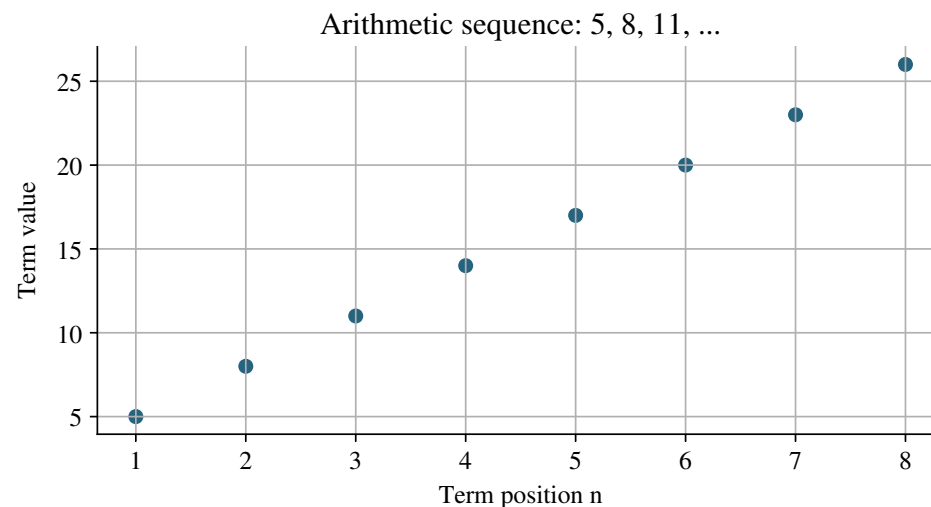
- Find a constant difference for an arithmetic sequence.
- Its n th term is $a + (n-1)d$.
- A term-to-term rule describes how to reach the next term; a position-to-term rule gives a term directly.

A worked case

$$u_n = a + (n - 1)d$$

■ Worked example

For 5, 8, 11, 14, ... the common difference is 3. The n th term is $5 + 3(n-1) = 3n + 2$. At $n=8$, $u_8 = 26$. To find the position of 62, solve $3n + 2 = 62$, giving $n=20$.



Where it goes wrong

- The first term has index 1, so the exponent is $n-1$.
- A sequence is a list; a series is a sum.
- A geometric sequence can alternate in sign and still converge.

■ Watch out

Not true: The first term of a sequence has index zero. A sum to infinity exists only when $-1 < r < 1$; for $r=2$ the terms grow without limit.

Applying it

- Generate several terms and check a proposed n th-term rule.
- Infinite geometric series and advanced sum formulae are excluded from this Foundation/Core lesson.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Common difference:** The constant added between consecutive terms of an arithmetic sequence.
- Find a constant difference for an arithmetic sequence.
- Watch for: the first term has index 1, so the exponent is $n-1$.

3.2 · Arithmetic and finite geometric sequences · Higher

Does the change add or multiply?

- A saving plan adds ¥30 more each week; a population model grows by 5% each year. Equal differences and equal ratios need different models.
- This lesson studies **common ratio**: The constant multiplier between consecutive terms of a geometric sequence.

The method

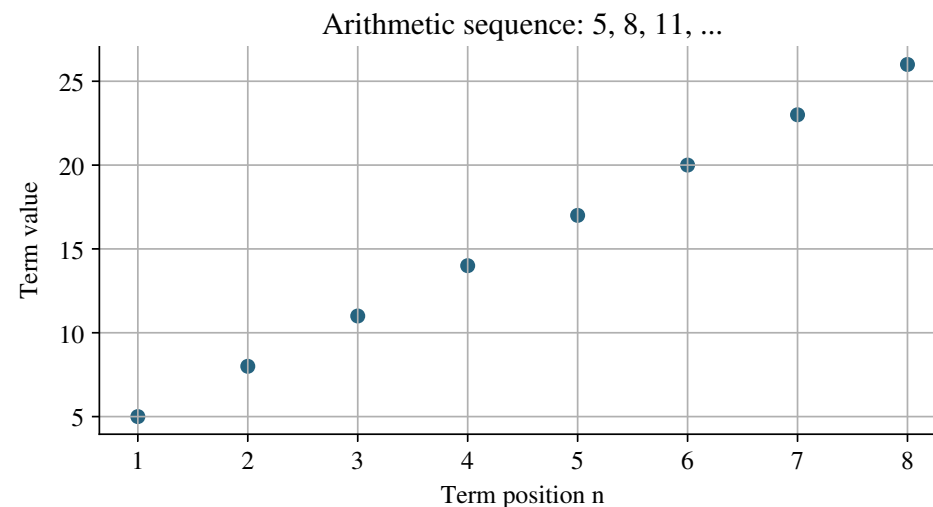
- An arithmetic sequence adds a constant difference and has n th term $a+(n-1)d$.
- A geometric sequence multiplies by a constant ratio and has n th term ar^{n-1} .
- Check the starting position and keep a surd ratio exact when one is supplied.

A worked case

$$u_n = a + (n-1)d, \quad v_n = ar^{n-1}$$

■ Worked example

For 5,8,11,... the n th term is $3n+2$, so $u_8=26$. For 2,6,18,... the common ratio is 3 and $u_n=2 \times 3^{n-1}$, giving $u_4=54$. The geometric sequence $1, \sqrt{2}, 2, 2\sqrt{2}, \dots$ has ratio $\sqrt{2}$. These are finite-term calculations; no infinite-series sum is used.



Where it goes wrong

- A geometric ratio is not a common difference.
- The exponent is $n-1$ when a is the first term at position 1.
- A finite pattern does not itself justify an infinite-sum formula.

■ Watch out

Not true: A common ratio and a common difference describe the same operation. A sum to infinity exists only when $-1 < r < 1$; for $r=2$ the terms grow without limit.

Applying it

- AQA A23–A25 Higher uses arithmetic rules, geometric progression patterns and quadratic n th terms.
- Arithmetic-series formulae and infinite-series sums are excluded from this GCSE lesson.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Common ratio:** The constant multiplier between consecutive terms of a geometric sequence.
- An arithmetic sequence adds a constant difference and has n th term $a+(n-1)d$.
- Watch for: A geometric ratio is not a common difference.

3.2 · Algebraic notation, substitution and vocabulary · Foundation

What does each symbol do?

- A delivery charge is 4 yuan plus 3 yuan for each kilometre. What do the separate terms in $4+3d$ describe?
- This lesson studies **coefficient** : The numerical factor multiplying a variable term.

The method

- A term is a part joined by addition or subtraction.
- In $4+3d$, 4 is a constant and 3 is the coefficient of d .
- The expression has no equality sign; $4+3d=19$ is an equation.
- A formula connects named quantities.
- In ab , multiplication is understood; a^2b means $a \times a \times b$, not $a \times b \times b$.

A worked case

$$a^2b = (-2)^2 \times 3 = 12$$

■ Worked example

For $d=5$, the charge is $4+3 \times 5=19$ yuan. For $a=-2, b=3$, $a^2b=(-2)^2 \times 3=12$. The fraction coefficient in $(\frac{3}{4})x$ gives 6 when $x=8$. An inequality $4+3d \leq 19$ describes all permitted distances, while an identity such as $2(x+3)=2x+6$ is true for every x .

Constant + variable charge

4

+

$3 \times d$

Where it goes wrong

- Put a negative substituted number in brackets before squaring.
- A coefficient is not an exponent.
- An expression can be evaluated but cannot be solved unless a condition or equation is supplied.

■ Watch out

Not true: In the expression $3x^2$, the coefficient of x^2 is 2. The coefficient is the number multiplying the term: in $3x^2$ it is 3; 2 is the power.

Applying it

- AQA A1–A3 require precise notation and vocabulary, including formulae from other subjects.
- Identify the inputs and units before substitution; keep fraction coefficients exact.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Coefficient:** The numerical factor multiplying a variable term.
- A term is a part joined by addition or subtraction.
- Watch for: put a negative substituted number in brackets before squaring.

3.2 · Algebraic notation, substitution and vocabulary · Higher

What does each symbol do?

- A delivery charge is 4 yuan plus 3 yuan for each kilometre. What do the separate terms in $4+3d$ describe?
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A worked case

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■ Worked example

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Applying it

- AQA A1–A3 require precise notation and vocabulary, including formulae from other subjects.
- Identify the inputs and units before substitution; keep fraction coefficients exact.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Coefficient:** The numerical factor multiplying a variable term.
- A term is a part joined by addition or subtraction.
- Watch for: put a negative substituted number in brackets before squaring.

3.2 · Collecting, expanding and factorising expressions · Foundation

Can two area descriptions agree?

- An L-shaped area has a symbolic width. Can two different expressions describe the same total area?
- This lesson studies **factorise** : Rewrite an expression as a product of factors.

The method

- Collect like terms, distribute over brackets and reverse expansion by factorising.
- Take out a common factor first.
- Expand products of two binomials and factorise simple monic quadratics and differences of squares.

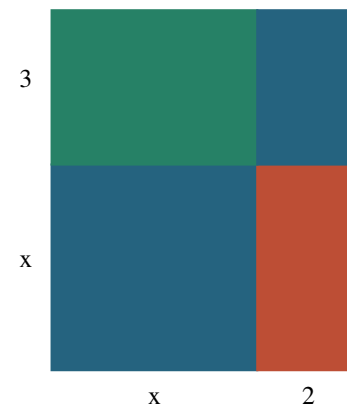
A worked case

$$(x + 2)(x + 3) = x^2 + 5x + 6$$

■ Worked example

$3x+4x=7x$; $3x+4y$ cannot be combined. Expand $2(x+3)=2x+6$ and $(x+2)(x+3)=x^2+5x+6$. Reverse the last identity to factorise x^2+5x+6 . Difference of squares gives $x^2-9=(x-3)(x+3)$. The same expansion rule works with given roots: $(\sqrt{2}+1)(\sqrt{2}-1)=2-1=1$.

$$(x + 2)(x + 3) = x^2 + 5x + 6$$



Where it goes wrong

- x^2 and x are unlike terms.
- A factor multiplies a whole expression; it is not a separate added term.
- Check a proposed factorisation by expanding it.

■ Watch out

Not true: The terms $3x$ and $4y$ can be collected to give $7xy$. Only a common factor of the whole numerator and the whole denominator can cancel: $(x+2)/x$ is not 2 .

Applying it

- AQA A4 Foundation includes collecting, single brackets, common factors, two-binomial expansion and monic quadratic factorisation.
- Higher adds non-monic factorisation, more binomials, surds and algebraic fractions.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Factorise:** Rewrite an expression as a product of factors.
- Collect like terms, distribute over brackets and reverse expansion by factorising.
- Watch for: x^2 and x are unlike terms.

3.2 · Collecting, expanding and factorising expressions · Higher

Can two area descriptions agree?

- An L-shaped area has a symbolic width. Can two different expressions describe the same total area?
- This lesson studies **factorise** : Rewrite an expression as a product of factors.

The method

- Collect only like terms.
- Distribute multiplication over every term in a bracket, including signs.
- Factorising reverses expansion.
- Use common factors first; then pairs for a quadratic.
- Algebraic fractions may cancel common factors, with forbidden denominator values stated.

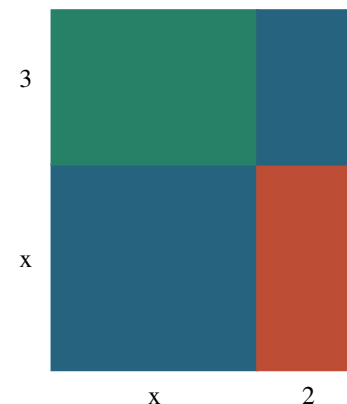
A worked case

$$(x + 2)(x + 3) = x^2 + 5x + 6$$

■ Worked example

$3x + 4x = 7x$, but $3x + 4y$ cannot be combined. Expand $2(x+3) = 2x+6$ and $(x+2)(x+3) = x^2 + 5x + 6$. Reverse the last result to factorise $x^2 + 5x + 6$. Difference of squares gives $x^2 - 9 = (x-3)(x+3)$. For Higher, $2x^2 + 5x + 2 = (2x+1)(x+2)$, and $(x^2 - 9)/(x-3) = x+3$ only when $x \neq 3$. Also $(x+1)(x+2)(x+3) = x^3 + 6x^2 + 11x + 6$.

$$(x + 2)(x + 3) = x^2 + 5x + 6$$



Where it goes wrong

- x^2 and x are unlike terms.
- Cancelling across a sum is invalid; factor the whole expression first.
- A simplified fraction must retain values excluded by its original denominator.

■ Watch out

Not true: A term can be cancelled from a numerator and denominator even if it is not a factor of the whole expression. Only a common factor of the whole numerator and the whole denominator can cancel: $(x+2)/x$ is not 2.

Applying it

- AQA A4 Foundation includes collecting, single brackets, common factors, two-binomial expansion and monic quadratic factorisation.
- Higher adds non-monic factorisation, more binomials, surds and algebraic fractions.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Factorise:** Rewrite an expression as a product of factors.
- Collect only like terms.
- Watch for: x^2 and x are unlike terms.

3.2 · Rearranging formulae and checking the subject · Foundation

Can we work backwards from the area?

- A triangle has known area and height, but its base is missing. How can the area formula be reversed?
- This lesson studies **subject** : The quantity written alone on one side of a formula.

The method

- Undo operations on both sides in reverse order.
- In $A = bh/2$, multiply both sides by 2 then divide by nonzero h to obtain $b = 2A/h$.
- When the desired variable occurs twice, collect it before dividing.
- State restrictions introduced by division.

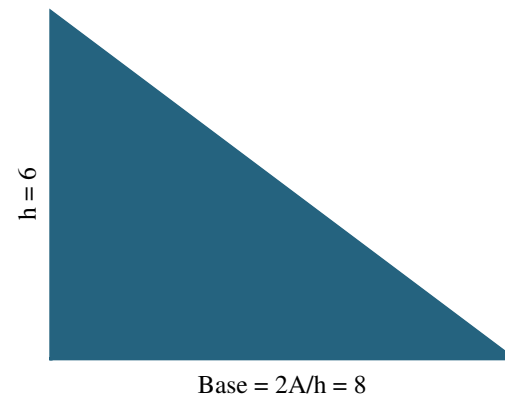
A worked case

$$A = \frac{bh}{2} \iff b = \frac{2A}{h} \quad (h \neq 0)$$

■ Worked example

For $A=24, h=6$ in $A=bh/2$, $b=2 \times 24/6=8$. From $v=u+at$, $a=(v-u)/t$; $v=19, u=4, t=5$ gives $a=3$. From $y=3x+2$, $x=(y-2)/3$. From $p=qx+r$, $x=(p-r)/q$ for $q \neq 0$. Substitute the result into the original equation to check it.

Undo the area formula to find the base



Where it goes wrong

- Doing an operation to only one side breaks the equality.
- The subject is a variable, not a numerical answer.
- Check the rearranged result by putting the found value into the original formula.

■ Watch out

Not true: To rearrange a formula, an operation may be applied to one side only. The same operation must be applied to both sides to keep the equation balanced.

Applying it

- AQA A5 includes standard and given formulae from other subjects, in words and symbols.
- Choose a valid unit conversion before substituting; a rearrangement does not itself change units.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Subject:** The quantity written alone on one side of a formula.
- Undo operations on both sides in reverse order.
- Watch for: doing an operation to only one side breaks the equality.

3.2 · Rearranging formulae and checking the subject · Higher

Can we work backwards from the area?

- A triangle has known area and height, but its base is missing. How can the area formula be reversed?
- This lesson studies **subject** : The quantity written alone on one side of a formula.

The method

- Undo operations on both sides in reverse order.
- In $A = bh/2$, multiply both sides by 2 then divide by nonzero h to obtain $b = 2A/h$.
- When the desired variable occurs twice, collect it before dividing.
- State restrictions introduced by division.

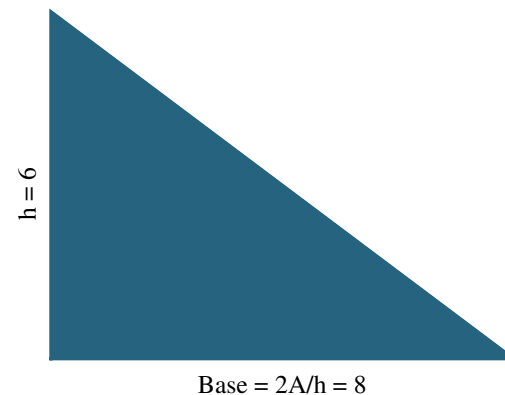
A worked case

$$A = \frac{bh}{2} \iff b = \frac{2A}{h} \quad (h \neq 0)$$

■ Worked example

For $A=24, h=6$, $b=2 \times 24/6=8$. From $v=u+at$, $a=(v-u)/t$; for $v=19, u=4, t=5$ this gives $a=3$. From $y=3x+2$, $x=(y-2)/3$. From $p=qx+r$, $x=(p-r)/q$ for $q \neq 0$. Higher may factor repeated subjects: $y=3x+px$ gives $x=y/(3+p)$ when $p \neq -3$.

Undo the area formula to find the base



Where it goes wrong

- Doing an operation to only one side breaks the equality.
- The subject is a variable, not a numerical answer.
- Check the rearranged result by putting the found value into the original formula.

■ Watch out

Not true: To rearrange a formula, an operation may be applied to one side only. The same operation must be applied to both sides to keep the equation balanced.

Applying it

- AQA A5 includes standard and given formulae from other subjects, in words and symbols.
- Choose a valid unit conversion before substituting; a rearrangement does not itself change units.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Subject:** The quantity written alone on one side of a formula.
- Undo operations on both sides in reverse order.
- Watch for: doing an operation to only one side breaks the equality.

3.2 · Identities, equivalence and algebraic arguments · Foundation

Does one successful test prove a claim?

- Two students test $x=1$ and get equal results. Does that prove their expressions agree for every x ?
- This lesson studies **equivalent expression**: An expression with the same value as another for every allowed input.

The method

- An equation may hold only for some values, while an identity holds for every allowed input.
- Establish equivalent expressions by valid expansion or factorisation.
- A few matching inputs are checks rather than a general argument.

A worked case

$$3(x+2) - x = 2x + 6$$

■ Worked example

Expand $3(x+2)-x=3x+6-x=2x+6$. This chain of valid steps shows the expressions agree for every x . However, $x^2=x$ holds only when $x=0$ or $x=1$; at $x=2$, $4 \neq 2$. Evaluating $(2n+1)^2$ at $n=3$ gives 49, but this one calculation does not establish an all-integers claim.

Valid expansion establishes equivalence

$$3(x+2) - x$$

$$3x + 6 - x$$

$$2x + 6$$

Where it goes wrong

- Start from an expression or the assumptions, not from the conclusion as though it were already true.
- An example can disprove an all-values claim, but one confirming example cannot prove it.

■ Watch out

Not true: Testing two expressions at one input proves that they are identical. Two different expressions can agree at one input; an identity must hold for every value, which needs algebra.

Applying it

- AQA A3/A6 Foundation distinguishes expression, equation and identity and argues equivalence using algebra.
- General parity and divisibility proofs belong to the Higher variant.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Equivalent expression:** An expression with the same value as another for every allowed input.
- An equation may hold only for some values, while an identity holds for every allowed input.
- Watch for: start from an expression or the assumptions, not from the conclusion as though it were already true.

3.2 · Identities, equivalence and algebraic arguments · Higher

Does one successful test prove a claim?

- Two students test $x=1$ and get equal results. Does that prove their expressions agree for every x ?
- This lesson studies **equivalent expression** : An expression with the same value as another for every allowed input.

The method

- An equation may be true only at certain values.
- An identity is true at every allowed value.
- Establish equivalence by valid expansion or factorisation; testing a few inputs is only a check.
- Higher proofs use a general integer or algebraic variable and a conclusion tied to its definition.

A worked case

$$3(x+2) - x = 2x + 6$$

■ Worked example

Expanding $3(x+2)-x$ gives $3x+6-x=2x+6$, proving equivalence for every x . But $x^2=x$ holds only for $x=0$ or 1 . A counterexample $x=2$ rejects an all-values claim. For Higher, an odd integer is $2n+1$; its square is $4n^2+4n+1=2(2n^2+2n)+1$, so it is odd for every integer n .

Valid expansion establishes equivalence

$$3(x+2) - x$$

$$3x + 6 - x$$

$$2x + 6$$

Where it goes wrong

- Start from an expression or the assumptions, not from the conclusion as though it were already true.
- An example can disprove an all-values claim, but one confirming example cannot prove it.

■ Watch out

Not true: Testing two expressions at one input proves that they are identical. Two different expressions can agree at one input; an identity must hold for every value, which needs algebra.

Applying it

- AQA A6 Foundation distinguishes equation/identity and argues equivalence.
- Higher extends this to algebraic proofs.
- State integer restrictions when using parity or consecutive integers.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Equivalent expression:** An expression with the same value as another for every allowed input.
- An equation may be true only at certain values.
- Watch for: start from an expression or the assumptions, not from the conclusion as though it were already true.

3.2 · Function machines and reversing operations · Foundation

Which input produced the output?

- A machine doubles a number then adds three. Which starting number would produce eleven?
- This lesson studies **input** : The value supplied to a rule before its operations are carried out.

The method

- Follow the operations in their stated order.
- To recover a starting input, undo the final operation first.
- A table pairs each input with its output.
- The same starting input must have only one output for the rule to be a function.

A worked case

$$x \mapsto 2x + 3$$

■ Worked example

Input 4 gives $2 \times 4 + 3 = 11$. To find the unknown input for output 11, subtract 3 to get 8 then divide by 2 to get 4. Inputs -1,0,1 give outputs 1,3,5. A table lists each input beside its output; the operations are always performed in the same order.

Forward order; reverse by undoing the last step first

Input 4

$\times 2 \rightarrow 8$

$+ 3 \rightarrow 11$

Where it goes wrong

- Reversing the rule does not mean repeating it.
- The last forward step is the first reverse step.
- Different operation orders can produce different outputs.

■ Watch out

Not true: Reversing double then add three means subtracting three only. Undo the steps in reverse order: subtract 3 first, then halve.

Applying it

- AQA A7 Foundation interprets simple functions as input-output rules.
- Higher also uses formal inverse and composite function notation in the separate function lesson.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Input:** The value supplied to a rule before its operations are carried out.
- Follow the operations in their stated order.
- Watch for: reversing the rule does not mean repeating it.

3.2 · Function machines and reversing operations · Higher

Which input produced the output?

- A machine doubles a number then adds three. Which starting number would produce eleven?
- This lesson studies **input** : The value supplied to a rule before its operations are carried out.

The method

- Follow the operations in their stated order.
- To recover a starting input, undo the final operation first.
- A table pairs each input with its output.
- The same starting input must have only one output for the rule to be a function.

A worked case

$$x \mapsto 2x + 3$$

■ Worked example

Input 4 gives $2 \times 4 + 3 = 11$. To reverse output 11, subtract 3 to get 8 then divide by 2 to get 4. Inputs -1, 0, 1 give outputs 1, 3, 5. If a second machine squares its input, passing 4 through the first then the second gives $11^2 = 121$; reversing their order gives $2 \times 16 + 3 = 35$.

Forward order; reverse by undoing the last step first

Input 4

$\times 2 \rightarrow 8$

$+ 3 \rightarrow 11$

Where it goes wrong

- Reversing the rule does not mean repeating it.
- The last forward step is the first reverse step.
- Different operation orders can produce different outputs.

■ Watch out

Not true: Reversing double then add three means subtracting three only. Undo the steps in reverse order: subtract 3 first, then halve.

Applying it

- AQA A7 Foundation interprets simple functions as input-output rules.
- Higher also uses formal inverse and composite function notation in the separate function lesson.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Input:** The value supplied to a rule before its operations are carried out.
- Follow the operations in their stated order.
- Watch for: reversing the rule does not mean repeating it.

3.2 · Domains, inverses and composition · Higher

Which inputs are allowed?

- A square-root model returns a real output only for some inputs. Its formula alone does not specify a complete function.
- This lesson studies **domain** : The set of allowed inputs to a function.

The method

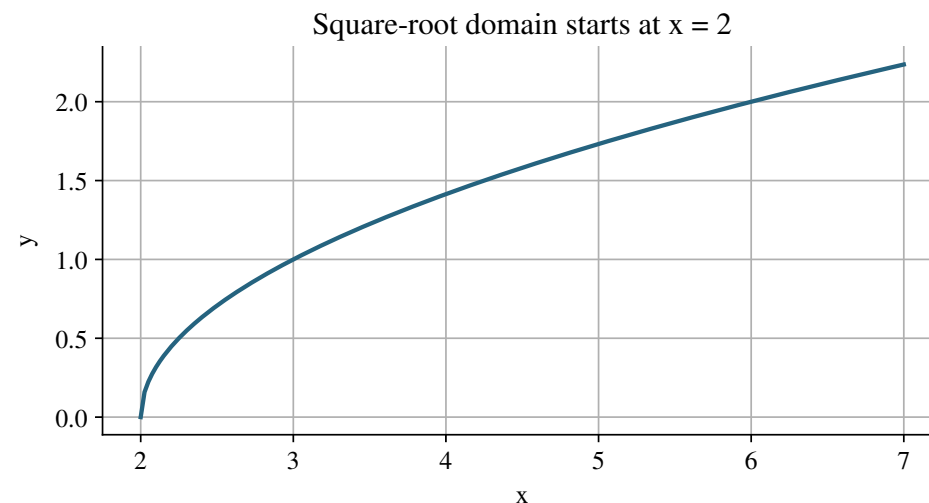
- State the domain and range.
- For an inverse, first ensure the function is one-to-one on its domain.
- Composition fg means apply g first, then f ; the intermediate output must be an allowed input to f .

A worked case

$$f(g(x)) = (f \circ g)(x), \quad f^{-1}(f(x)) = x$$

■ Worked example

For $f(x)=\sqrt{x-2}$, $x \geq 2$ and the range is $y \geq 0$. From $y=\sqrt{x-2}$, $x=y^2+2$. Thus f inverse(x)= x^2+2 with $x \geq 0$. For $g(x)=x+3$, $fg(1)=f(4)=\sqrt{2}$.



Where it goes wrong

- Squaring can introduce extraneous solutions.
- Restricting a parabola's domain is essential before claiming an inverse.
- A horizontal translation inside f has the opposite sign to the graph's movement.

■ Watch out

Not true: Every quadratic function on all real numbers has an inverse function. A quadratic on all real numbers is many-to-one ($f(1)=f(-1)$ for x^2), so its domain must be restricted first.

Applying it

- Check $f(f \text{ inverse}(x))=x$ on the inverse domain.
- Use a sketch to test whether a horizontal line meets the original graph more than once.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Domain:** The set of allowed inputs to a function.
- State the domain and range.
- Watch for: squaring can introduce extraneous solutions.

3.2 · Coordinate quadrants and basic graph families · Foundation

What happens near the forbidden input?

- A table of a cube and a reciprocal gives very different values near zero. Can their graphs have the same shape?
- This lesson studies **asymptote** : A line approached by a graph without a finite crossing in the stated example.

The method

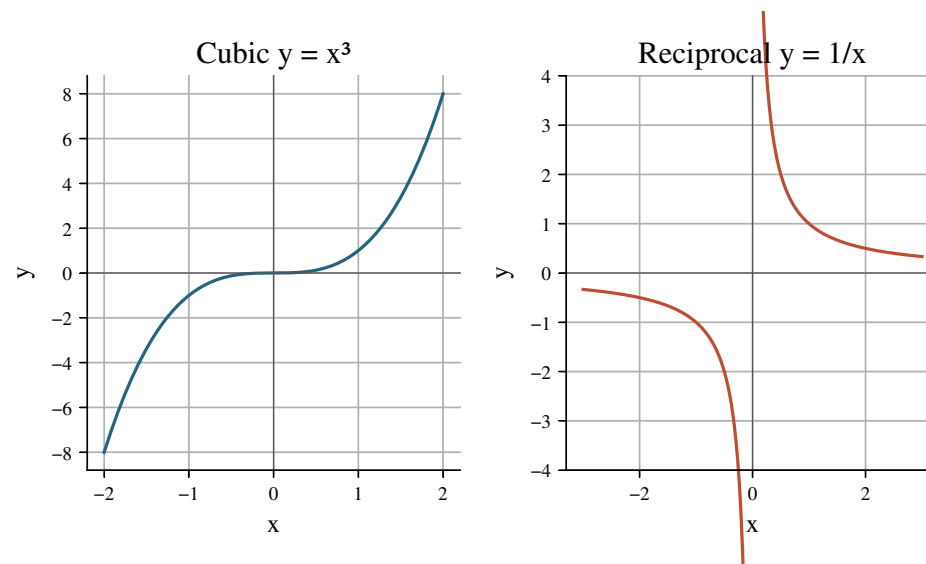
- Coordinates are ordered (x,y).
- Signs identify the four quadrants.
- Use a value table, intercepts and symmetry to sketch a line, quadratic, cubic or reciprocal.
- For $y=1/x$, zero is excluded and the axes are asymptotes.
- Approximate intersections give graphical solutions.

A worked case

$$y = x^3, \quad y = \frac{1}{x} \quad (x \neq 0)$$

■ Worked example

For $y=x^3$, inputs -2,-1,0,1,2 give -8,-1,0,1,8. For $y=1/x$, inputs -2,-1,1,2 give -1/2,-1,1,1/2; never substitute zero. The point (-2,3) is in quadrant II. The graphs $y=x^2$ and $y=4$ meet at $x=-2$ and $x=2$. A quadratic $y=(x-3)(x-7)$ has roots 3,7 and symmetry line $x=5$.



Where it goes wrong

- Join a reciprocal branch smoothly on its own side of zero; never draw a segment through the excluded input.
- A graph table needs enough points to reveal a turning point or shape.

■ Watch out

Not true: The graph of $y=1/x$ has a point at $x=0$. $1/0$ is undefined, so $y=1/x$ has no point at $x=0$; the y-axis is an asymptote.

Applying it

- AQA A8/A11/A12 Foundation includes all quadrants, lines, quadratics, simple cubics and reciprocals.
- Higher exponential and degree-based trigonometric families are treated separately.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Asymptote:** A line approached by a graph without a finite crossing in the stated example.
- Coordinates are ordered (x,y).
- Watch for: join a reciprocal branch smoothly on its own side of zero; never draw a segment through the excluded input.

3.2 · Coordinate quadrants and basic graph families · Higher

What happens near the forbidden input?

- A table of a cube and a reciprocal gives very different values near zero. Can their graphs have the same shape?
- This lesson studies **asymptote** : A line approached by a graph without a finite crossing in the stated example.

The method

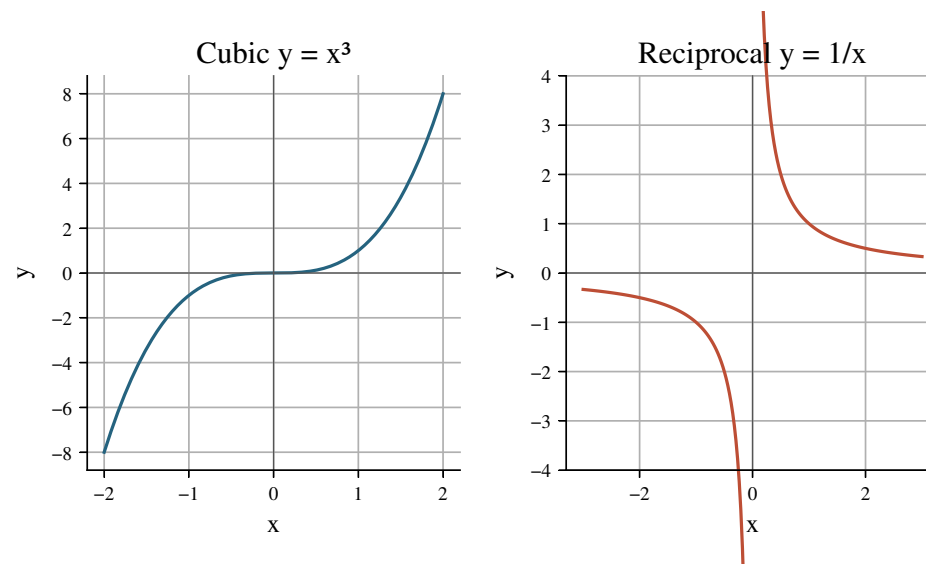
- Coordinates are ordered (x,y).
- Signs identify the four quadrants.
- Use a value table, intercepts and symmetry to sketch a line, quadratic, cubic or reciprocal.
- For $y=1/x$, zero is excluded and the axes are asymptotes.
- Approximate intersections give graphical solutions.

A worked case

$$y = x^3, \quad y = \frac{1}{x} \quad (x \neq 0)$$

■ Worked example

For $y=x^3$, inputs -2,-1,0,1,2 give -8,-1,0,1,8. For $y=1/x$, inputs -2,-1,1,2 give -1/2,-1,1,1/2; never substitute zero. The point (-2,3) is in quadrant II. The graphs $y=x^2$ and $y=4$ meet at $x=-2$ and $x=2$. A quadratic $y=(x-3)(x-7)$ has roots 3,7 and symmetry line $x=5$.



Where it goes wrong

- Join a reciprocal branch smoothly on its own side of zero; never draw a segment through the excluded input.
- A graph table needs enough points to reveal a turning point or shape.

■ Watch out

Not true: The graph of $y=1/x$ has a point at $x=0$. $1/0$ is undefined, so $y=1/x$ has no point at $x=0$; the y-axis is an asymptote.

Applying it

- AQA A8/A11/A12 Foundation includes all quadrants, lines, quadratics, simple cubics and reciprocals.
- Higher exponential and degree-based trigonometric families are treated separately.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Asymptote:** A line approached by a graph without a finite crossing in the stated example.
- Coordinates are ordered (x,y).
- Watch for: join a reciprocal branch smoothly on its own side of zero; never draw a segment through the excluded input.

3.2 · Exponential and degree-based trigonometric graphs · Higher

Which graph repeats and which keeps growing?

- A tide repeats, while a doubling population keeps growing. Which graph feature distinguishes these patterns?
- This lesson studies **period**: The input interval after which a repeating graph has the same values again.

The method

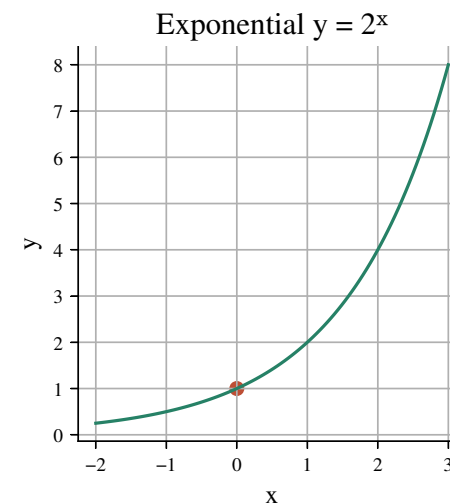
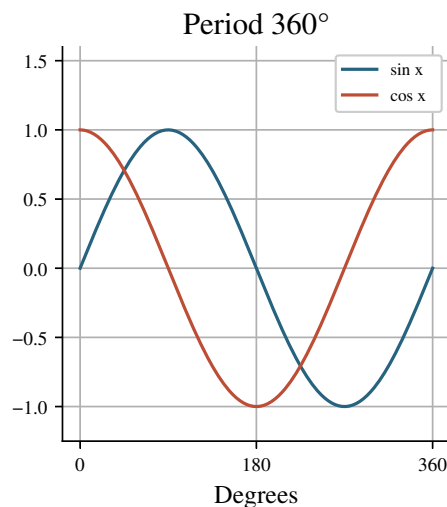
- For $y=k^x$ with $k>0, k\neq 1$, the y-intercept is 1; $k>1$ gives growth and $0<k<1$ gives decay.
- In degrees, sine and cosine repeat every 360° with range $[-1,1]$; tangent repeats every 180° and is undefined at $90^\circ+180^\circ n$.
- Label axes in degrees.

A worked case

$$\sin 270^\circ = -1, \quad \tan 45^\circ = 1$$

■ Worked example

For $y=2^x$, $x=-1,0,1,3$ give $1/2,1,2,8$. Sine has values 0,1,0,-1,0 at $0^\circ,90^\circ,180^\circ,270^\circ,360^\circ$. Cosine starts at 1; $\cos 180^\circ=-1$. Tangent has $\tan 45^\circ=1$ but no finite value at 90° . For $k=1/2$, increasing x produces decay, never a negative output.



Where it goes wrong

- Degrees and radians are different units.
- A steep tangent branch is not a finite point at its asymptote.
- Exponential growth is not repeated addition of a constant amount.

■ Watch out

Not true: The sine graph and tangent graph both have period 360° as their smallest positive period. Sine has period 360° , but tangent repeats every 180° .

Applying it

- AQA A12 Higher requires exponential functions with positive bases and sine/cosine/tangent for angles of any size in degrees.
- No differentiation or radian sector formula is introduced here.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Period:** The input interval after which a repeating graph has the same values again.
- For $y=k^x$ with $k>0, k \neq 1$, the y-intercept is 1; $k>1$ gives growth and $0<k<1$ gives decay.
- Watch for: degrees and radians are different units.

3.2 · Translations and reflections of function graphs · Higher

Does changing the input move the graph left or right?

- A graph starts with its minimum at $(0,0)$. How does changing the input to $x-3$ move the minimum?
- This lesson studies **graph translation**: Move every point of a graph by the same horizontal and vertical displacement.

The method

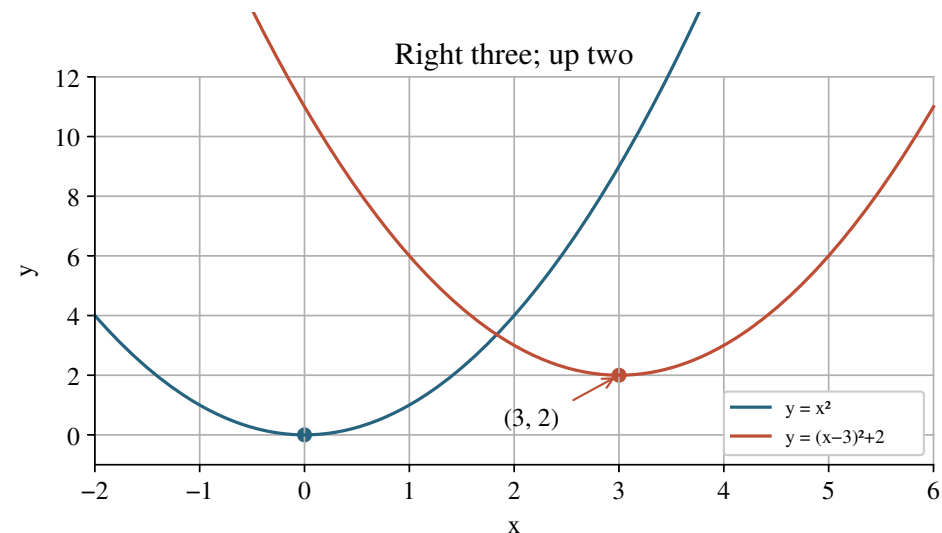
- $y=f(x)+a$ shifts the output up by a . $y=f(x-a)$ shifts the graph right by a . $y=-f(x)$ reflects in the x-axis; $y=f(-x)$ reflects in the y-axis.
- Transform the points as well as the formula, checking a known feature.

A worked case

$$y = (x - 3)^2 + 2$$

■ Worked example

With $f(x)=x^2$, $y=f(x-3)+2=(x-3)^2+2$ has vertex $(3,2)$. The point $(1,1)$ on f moves to $(4,3)$. At $x=4$ the new output is 3. The reflection $y=-x^2$ turns the minimum at the origin into a maximum. For a nonsymmetric graph, $f(-x)$ mirrors each x-coordinate, not each y-coordinate.



Where it goes wrong

- A positive number added inside the input, $f(x+3)$, moves the graph left, not right.
- A reflection in the y-axis changes x ; a reflection in the x-axis changes y .

■ Watch out

Not true: The graph $y=f(x+3)$ moves three units to the right. $f(x+3)$ moves the graph three units to the left.

Applying it

- AQA A13 Higher covers translations and reflections of a given function.
- Use features and transformed points to justify the sketch; do not substitute a geometric enlargement for this graph transformation.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Graph translation:** Move every point of a graph by the same horizontal and vertical displacement.
- $y=f(x)+a$ shifts the output up by a . $y=f(x-a)$ shifts the graph right by a . $y=-f(x)$ reflects in the x -axis; $y=f(-x)$ reflects in the y -axis.
- Watch for: A positive number added inside the input, $f(x+3)$, moves the graph left, not right.

3.2 · Distance-time graphs and contextual intersections · Foundation

Where is the waiting time on the graph?

- A delivery rider travels, waits, then travels again. Which parts of a distance-time graph reveal the waiting time?
- This lesson studies **stationary** : Not changing position over the time interval described.

The method

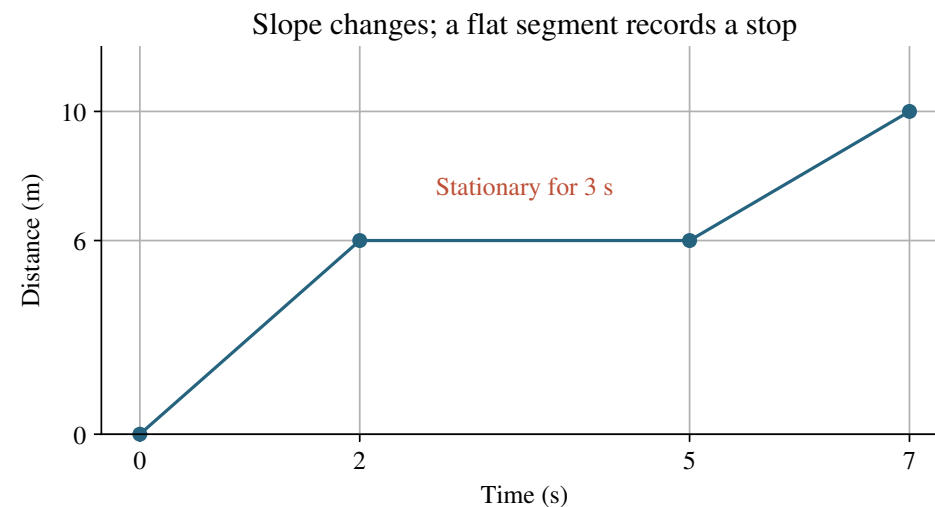
- Read the axis quantities and units before interpreting shape.
- On a distance-time graph, slope is speed for a segment with increasing distance; a horizontal segment means no distance change.
- An intersection of two charge graphs gives equal costs.
- A curved section requires a local rather than one fixed slope.

A worked case

$$v = \frac{\Delta d}{\Delta t}$$

■ Worked example

The distance points $(0,0), (2,6), (5,6), (7,10)$, in seconds and metres, give speed $6/2=3$ m/s for the first section, a 3-second stop, then speed $(10-6)/(7-5)=2$ m/s. Average speed for the full interval is $10/7$ m/s, including the stop. Charges $20+3x$ and $44+x$ meet at $x=12$, at cost 56. If speed increases from 0 to 10 m/s over 5 seconds, its average acceleration is $10/5=2$ m/s²; a speed-time graph measures this through its slope.



Where it goes wrong

- A horizontal distance graph does not mean fast motion.
- A graph height gives distance, while slope gives its rate.
- Average speed includes every elapsed interval, including waiting.

■ Watch out

Not true: A horizontal segment on a distance-time graph means the object is moving at its greatest speed. A horizontal segment means the distance is not changing, so the object is stationary.

Applying it

- AQA A14 uses real contexts and graphical solutions.
- Foundation can interpret a plotted non-standard function; Higher also interprets exponential models and nonlinear graph estimates in the next lesson.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Stationary:** Not changing position over the time interval described.
- Read the axis quantities and units before interpreting shape.
- Watch for: A horizontal distance graph does not mean fast motion.

3.2 · Distance-time graphs and contextual intersections · Higher

Where is the waiting time on the graph?

- A delivery rider travels, waits, then travels again. Which parts of a distance-time graph reveal the waiting time?
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The method

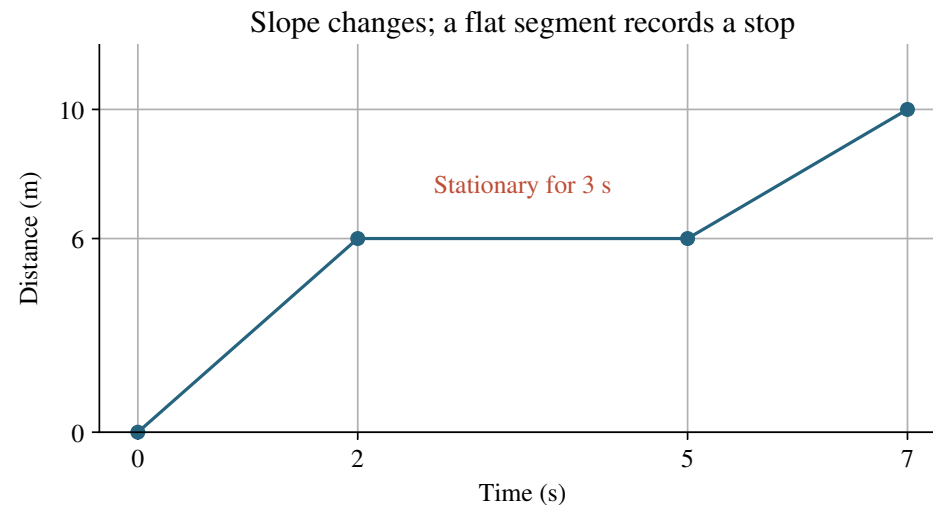
- Read the axis quantities and units before interpreting shape.
- On a distance-time graph, slope is speed for a segment with increasing distance; a horizontal segment means no distance change.
- An intersection of two charge graphs gives equal costs.
- A curved section requires a local rather than one fixed slope.

A worked case

$$v = \frac{\Delta d}{\Delta t}$$

■ Worked example

The distance points (0,0),(2,6),(5,6),(7,10), in seconds and metres, give speed $6/2=3$ m/s for the first section, a 3-second stop, then speed $(10-6)/(7-5)=2$ m/s. Average speed for the full interval is $10/7$ m/s, including the stop. Charges $20+3x$ and $44+x$ meet at $x=12$, at cost 56. If speed increases from 0 to 10 m/s over 5 seconds, its average acceleration is $10/5=2$ m/s²; a speed-time graph measures this through its slope.



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- Average speed includes every elapsed interval, including waiting.

■ Watch out

Not true: A horizontal segment on a distance-time graph means the object is moving at its greatest speed. A horizontal segment means the distance is not changing, so the object is stationary.

Applying it

- AQA A14 uses real contexts and graphical solutions.
- Foundation can interpret a plotted non-standard function; Higher also interprets exponential models and nonlinear graph estimates in the next lesson.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Stationary:** Not changing position over the time interval described.
- Read the axis quantities and units before interpreting shape.
- Watch for: A horizontal distance graph does not mean fast motion.

3.2 · Graphical gradients and area estimates · Higher

Which graphical quantity answers the question?

- A vehicle speeds up along a curve. Is the slope between its start and finish enough to describe its speed halfway?
- This lesson studies **tangent gradient**: The slope of a tangent, used to estimate a curve's rate at one input.

The method

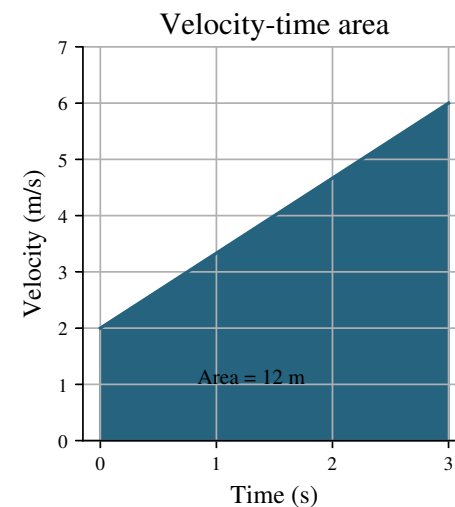
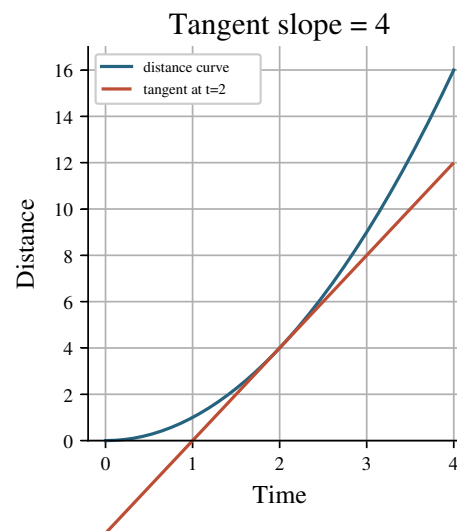
- A chord gives an average rate between two inputs; a tangent estimates the local rate.
- Read two well-separated points on the drawn tangent to reduce measurement error.
- On a velocity-time graph, area represents displacement.
- Split it into triangles/trapezia or estimate curved area using narrow strips.

A worked case

$$\frac{9-1}{3-1} = 4, \quad A = \frac{2+6}{2} \times 3 = 12$$

■ Worked example

For a distance curve $d=t^2$, the chord from (1,1) to (3,9) has gradient $(9-1)/(3-1)=4$. A tangent at (2,4) passes through (1,0) and (3,8), giving gradient 4 without calculus. A velocity-time trapezium with endpoint velocities 2 and 6 m/s over 3 s has area $(2+6) \times 3/2 = 12$ m. Smaller strips can improve a curved-area estimate; curvature affects over/underestimation. A cost-versus-quantity tangent through (2,12) and (6,28) gives a local rate $(28-12)/(6-2)=4$ yuan per extra item, linking graph slope to a financial interpretation.



Where it goes wrong

- Tangent estimates and curve chords use different pairs of points.
- Area under a distance-time graph is not distance travelled.
- A strip estimate is approximate unless the graph is linear on each strip.

■ Watch out

Not true: Area under any graph means the distance travelled. The area under a speed-time graph is distance; under other graphs the area means something else, or nothing useful.

Applying it

- AQA A15 and R15 Higher require graph-based rates and area interpretation.
- This lesson uses graphical reasoning and geometric areas, not symbolic differentiation/integration.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Tangent gradient:** The slope of a tangent, used to estimate a curve's rate at one input.
- A chord gives an average rate between two inputs; a tangent estimates the local rate.
- Watch for: tangent estimates and curve chords use different pairs of points.

3.2 · Origin-centred circles and tangent equations · Higher

Which line just touches the circular path?

- A circular path has a sensor at (3,4). Which straight line just touches the path at that point?
- This lesson studies **radius gradient** : The slope of a radius joining the centre to a point on the circle.

The method

- For an origin-centred circle, $x^2 + y^2 = r^2$.
- A point is on the circle if its squared coordinates sum to r^2 .
- A tangent is perpendicular to the radius there.
- Use a negative reciprocal gradient when both gradients are finite; handle horizontal/vertical cases directly.

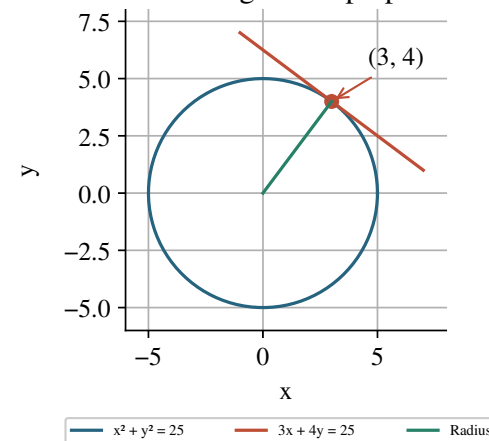
A worked case

$$x^2 + y^2 = 25, \quad 3x + 4y = 25$$

■ Worked example

At (3,4), $r^2 = 3^2 + 4^2 = 25$, so $x^2 + y^2 = 25$. The radius gradient is $4/3$, hence tangent gradient $-3/4$. Its equation $y - 4 = (-3/4)(x - 3)$ simplifies to $3x + 4y = 25$. At (5,0) the radius is horizontal and the tangent is the vertical line $x = 5$.

Radius and tangent are perpendicular



Where it goes wrong

- The radius and tangent share a point but different directions.
- Do not use the negative reciprocal of zero; a horizontal radius has a vertical tangent.
- Check the tangent passes through its contact point.

■ Watch out

Not true: The tangent at a circle point has the same gradient as the radius to that point. The tangent is perpendicular to the radius, so its gradient is the negative reciprocal of the radius gradient.

Applying it

- AQA A16 Higher concerns circles centred at the origin.
- Offset-centre circle formulae and circle calculus are not needed for this lesson.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Radius gradient:** The slope of a radius joining the centre to a point on the circle.
- For an origin-centred circle, $x^2 + y^2 = r^2$.
- Watch for: the radius and tangent share a point but different directions.

3.2 · Numerical iteration for equation roots · Higher

Will repeated calculation settle down?

- An equation has no convenient factorisation. Can repeated calculation locate a solution without guessing random values?
- This lesson studies **iteration** : Repeat a rule, using the previous output as the next input.

The method

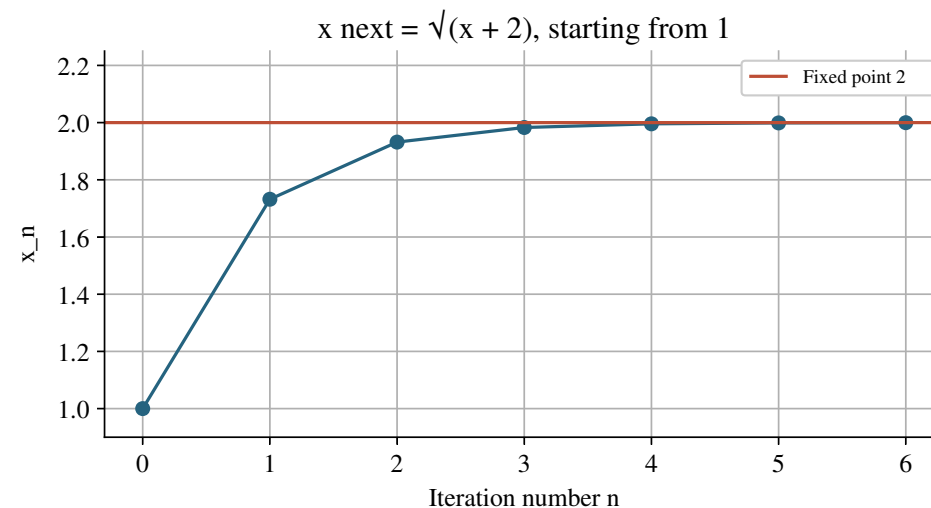
- Rearrange an equation as $x = g(x)$, choose x_0 and use $x_{(n+1)} = g(x_n)$.
- Record sufficient working precision.
- A stable-looking sequence must still be checked in the original equation; not every rearrangement converges.
- A sign change across continuous inputs can check a rounded root.

A worked case

$$x_{n+1} = \sqrt{x_n + 2}, \quad x_0 = 1$$

■ Worked example

For $x^2 - x - 2 = 0$, use $x_{\text{next}} = \sqrt{x+2}$ with $x_0 = 1$: $x_1 = \sqrt{3} \approx 1.73205$, $x_2 \approx 1.93185$, $x_3 \approx 1.98289$. The positive fixed point is 2 because $2 = \sqrt{4}$ and $2^2 - 2 - 2 = 0$. The rearrangement only seeks a nonnegative root; the original equation also has root -1. Using $x_{\text{next}} = x^2 - 2$ from 3 instead gives 7 then 47, so a different rearrangement can diverge.



Where it goes wrong

- Use the previous iterate, not the starting value every time.
- Keep more digits than the final answer.
- A sign change needs continuity; a jump across a vertical asymptote does not guarantee a root.

■ Watch out

Not true: Every rearrangement of an equation converges from every starting value. An iteration converges only when the rearranged function is not too steep near the root ($|g'(x)| < 1$).

Applying it

- AQA A20 Higher uses numerical iteration and suffix notation.
- Use the calculator to follow the stated rule, then report the requested rounding with a check.
- Newton derivatives are outside this GCSE lesson.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.

■ Key idea

- **Iteration:** Repeat a rule, using the previous output as the next input.
- Rearrange an equation as $x=g(x)$, choose x_0 and use $x_{(n+1)}=g(x_n)$.
- Watch for: use the previous iterate, not the starting value every time.

3.2 · Linear inequalities and number-line solutions · Foundation

Which purchases fit within the budget?

- A student can spend at most 17 yuan on a fixed 5-yuan charge plus 3 yuan per item. Is one equality enough to describe every allowed purchase?
- This lesson studies **solution set** : All inputs that satisfy the stated condition.

The method

- Solve a linear inequality using the same balance operations as an equation.
- Multiplying or dividing by a negative reverses its direction.
- Use a closed endpoint for $\leq$ or $\geq$ and an open endpoint for $<$ or $>$.
- Intersect restrictions to find their common permitted inputs.

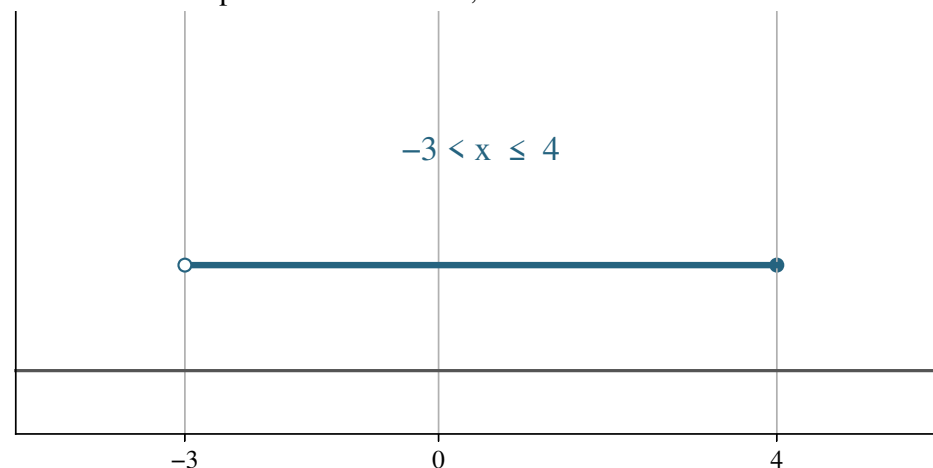
A worked case

$$3x + 5 \leq 17 \iff x \leq 4$$

■ Worked example

$3x+5\leq 17$ gives $x\leq 4$. For nonnegative whole items, the permitted values are 0,1,2,3,4. Also $-2x<6$ gives $x>-3$ after division by -2. The combined restriction $-3< x\leq 4$ has an open circle at -3 and a closed circle at 4. A value $x=5$ fails the original budget because $3\times 5+5=20$.

Open means excluded; closed means included



Where it goes wrong

- A reversed sign is needed only when multiplying or dividing by a negative, not when adding a negative.
- Include the physical domain: negative or fractional item counts may be meaningless.

■ Watch out

Not true: Dividing an inequality by a negative number leaves its direction unchanged. Dividing by a negative number reverses the inequality: $-2x < 6$ gives $x > -3$.

Applying it

- AQA A22 Foundation requires one-variable linear inequalities and number lines.
- Higher quadratic and two-variable regions are in a separate lesson; avoid replacing the inequality with a single boundary value.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Solution set:** All inputs that satisfy the stated condition.
- Solve a linear inequality using the same balance operations as an equation.
- Watch for: A reversed sign is needed only when multiplying or dividing by a negative, not when adding a negative.

3.2 · Linear inequalities and number-line solutions · Higher

Which purchases fit within the budget?

- A student can spend at most 17 yuan on a fixed 5-yuan charge plus 3 yuan per item. Is one equality enough to describe every allowed purchase?
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The method

- Solve a linear inequality using the same balance operations as an equation.
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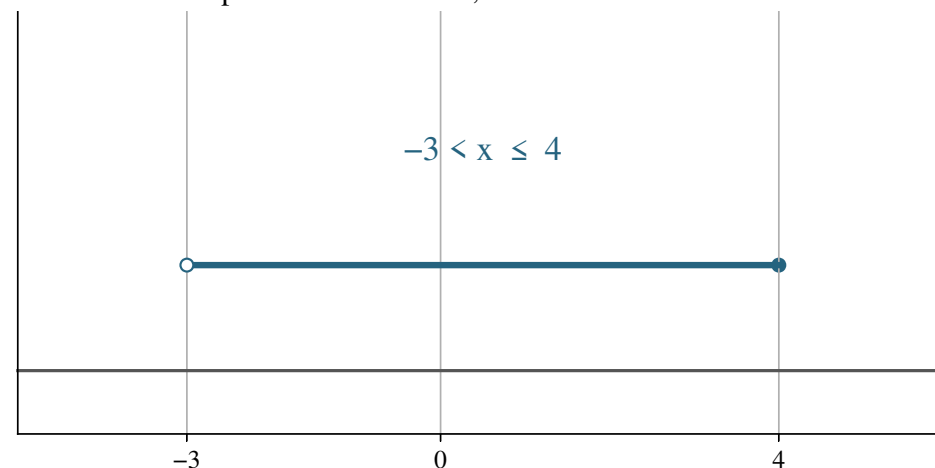
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■ Watch out

Not true: Dividing an inequality by a negative number leaves its direction unchanged. Dividing by a negative number reverses the inequality: $-2x < 6$ gives $x > -3$.

Applying it

- AQA A22 Foundation requires one-variable linear inequalities and number lines.
- Higher quadratic and two-variable regions are in a separate lesson; avoid replacing the inequality with a single boundary value.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Solution set:** All inputs that satisfy the stated condition.
- Solve a linear inequality using the same balance operations as an equation.
- Watch for: A reversed sign is needed only when multiplying or dividing by a negative, not when adding a negative.

3.2 · Quadratic inequalities and two-variable regions · Higher

Where do all the restrictions overlap?

- A design must lie above one line and below another. How can a graph show every feasible pair at once?
- This lesson studies **boundary line** : The equality line separating values that satisfy an inequality from those that do not.

The method

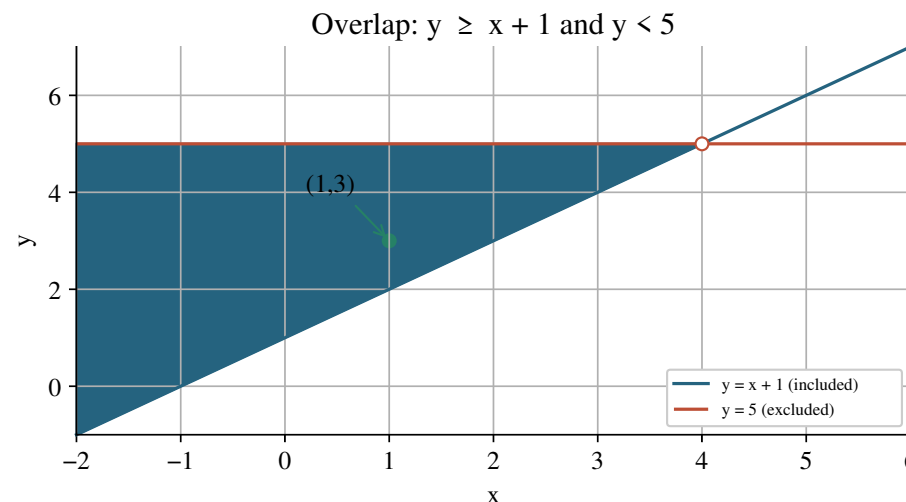
- For a quadratic inequality, locate roots and test the sign in each interval.
- For a two-variable inequality, draw its equality boundary; use a dashed line if equality is excluded and a solid line if included.
- Test a point on each side and intersect the allowed regions.
- Write the domain or set notation clearly.

A worked case

$$(x - 3)(x - 7) \leq 0 \iff x \in [3, 7]$$

■ Worked example

$(x-3)(x-7) \leq 0$ holds on $3 \leq x \leq 7$, since the factors have opposite signs between the roots and the endpoints give zero. For $y \geq x+1$ and $y < 5$, shade on/above the solid line $y=x+1$ and below the dashed line $y=5$. Their meeting input is $x=4$, but $(4,5)$ is excluded by $y < 5$. The point $(1,3)$ satisfies both inequalities.



Where it goes wrong

- For an upward-opening quadratic, positive values lie outside the roots, not between them.
- A boundary intersection is not necessarily included.
- Use a test point not on the boundary itself.

■ Watch out

Not true: A dashed inequality boundary includes every point on its line. A dashed boundary is not included; a solid boundary is included.

Applying it

- AQA A22 Higher includes quadratic inequalities in one variable and linear inequalities in two variables.
- The graphical overlap is the solution region, not one selected point.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Boundary line:** The equality line separating values that satisfy an inequality from those that do not.
- For a quadratic inequality, locate roots and test the sign in each interval.
- Watch for: for an upward-opening quadratic, positive values lie outside the roots, not between them.

3.2 · Figurate, geometric and Fibonacci-type sequences · Foundation

Does the dot pattern add or multiply?

- A growing dot pattern adds a longer row each time. How can a diagram distinguish it from repeated multiplication?
- This lesson studies **recursive rule** : A rule that generates new terms from preceding terms.

The method

- Record positions and terms separately.
- Triangular numbers add 1,2,3,...; square and cube numbers use n^2 and n^3 .
- A geometric sequence multiplies by a common positive ratio.
- A Fibonacci-type sequence starts with stated terms and then adds its two predecessors.
- A supplied recursive rule must include enough initial values.

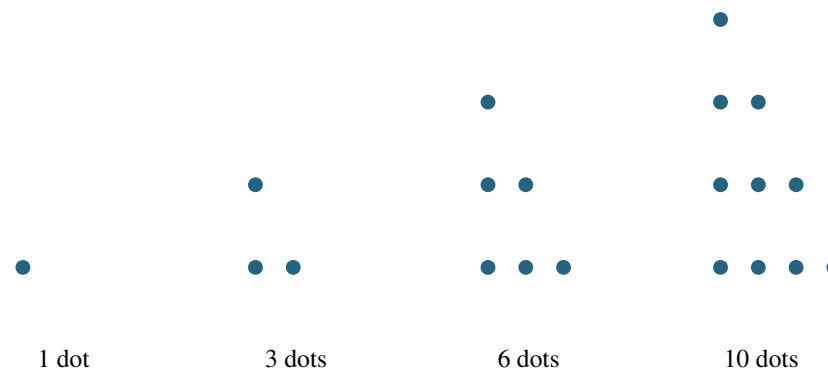
A worked case

$$u_n = 2 \times 3^{n-1}$$

■ Worked example

Triangular terms are 1,3,6,10,15; square terms 1,4,9,16,25; cube terms 1,8,27,64,125. The geometric sequence 2,6,18,54,... has ratio 3 and n th term $2 \times 3^{(n-1)}$. Starting 2,3 and adding the previous two gives 2,3,5,8,13. For $u_{\text{next}} = 2u + 1$ from $u_1 = 1$, the next terms are 3,7,15.

Triangular patterns add a new row each time



Where it goes wrong

- A nonconstant first difference does not mean a pattern is random.
- A geometric ratio is not a common difference.
- Check the starting index when writing a position rule.

■ Watch out

Not true: Every sequence with increasing terms has a constant difference. Increasing sequences can have changing differences, such as 1, 4, 9, 16.

Applying it

- AQA A23/A24 Foundation includes these patterns and positive rational geometric ratios.
- Higher also permits surd ratios and derives quadratic n th terms in the next lesson.
- Infinite-series sums are not part of this GCSE lesson.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Recursive rule:** A rule that generates new terms from preceding terms.
- Record positions and terms separately.
- Watch for: A nonconstant first difference does not mean a pattern is random.

3.2 · Figurate, geometric and Fibonacci-type sequences · Higher

Does the dot pattern add or multiply?

- A growing dot pattern adds a longer row each time. How can a diagram distinguish it from repeated multiplication?
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The method

- Record positions and terms separately.
- Triangular numbers add 1,2,3,...; square and cube numbers use n^2 and n^3 .
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- A Fibonacci-type sequence starts with stated terms and then adds its two predecessors.
- A supplied recursive rule must include enough initial values.

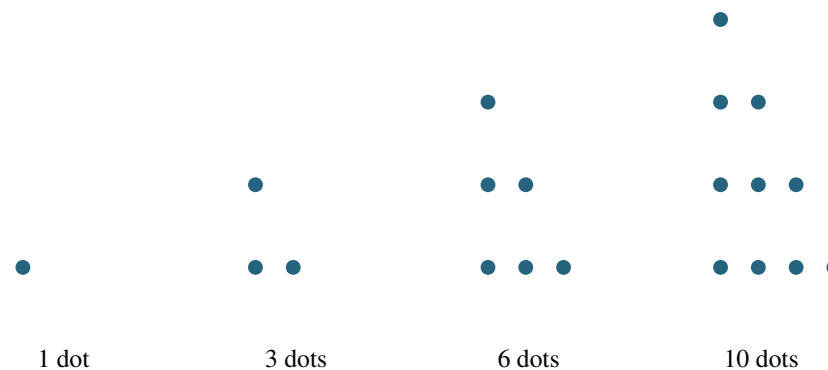
A worked case

$$u_n = 2 \times 3^{n-1}$$

■ Worked example

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Not true: Every sequence with increasing terms has a constant difference. Increasing sequences can have changing differences, such as 1, 4, 9, 16.

Applying it

- AQA A23/A24 Foundation includes these patterns and positive rational geometric ratios.
- Higher also permits surd ratios and derives quadratic nth terms in the next lesson.
- Infinite-series sums are not part of this GCSE lesson.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Recursive rule:** A rule that generates new terms from preceding terms.
- Record positions and terms separately.
- Watch for: A nonconstant first difference does not mean a pattern is random.

3.2 · Quadratic nth terms and surd-ratio progressions · Higher

What does the second difference reveal?

- The terms 3,8,15,24 grow by 5,7,9. What rule predicts the next term without building every earlier term?
- This lesson studies **second difference** : The difference between consecutive first differences of a sequence.

The method

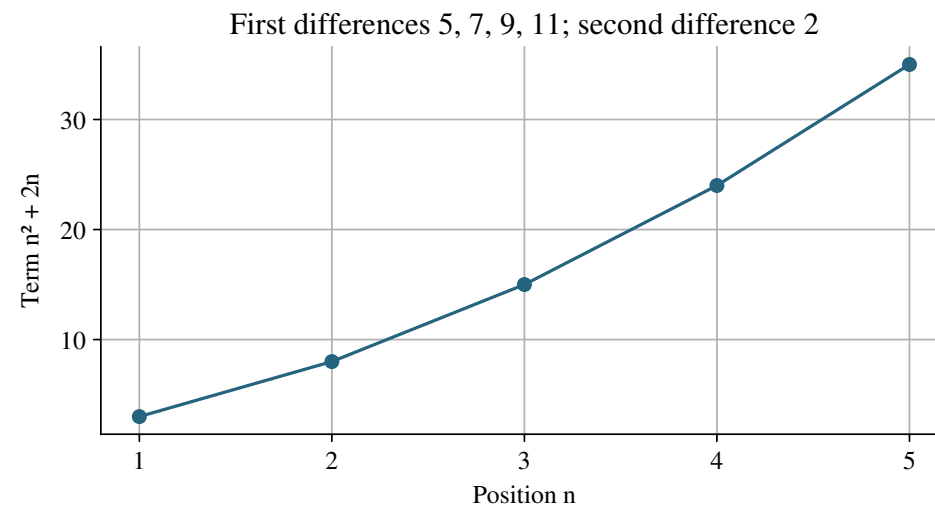
- A constant second difference signals a quadratic rule an^2+bn+c .
- Its value is $2a$.
- Subtract an^2 from the terms; fit the remaining linear rule and test at several positions.
- A geometric sequence with a surd ratio still multiplies by the same exact factor; keep root values exact.

A worked case

$$u_n = n^2 + 2n, \quad \Delta^2 u_n = 2$$

■ Worked example

For 3,8,15,24, first differences are 5,7,9 and second difference 2, so $a = 1$. Subtract n^2 at $n=1,2,3,4$ to get 2,4,6,8= $2n$. Thus $u_n=n^2+2n$ and $u_5=35$. For $1,\sqrt{2},2,2\sqrt{2},\dots$ the ratio is $\sqrt{2}$ and $u_n=(\sqrt{2})^{(n-1)}$. Check $u_3=2$ rather than rounding the root repeatedly.



Where it goes wrong

- The second difference equals $2a$, not a .
- A quadratic rule must be checked against the first terms; there can be a nonzero constant c .
- A finite pattern alone does not prove a unique rule without the stated sequence family.

■ Watch out

Not true: The coefficient of n^2 equals the second difference. The coefficient of n^2 is half the second difference.

Applying it

- AQA A25 Higher derives quadratic nth terms; A24 permits surd-ratio sequences.
- Foundation recognises and generates quadratic patterns without this general derivation.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Second difference:** The difference between consecutive first differences of a sequence.
- A constant second difference signals a quadratic rule an^2+bn+c .
- Watch for: the second difference equals $2a$, not a .

3.2 · Algebraic fractions and excluded values · Higher

Cancelling a factor does not put an undefined input back into the original formula.

- A cancelled fraction can look like a straight line and still have one missing point.
- This lesson studies **excluded value**: An input for which an original denominator or divisor is zero.

The method

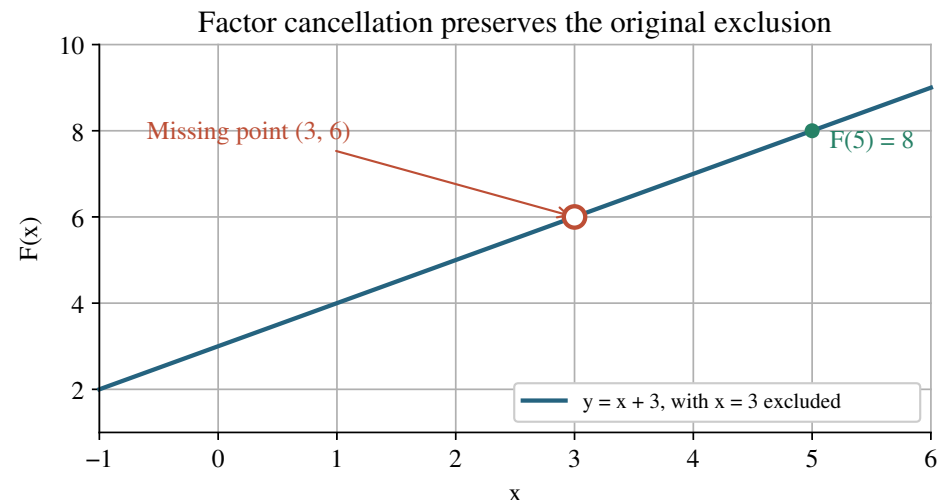
- Factor every numerator and denominator before cancelling a common factor.
- A term joined by addition is not a cancellable factor.
- Record exclusions from the original expression first.
- For addition or subtraction use a common denominator, retaining brackets around the entire numerator.
- Multiply factored numerators and denominators.
- To divide, multiply by the reciprocal and exclude inputs making the divisor zero as well as inputs making any original fraction undefined.

A worked case

$$\frac{x^2 - 9}{x - 3} = x + 3 \quad (x \neq 3)$$

■ Worked example

For $F(x) = (x^2 - 9)/(x - 3)$, factor $x^2 - 9 = (x - 3)(x + 3)$. Thus $F(x) = x + 3$ for $x \neq 3$; $F(5) = 8$, but $F(3)$ is undefined, not 6. For addition, $1/(x - 1) + 2/(x + 1) = [(x + 1) + 2(x - 1)]/[(x - 1)(x + 1)] = (3x - 1)/(x^2 - 1)$, with $x \neq \pm 1$. At $x = 3$, both forms give 1. For subtraction, the numerator is $(x + 1) - 2(x - 1) = 3 - x$, so the minus sign acts on both terms. Multiplication $[(x - 1)/(x + 2)] \times [(x + 2)/(x + 1)]$ gives $(x - 1)/(x + 1)$, but the original excludes $x = -2$ and $x = -1$. Division $[(x^2 - 1)/(x^2 + 3x + 2)] \div [(x - 1)/(x + 2)]$ gives 1, yet $x = -2, -1, 1$ are excluded: $x = 1$ makes the divisor zero. Finally $F(x) = 6$ reduces to $x + 3 = 6$. Its only candidate $x = 3$ is forbidden, so this equation has no solution. Check candidates in the original equation before reporting them.



Where it goes wrong

- Never cancel the x in $(x + 2)/x$.
- A simplified denominator does not show all original restrictions.
- In a division task, check when the divisor is zero.
- Cross-multiplication can produce an excluded candidate; a formal root is not automatically a solution.

■ Watch out

Not true: Cancelling a denominator factor makes its excluded input valid. An input that makes the original denominator zero stays excluded, even after cancelling.

Applying it

- AQA A4 Higher includes algebraic fractions and valid cancellation.
- The equation example checks whether a candidate is inside the original domain.
- Explain each factor cancellation, retain original restrictions and check every equation candidate.
- These operations extend the existing manipulation example into a complete worked arithmetic sequence.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.2.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Excluded value:** An input for which an original denominator or divisor is zero.
- Factor every numerator and denominator before cancelling a common factor.
- Watch for: never cancel the x in $(x+2)/x$.

3.3 · Percentages, ratio and proportional reasoning · Foundation

Can we recover the original price?

- A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price.
- This lesson studies **multiplier** : A factor that performs a percentage change in one multiplication.

The method

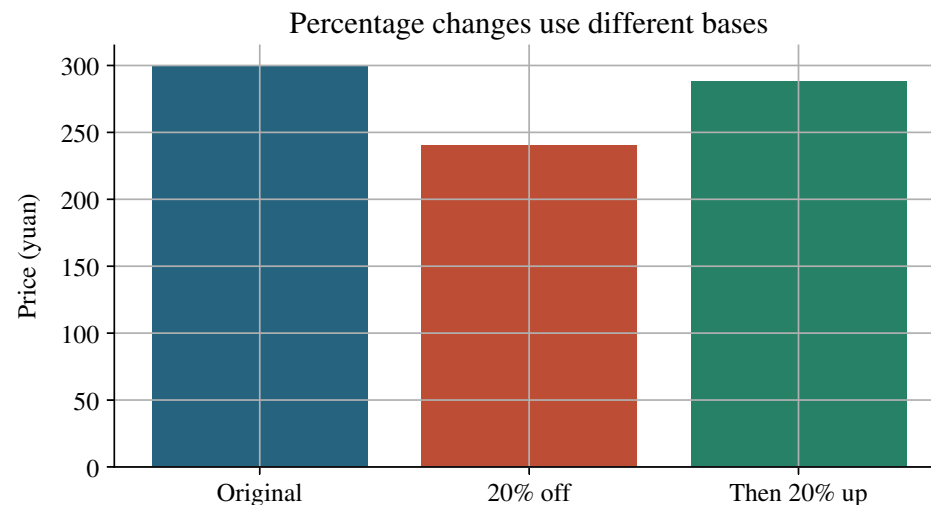
- A $p\%$ increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$.
- Reverse a percentage by dividing by the multiplier.
- In a ratio, first find the total number of parts.

A worked case

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100}\right)$$

■ Worked example

Let the original price be P . The model is sale price $= 0.8P$. Hence $P = 240/0.8 = 300$. A later 20% increase gives $240 \times 1.2 = 288$, so the two changes do not cancel.



Where it goes wrong

- A percentage uses a stated base.
- Subtracting the percentages loses that base.
- For compound change, multiply the multipliers; do not add the percentages.

■ Watch out

Not true: A 20% decrease followed by a 20% increase restores the starting price. The 20% increase is taken of a smaller amount: $100 \times 0.8 \times 1.2 = 96$, not 100.

Applying it

- Use percentage multipliers and divide a total into ratio parts.
- This Foundation/Core lesson uses linear proportional contexts, not the advanced regression methods.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.3.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Multiplier:** A factor that performs a percentage change in one multiplication.
- A $p\%$ increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$.
- Watch for: A percentage uses a stated base.

3.3 · Percentages, ratio and proportional reasoning · Higher

Can we recover the original price?

- A coat is reduced by 20% to ¥240. The discount applies to the original price, not to the sale price.
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The method

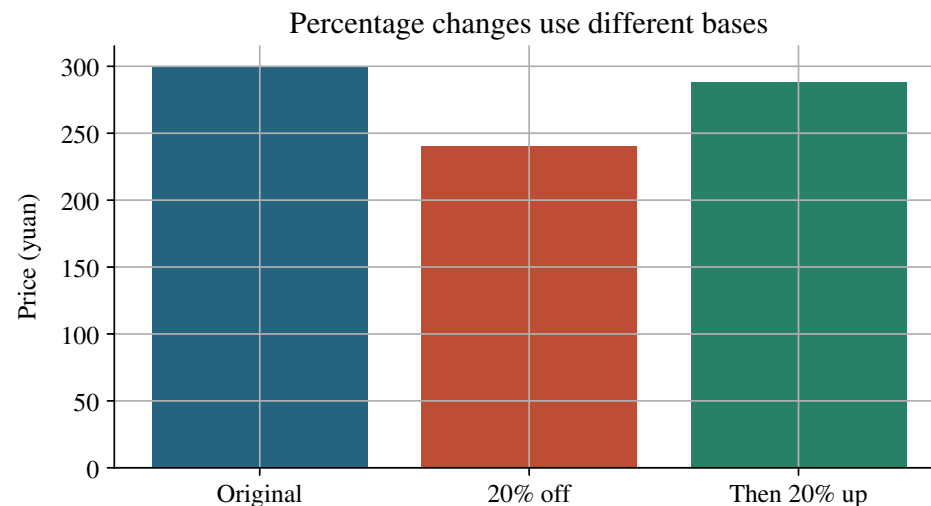
- A $p\%$ increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$.
- Reverse a percentage by dividing by the multiplier.
- In a ratio, first find the total number of parts.

A worked case

$$P_{\text{new}} = P_{\text{old}} \left(1 + \frac{r}{100} \right)$$

■ Worked example

Let the original price be P . The model is sale price $= 0.8P$. Hence $P = 240/0.8 = 300$. A later 20% increase gives $240 \times 1.2 = 288$, so the two changes do not cancel.



Where it goes wrong

- A percentage uses a stated base.
- Subtracting the percentages loses that base.
- For compound change, multiply the multipliers; do not add the percentages.

■ Watch out

Not true: A 20% decrease followed by a 20% increase restores the starting price. The 20% increase is taken of a smaller amount: $100 \times 0.8 \times 1.2 = 96$, not 100.

Applying it

- For direct proportion use $y=kx$; for inverse proportion use $y=k/x$.
- Calculate k from a known pair before using a new value.
- State what you held constant.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.3.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Multiplier:** A factor that performs a percentage change in one multiplication.
- A $p\%$ increase has multiplier $1+p/100$; a decrease has multiplier $1-p/100$.
- Watch for: A percentage uses a stated base.

3.3 · Ratios, equivalent proportions and mixtures · Foundation

A 700 ml batch and a doubled batch should taste the same, although their volumes differ.

- A recipe uses concentrate and water in the ratio 2:5.
- This lesson studies **part-to-whole ratio** : A comparison of one share with the combined quantity.

The method

- Put quantities in the same units before simplifying a ratio.
- Divide every part by the same nonzero factor.
- For $a:b$ the whole has $a+b$ parts; the first share is $a/(a+b)$ of the whole but a/b of the other share.
- Equivalent ratios have the same multiplicative relationship: $a/b=c/d$ gives $ad=bc$ when denominators are nonzero.

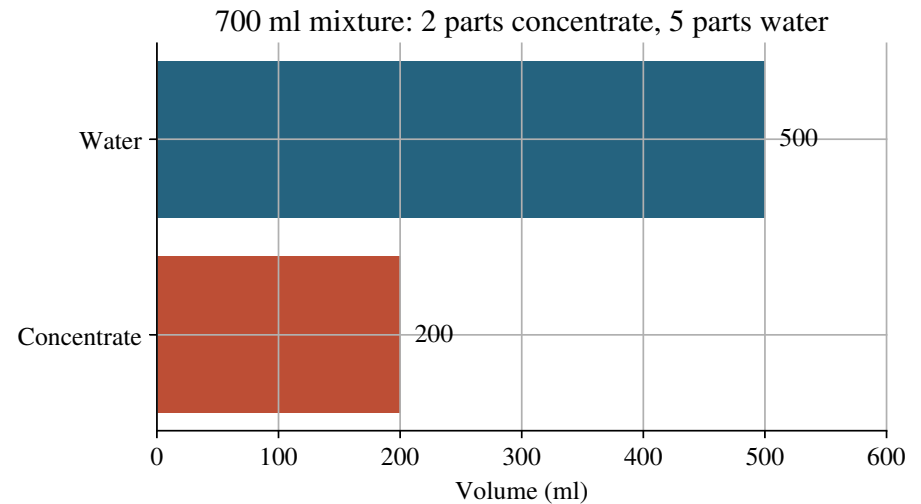
A worked case

$$\frac{a}{b} = \frac{c}{d} \iff ad = bc, \quad C = \frac{2}{7}V, \quad W = \frac{5}{2}C$$

■ Worked example

For 700 ml at 2:5, each part is 100 ml: concentrate 200 ml and water 500 ml. Concentrate:whole=2:7, while concentrate/water=2/5. Water/concentrate=5/2, a fraction greater than 1. Doubling both gives 400:1000=2:5. If concentrate is x and water y , the recipe requires $y=(5/2)x$, a straight line through the origin. For 300 ml concen-

trate, water is 750 ml and the total is 1050 ml. A batch with 200 ml concentrate and 600 ml water has ratio 1:3 and is weaker. To simplify 1.5 litres:500 ml, first write 1500:500=3:1.



Where it goes wrong

- Adding the same amount to both shares generally changes the ratio.
- Do not divide by 2+7 when the stated ratio is already part:whole 2:7; the whole is seven parts.
- A concentration fraction uses total volume as denominator.

■ Watch out

Not true: Adding 100 ml to both shares preserves a mixture ratio. Adding the same amount to both parts changes the ratio: 1:2 becomes 101:102 after adding 100 ml.

Applying it

- AQA R3–R8 connects ratio notation, fractions, equivalent proportions and linear functions.
- Check that both shares sum to the stated total, and name which quantity is compared with which.
- Use a ratio table to scale a mixture without assuming a fixed additive difference.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.3.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Part-to-whole ratio:** A comparison of one share with the combined quantity.
- Put quantities in the same units before simplifying a ratio.
- Watch for: adding the same amount to both shares generally changes the ratio.

3.3 · Ratios, equivalent proportions and mixtures · Higher

A 700 ml batch and a doubled batch should taste the same, although their volumes differ.

- A recipe uses concentrate and water in the ratio 2:5.
- This lesson studies **part-to-whole ratio** : A comparison of one share with the combined quantity.

The method

- Put quantities in the same units before simplifying a ratio.
- Divide every part by the same nonzero factor.
- For a:b the whole has a+b parts; the first share is $a/(a+b)$ of the whole but a/b of the other share.
- Equivalent ratios have the same multiplicative relationship: $a/b=c/d$ gives $ad=bc$ when denominators are nonzero.

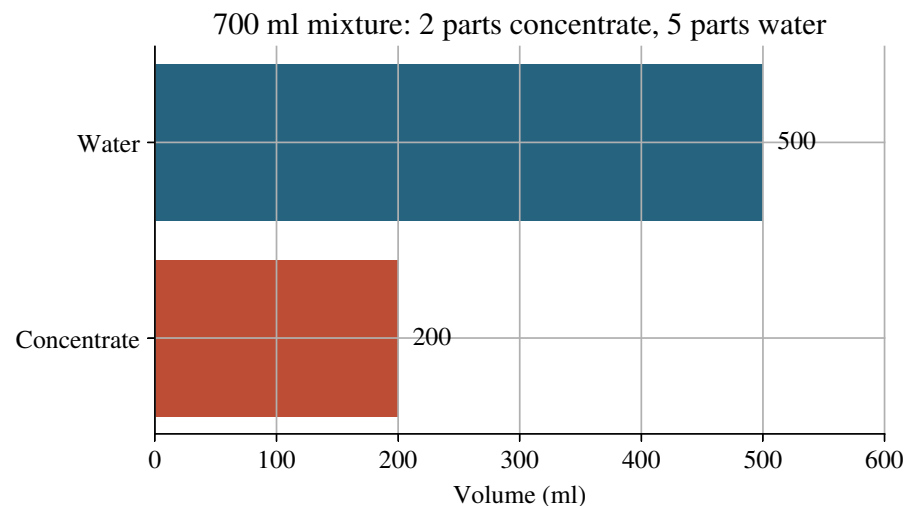
A worked case

$$\frac{a}{b} = \frac{c}{d} \iff ad = bc, \quad C = \frac{2}{7}V, \quad W = \frac{5}{2}C$$

■ Worked example

For 700 ml at 2:5, each part is 100 ml: concentrate 200 ml and water 500 ml. Concentrate:whole=2:7, while concentrate/water=2/5. Water/concentrate=5/2, a fraction greater than 1. Doubling both gives 400:1000=2:5. If concentrate is x and water y , the recipe requires $y=(5/2)x$, a straight line through the origin. For 300 ml concentrate, water is 750 ml and the total is 1050 ml. A batch with 200 ml concentrate and

600 ml water has ratio 1:3 and is weaker. To simplify 1.5 litres:500 ml, first write 1500:500=3:1.



Where it goes wrong

- Adding the same amount to both shares generally changes the ratio.
- Do not divide by 2+7 when the stated ratio is already part:whole 2:7; the whole is seven parts.
- A concentration fraction uses total volume as denominator.

■ Watch out

Not true: Adding 100 ml to both shares preserves a mixture ratio. Adding the same amount to both parts changes the ratio: 1:2 becomes 101:102 after adding 100 ml.

Applying it

- AQA R3–R8 connects ratio notation, fractions, equivalent proportions and linear functions.
- Check that both shares sum to the stated total, and name which quantity is compared with which.
- Use a ratio table to scale a mixture without assuming a fixed additive difference.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.3.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Part-to-whole ratio:** A comparison of one share with the combined quantity.
- Put quantities in the same units before simplifying a ratio.
- Watch for: adding the same amount to both shares generally changes the ratio.

3.3 · Scale drawings and maps · Foundation

The written scale compares centimetres with centimetres, not centimetres with kilometres.

- A map at 1:25,000 shows a 4 cm route.
- This lesson studies **scale** : The ratio of a drawing length to its corresponding real length.

The method

- In a scale 1:n, one drawing unit represents n of the same real unit.
- Multiply a drawing length by n to obtain the real length; divide a real length by n to draw it.
- Then convert the unit.
- Measure only when the diagram explicitly supplies an accurate scale.

A worked case

$$L_{\text{real}} = nL_{\text{drawing}}$$

■ Worked example

At 1:25,000, 4 cm represents 100,000 cm=1000 m=1 km. A real 1.5 km path is 150,000 cm, so it measures $150,000/25,000=6$ cm on the map. A room 6 m by 4 m drawn at 1:100 becomes 6 cm by 4 cm because each metre is 100 cm. Its drawing diagonal is $\sqrt{(6^2+4^2)}\approx 7.21$ cm and the real diagonal is about 7.21 m. Enlarging the printed map changes its numerical scale: doubling drawing lengths halves the scale denominator. A scale bar printed with the map enlarges with it.

Match units before applying the scale

4 cm on map



1 km on ground at 1:25,000

Where it goes wrong

- A ratio compares matching units.
- Never measure a diagram marked not to scale.
- A photocopied numerical scale can become invalid even though its scale bar still works.

■ Watch out

Not true: A scale of 1:25000 means 1 cm represents 25000 km. The scale has no units, so 1 cm represents 25 000 cm, which is 250 m.

Applying it

- AQA R2 includes maps, scale factors and geometric problems.
- Label drawing and real dimensions separately; reverse the calculation to verify the scale.
- Use an exact ratio until the context asks for rounding.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.3.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Scale:** The ratio of a drawing length to its corresponding real length.
- In a scale 1:n, one drawing unit represents n of the same real unit.
- Watch for: A ratio compares matching units.

3.3 · Scale drawings and maps · Higher

The written scale compares centimetres with centimetres, not centimetres with kilometres.

- A map at 1:25,000 shows a 4 cm route.
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A worked case

$$L_{\text{real}} = nL_{\text{drawing}}$$

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■ Key idea

- **Scale:** The ratio of a drawing length to its corresponding real length.
- In a scale 1:n, one drawing unit represents n of the same real unit.
- Watch for: A ratio compares matching units.

3.3 · Direct and inverse proportional relationships · Foundation

Sharing one fixed job among twice as many equally productive workers halves its duration.

- Buying twice as much fabric at a fixed price doubles the bill.
- This lesson studies **constant of proportionality**: The fixed multiplier in a stated proportional relationship.

The method

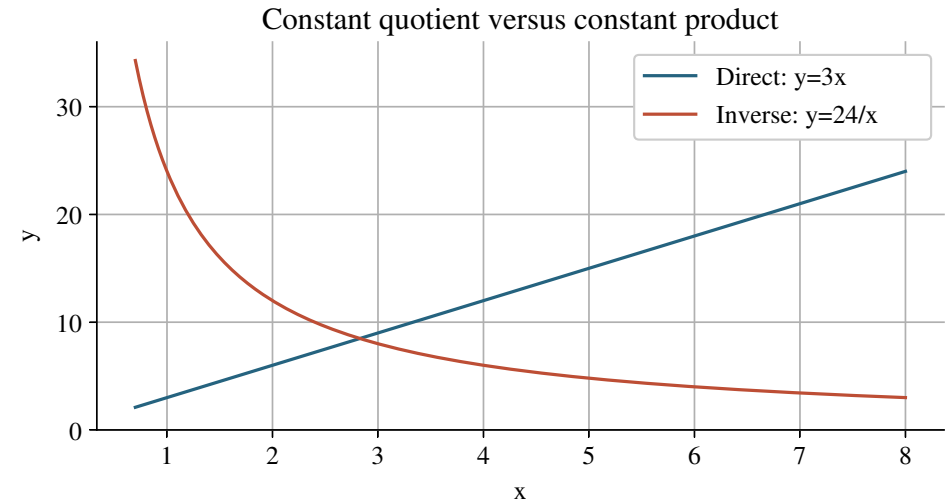
- Direct proportion $y=kx$ preserves y/x for nonzero x and gives a straight line through the origin.
- Inverse proportion $y=k/x$ preserves xy and gives a reciprocal curve, with $x=0$ excluded.
- State the fixed assumptions: a start-up charge breaks direct proportion, while changing productivity can break inverse proportion.
- Higher constructs the equation from a known pair; Foundation interprets and uses a given equation.

A worked case

$$y = kx \quad \text{or} \quad y = \frac{k}{x}, \quad x \neq 0$$

■ Worked example

For a given fabric cost $C=3L$, lengths 2,4,6 m cost 6,12,18 yuan: $C/L=3$. A 5-yuan fixed fee instead gives $C=5+3L$, which is linear but not directly proportional. For a given fixed-job model $t=24/w$, workers 2,4,6 need 12,6,4 hours, and $wt=24$ worker-hours. Doubling workers halves time. The inverse model assumes equally productive workers on one fixed job.



Where it goes wrong

- An increasing graph need not be direct proportion.
- An inverse relationship is proportional to $1/x$, not to $-x$.
- Keep a product constant for inverse proportion and a quotient constant for direct proportion.

■ Watch out

Not true: Every straight line represents direct proportion. Direct proportion needs a straight line through the origin; $y=2x+3$ is not direct proportion.

Applying it

- AQA R10/R13/R14 covers numerical, algebraic and graphical direct/inverse proportion.
- Interpret the units of k and the gradient.

- In Foundation use the supplied equations; Higher also constructs and interprets them from the context.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.3.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Constant of proportionality:** The fixed multiplier in a stated proportional relationship.
- Direct proportion $y=kx$ preserves y/x for nonzero x and gives a straight line through the origin.
- Watch for: an increasing graph need not be direct proportion.

3.3 · Direct and inverse proportional relationships · Higher

Sharing one fixed job among twice as many equally productive workers halves its duration.

- Buying twice as much fabric at a fixed price doubles the bill.
- This lesson studies **constant of proportionality**: The fixed multiplier in a stated proportional relationship.

The method

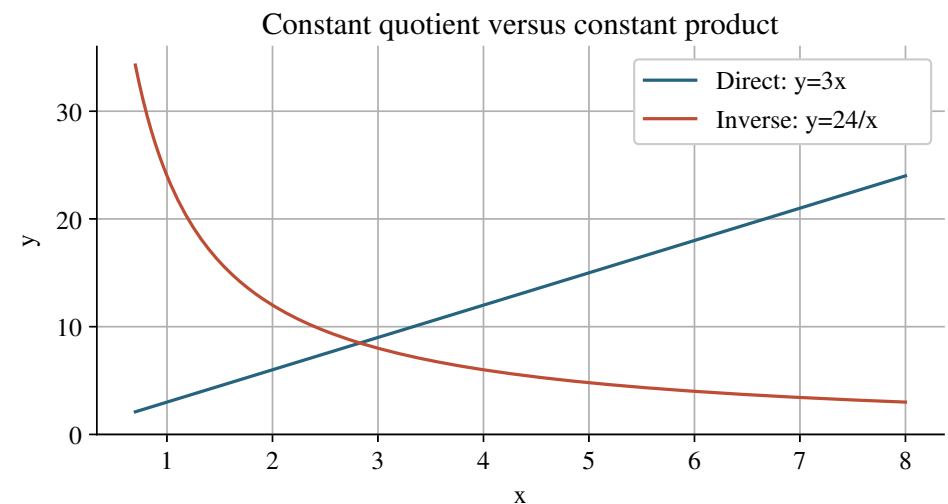
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- State the fixed assumptions: a start-up charge breaks direct proportion, while changing productivity can break inverse proportion.
- Higher constructs the equation from a known pair; Foundation interprets and uses a given equation.

A worked case

$$y = kx \quad \text{or} \quad y = \frac{k}{x}, \quad x \neq 0$$

■ Worked example

For a given fabric cost $C=3L$, lengths 2,4,6 m cost 6,12,18 yuan: $C/L=3$. A 5-yuan fixed fee instead gives $C=5+3L$, which is linear but not directly proportional. For a given fixed-job model $t=24/w$, workers 2,4,6 need 12,6,4 hours, and $wt=24$ worker-hours. Doubling workers halves time. Higher construction: if y is directly proportional to x and $y=18$ at $x=6$, $k=18/6=3$, hence $y=3x$. If t is inversely proportional to w and $t=6$ at $w=4$, $k=tw=24$, hence $t=24/w$. These models assume a constant rate and exclude impossible negative worker counts.



Where it goes wrong

- An increasing graph need not be direct proportion.
- An inverse relationship is proportional to $1/x$, not to $-x$.
- Keep a product constant for inverse proportion and a quotient constant for direct proportion.

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Not true: Every straight line represents direct proportion. Direct proportion needs a straight line through the origin; $y=2x+3$ is not direct proportion.

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- AQA R10/R13/R14 covers numerical, algebraic and graphical direct/inverse proportion.
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In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.3.
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■ Key idea

- **Constant of proportionality:** The fixed multiplier in a stated proportional relationship.
- Direct proportion $y=kx$ preserves y/x for nonzero x and gives a straight line through the origin.
- Watch for: an increasing graph need not be direct proportion.

3.3 · Rates, unit prices, density and pressure · Foundation

Comparing the sticker prices alone cannot tell which gives more material for each yuan.

- Two packages have different sizes and prices.
- This lesson studies **density**: Mass per unit volume, with both units stated.

The method

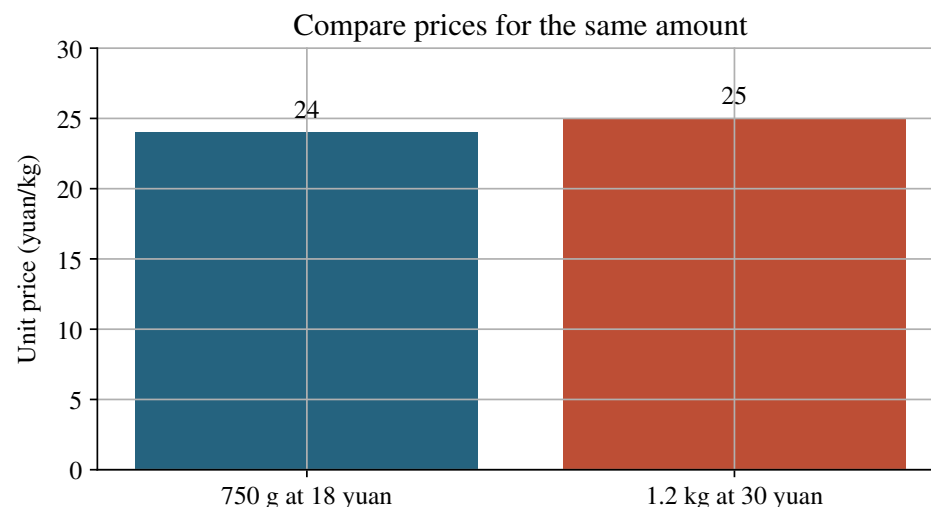
- A compound rate divides one quantity by another: speed= d/t , pay=earnings/time, unit price=cost/amount, density=mass/volume and pressure=force/area.
- Rearrange these equations before substituting.
- Convert each dimension separately; converting cm^2 to m^2 uses a squared length factor.
- Comparisons must use the same units and conditions.

A worked case

$$\rho = \frac{m}{V}, \quad p = \frac{F}{A}, \quad m = \rho V$$

■ Worked example

A 750 g pack costing 18 yuan has unit price $18/0.75=24$ yuan/kg. A 1.2 kg pack at 30 yuan costs 25 yuan/kg, so the first is better value if quality and waste are equal. A block of mass 540 g and volume 200 cm^3 has density 2.7 g/cm^3 ; a 50 cm^3 piece of that material has mass 135 g. Since $1 \text{ g}=0.001 \text{ kg}$ and $1 \text{ cm}^3=0.000001 \text{ m}^3$, $2.7 \text{ g/cm}^3=2700 \text{ kg/m}^3$. A 120 N force over 0.03 m^2 gives pressure $4000 \text{ N/m}^2=4000 \text{ Pa}$. At fixed force, halving the area doubles pressure. An hourly pay rate of 48 yuan/hour gives 120 yuan for 2.5 hours.



Where it goes wrong

- Volume conversions cube the length factor and area conversions square it.
- A density of 2.7 g/cm^3 is not 2.7 kg/m^3 .
- A cheaper package may cost more per kilogram; include only comparable products.

■ Watch out

Not true: Converting density from g/cm^3 to kg/m^3 leaves its numerical value unchanged. $1 \text{ g/cm}^3=1000 \text{ kg/m}^3$, so the number changes by a factor of 1000.

Applying it

- AQA R1/R11 includes speed, pay, pricing, density and pressure in numerical and algebraic contexts.
- Show the rearrangement with named quantities and carry units through the answer; use an inverse check such as $\text{density} \times \text{volume} = \text{mass}$.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.3.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Density:** Mass per unit volume, with both units stated.
- A compound rate divides one quantity by another: $\text{speed} = d/t$, $\text{pay} = \text{earnings}/\text{time}$, $\text{unit price} = \text{cost}/\text{amount}$, $\text{density} = \text{mass}/\text{volume}$ and $\text{pressure} = \text{force}/\text{area}$.
- Watch for: volume conversions cube the length factor and area conversions square it.

3.3 · Rates, unit prices, density and pressure · Higher

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The method

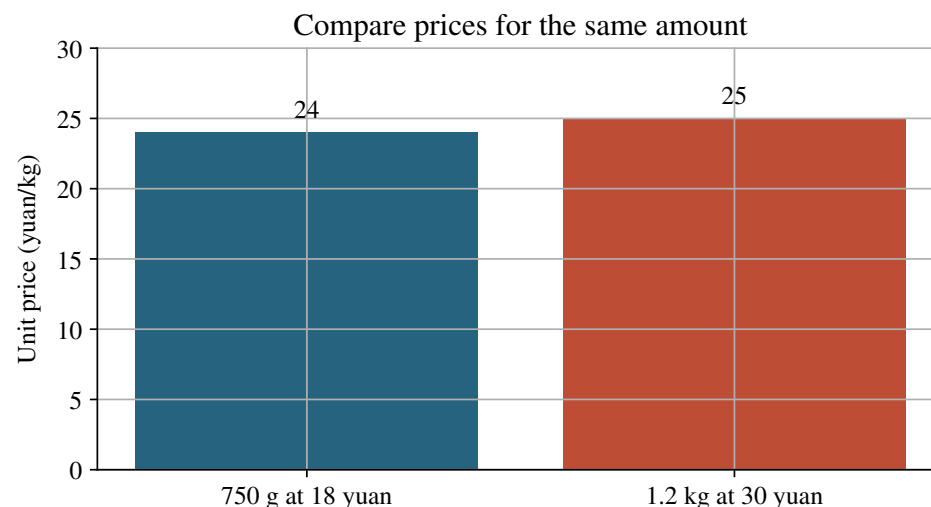
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Applying it

- AQA R1/R11 includes speed, pay, pricing, density and pressure in numerical and algebraic contexts.
- Show the rearrangement with named quantities and carry units through the answer; use an inverse check such as $\text{density} \times \text{volume} = \text{mass}$.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.3.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

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- Watch for: volume conversions cube the length factor and area conversions square it.

3.3 · Length, area and volume scale factors · Foundation

Each rectangular face doubles in two directions, while its volume doubles in three directions.

- A model doubles every length.
- This lesson studies **area scale factor**: The squared length factor between similar shapes.

The method

- For similar shapes with corresponding length factor k , area factor is k^2 and volume factor is k^3 .
- Name the direction of the comparison: model to real or real to model.
- Recover k from an area ratio by a square root and from a volume ratio by a cube root.
- A change in only one dimension does not create similar solids.
- Higher links corresponding sides to equal trigonometric ratios.

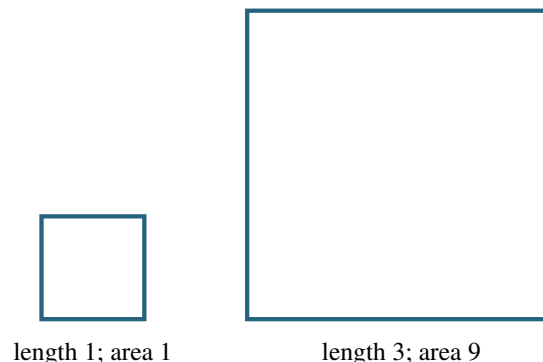
A worked case

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

■ Worked example

Enlarging a box from $2 \times 3 \times 4$ to $6 \times 9 \times 12$ multiplies lengths by 3. Its volume changes from 24 to 648, a factor of 27; a 2×3 face changes area from 6 to 54, a factor of 9. Length ratio 2:5 corresponds to area ratio 4:25 and volume ratio 8:125. If similar shapes have areas 20 and 80 cm^2 , the larger-to-smaller length factor is $\sqrt{(80/20)} = 2$. If similar solids have volumes 16 and 128 cm^3 , $k = \sqrt[3]{8} = 2$.

A length factor of 3 acts in both area dimensions



Where it goes wrong

- Areas do not scale by k , and volumes do not scale by k^2 .
- Similarity needs every corresponding length to share the same factor.
- Reversing a ratio requires the reciprocal factor.

■ Watch out

Not true: Doubling all lengths of a solid doubles its volume. Volume scales by the cube of the length factor: doubling lengths makes the volume 8 times larger.

Applying it

- AQA R12 Foundation includes ratios and scale factors for lengths, areas and volumes.
- Higher adds links to similarity including trigonometric ratios.
- Check dimensions and compare a simple box or rectangle before applying the general factor.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.3.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Area scale factor:** The squared length factor between similar shapes.
- For similar shapes with corresponding length factor k , area factor is k^2 and volume factor is k^3 .
- Watch for: areas do not scale by k , and volumes do not scale by k^2 .

3.3 · Length, area and volume scale factors · Higher

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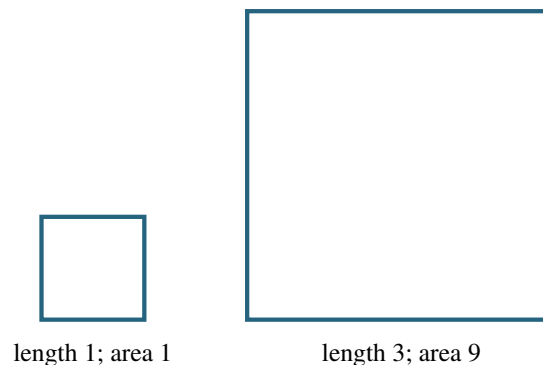
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A length factor of 3 acts in both area dimensions



Where it goes wrong

- Areas do not scale by k , and volumes do not scale by k^2 .
- Similarity needs every corresponding length to share the same factor.
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- AQA R12 Foundation includes ratios and scale factors for lengths, areas and volumes.
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In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.3.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

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- For similar shapes with corresponding length factor k , area factor is k^2 and volume factor is k^3 .
- Watch for: areas do not scale by k , and volumes do not scale by k^2 .

3.3 · Simple interest, compound growth and decay · Foundation

Simple interest adds the same 100 yuan each year; compound interest earns interest on earlier interest too.

- A 1000-yuan deposit earns 10% a year.
- This lesson studies **compound interest** : Interest calculated on the updated balance each period.

The method

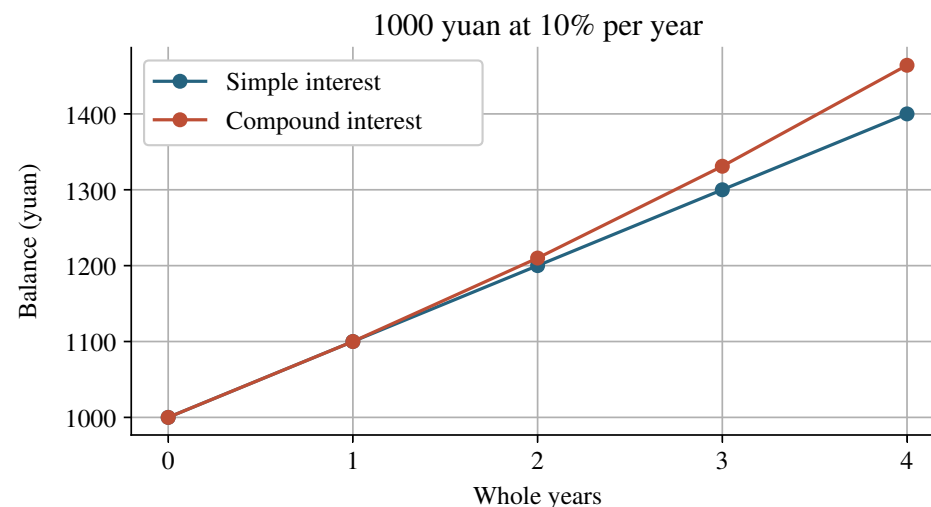
- A percentage r corresponds to decimal $r/100$; increases use multiplier $1+r/100$ and decreases $1-r/100$.
- Simple interest on principal P for n periods at rate r is $Prn/100$, giving balance $P(1+rn/100)$.
- Compound balance is $P(1+r/100)^n$.
- Decay uses the corresponding decreasing multiplier.
- Use matching rate and period units, and divide by the multiplier to recover an original value.

A worked case

$$B_n = P \left(1 + \frac{r}{100} \right)^n$$

■ Worked example

At 10% per year, 1000 yuan earns simple interest 100 per year: after two years interest is 200 and balance is 1200. Compound balances are 1100 after year one and 1210 after year two. A machine worth 800 yuan losing 20% each year becomes 640 then 512, not 480: the second loss is 20% of 640. An 800-yuan sale price after a 20% reduction corresponds to original price $800/0.8=1000$. Growth from 50 to 65 is $(65-50)/50 \times 100=30\%$; 65 is 130% of 50. If a compound balance must first exceed 1300 at 10%, year two gives 1210 and year three 1331, so three whole years are needed.



Where it goes wrong

- Use the original value as the denominator for percentage change.
- Repeated 20% decreases do not subtract 40% of the initial amount.
- A rate per year cannot be treated as a rate per month without a specified conversion.

■ Watch out

Not true: Simple and compound interest give the same balance after two years. Compound interest also earns interest on interest, so after two years its balance is larger.

Applying it

- AQA R9/R16 includes percentage comparisons, original values, simple interest and repeated growth/decay.
- State whether the task asks for interest alone or total balance, and interpret whole-period threshold answers.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.3.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Compound interest:** Interest calculated on the updated balance each period.
- A percentage r corresponds to decimal $r/100$; increases use multiplier $1+r/100$ and decreases $1-r/100$.
- Watch for: use the original value as the denominator for percentage change.

3.3 · Simple interest, compound growth and decay · Higher

Simple interest adds the same 100 yuan each year; compound interest earns interest on earlier interest too.

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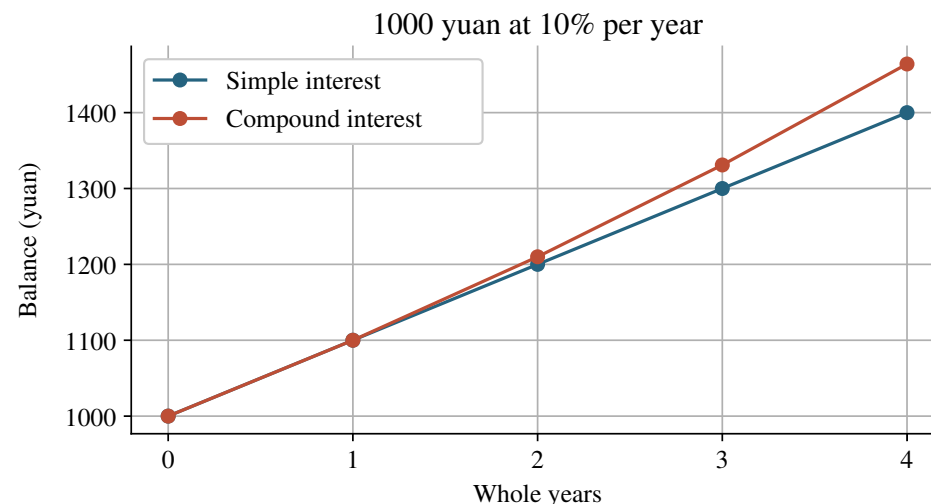
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- Use matching rate and period units, and divide by the multiplier to recover an original value.

A worked case

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Not true: Simple and compound interest give the same balance after two years. Compound interest also earns interest on interest, so after two years its balance is larger.

Applying it

- AQA R9/R16 includes percentage comparisons, original values, simple interest and repeated growth/decay.
- State whether the task asks for interest alone or total balance, and interpret whole-period threshold answers.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.3.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Compound interest:** Interest calculated on the updated balance each period.
- A percentage r corresponds to decimal $r/100$; increases use multiplier $1+r/100$ and decreases $1-r/100$.
- Watch for: use the original value as the denominator for percentage change.

3.3 · Iterative processes with repeated deposits · Higher

The deposit comes after interest, so the order changes the balance.

- A saver receives 10% interest and then adds 50 yuan each year.
- This lesson studies **recurrence relation**: A rule obtaining a later value from the preceding value.

The method

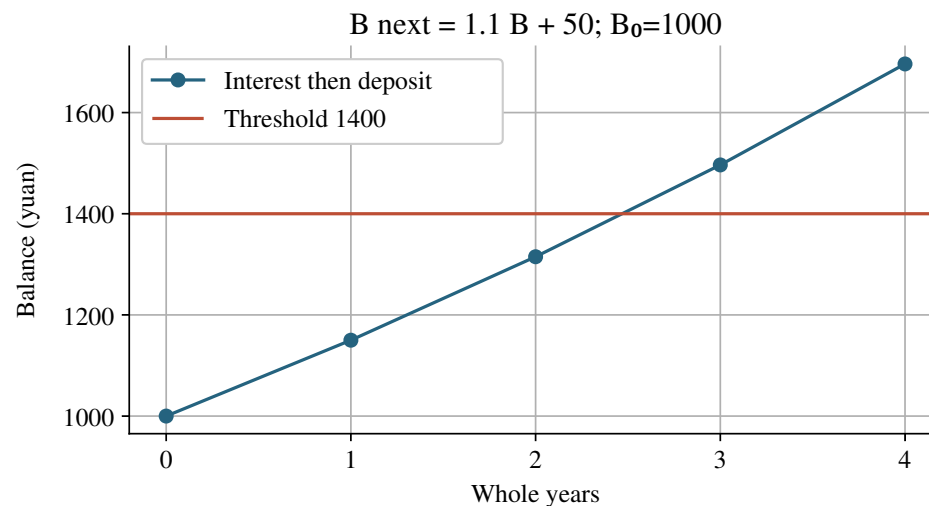
- Write an update rule and an initial value.
- Apply the operations in their stated order, using the previous output as the next input.
- A recurrence $B_{\text{next}}=1.1B+50$ differs from $1.1(B+50)$.
- Tables can locate a first whole-period threshold; verify both the preceding and crossing values.
- Iteration can model growth, decay or other repeated processes, not just solve equations.

A worked case

$$B_{n+1} = 1.1B_n + 50, \quad B_0 = 1000$$

■ Worked example

With $B_0=1000$ and $B_{n+1}=1.1B_n+50$, $B_1=1150$, $B_2=1315$ and $B_3=1496.5$. The balance first exceeds 1400 after three years because $1315 \leq 1400 < 1496.5$. If the deposit preceded interest, $B_1=1.1 \times 1050=1155$, five yuan greater. A decay-and-top-up rule $V_{\text{next}}=0.8V+20$, starting at 200, gives 180 then 164. A fixed point satisfies $V=0.8V+20$, hence $V=100$; values above 100 decrease toward it. A fixed point is a value preserved by the update, not a claim that every finite step reaches it exactly.



Where it goes wrong

- Preserve operation order and use the updated value each time.
- Do not round early or confuse the initial value with the first updated value.
- A continuous fractional-period estimate does not answer a whole-period question by itself.

■ Watch out

Not true: Depositing before interest gives the same result as depositing after interest. A deposit made before interest earns interest for that period, so the balance is larger.

Applying it

- AQA R16 Higher extends growth/decay to general iterative processes.
- State the model assumptions, initial value, recurrence and threshold interpretation.
- Use substitution to check a proposed fixed point without calculus.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.3.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Recurrence relation:** A rule obtaining a later value from the preceding value.
- Write an update rule and an initial value.
- Watch for: preserve operation order and use the updated value each time.

3.4 · Angles, lengths and area · Foundation

Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor**: The multiplier that relates corresponding lengths in similar shapes.

The method

- Use angle facts with a stated reason.
- Similar shapes have equal corresponding angles and proportional corresponding lengths.
- Areas of rectangles and triangles come from their dimensions; compound shapes can be split into simpler parts.

A worked case

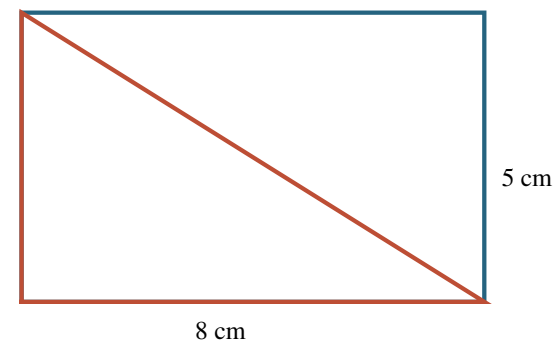
$$A_{\text{rectangle}} = LW, \quad A_{\text{triangle}} = \frac{1}{2}bh$$

■ Worked example

A rectangle of length 8 cm and width 5 cm has area $A=LW=40 \text{ cm}^2$. A triangle on the same base and height has area $A=bh/2=20 \text{ cm}^2$. For a pentagon, the interior-angle sum is $(5-2)\times 180=540^\circ$.

Same base and height, half the area

Rectangle: 40 cm^2 ; triangle: 20 cm^2



Where it goes wrong

- Equal angles alone establish similarity, not equal size.
- Use corresponding lengths in the same order.
- Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

■ Watch out

Not true: A triangle and rectangle with the same base and height have the same area. Volume scales by the cube of the length factor: doubling every length multiplies the volume by 8.

Applying it

- Use a labelled sketch and appropriate units.
- This Foundation/Core lesson does not test area/volume scale factors or advanced circle-theorem proofs.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Scale factor:** The multiplier that relates corresponding lengths in similar shapes.
- Use angle facts with a stated reason.
- Watch for: equal angles alone establish similarity, not equal size.

3.4 · Angle reasoning, similarity and mensuration · Higher

Does volume scale like length?

- A model has lengths one third of the real object. How much smaller are its area and volume?
- This lesson studies **scale factor** : The multiplier that relates corresponding lengths in similar shapes.

The method

- For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 .
- State angle reasons explicitly.
- A circle's tangent is perpendicular to the radius at the contact point.

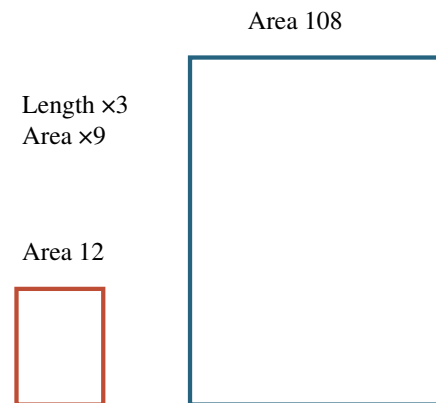
A worked case

$$\frac{A_2}{A_1} = k^2, \quad \frac{V_2}{V_1} = k^3$$

■ Worked example

If model-to-real length factor is 3, a model area of 12 cm^2 gives $12 \times 3^2 = 108 \text{ cm}^2$ and a model volume of 8 cm^3 gives $8 \times 3^3 = 216 \text{ cm}^3$. A cylinder with $r=3, h=5$ has volume $\pi r^2 h = 45\pi$.

A length scale factor is squared for area



Where it goes wrong

- Equal angles alone establish similarity, not equal size.
- Use corresponding lengths in the same order.
- Convert linear units before calculating area or volume, or square/cube the conversion factor correctly.

■ Watch out

Not true: Doubling every length of a solid doubles its volume. Volume scales by the cube of the length factor: doubling every length multiplies the volume by 8.

Applying it

- A geometric proof should name the relevant theorem, identify the equal angle or ratio, and draw the conclusion.
- A scale drawing is evidence only when the task permits measurement.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Scale factor:** The multiplier that relates corresponding lengths in similar shapes.
- For similar shapes with length scale factor k , areas scale by k^2 and volumes by k^3 .
- Watch for: equal angles alone establish similarity, not equal size.

3.4 · Right-angled triangles in two dimensions · Foundation

Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse** : The side opposite the right angle in a right-angled triangle.

The method

- Use Pythagoras in a right triangle and use sine, cosine or tangent with the sides labelled relative to the chosen angle.

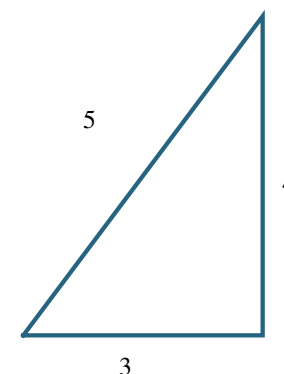
A worked case

$$a^2 + b^2 = c^2, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

■ Worked example

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan \theta = 4/3$, so $\theta \approx 53.1^\circ$. A right triangle with legs 6 and 8 has area $6 \times 8 / 2 = 24$.

Label the known sides before choosing a rule



Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

■ Watch out

Not true: Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

Applying it

- This Foundation/Core lesson uses right-angled triangles only.
- Sine and cosine rules for non-right triangles belong to the advanced-tier lesson.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Hypotenuse:** The side opposite the right angle in a right-angled triangle.
- Use Pythagoras in a right triangle and use sine, cosine or tangent with the sides labelled relative to the chosen angle.
- Watch for: label sides relative to the chosen angle.

3.4 · Right triangles and non-right triangles · Higher

Which side does the ladder need?

- A ladder reaches a height of 4 m while its foot is 3 m from a wall. Which sides are known, and which angle do we need?
- This lesson studies **hypotenuse**: The side opposite the right angle in a right-angled triangle.

The method

- In a right triangle $a^2 + b^2 = c^2$; $\sin\theta = \text{opposite/hypotenuse}$, $\cos\theta = \text{adjacent/hypotenuse}$ and $\tan\theta = \text{opposite/adjacent}$.
- For other triangles, use the sine or cosine rule, or $\text{area} = ab \sin C/2$.

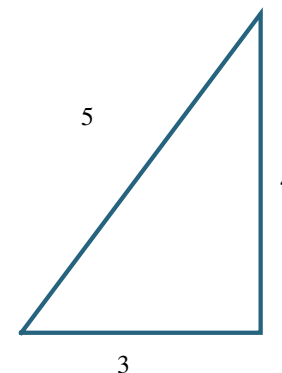
A worked case

$$a^2 + b^2 = c^2, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

■ Worked example

The ladder length is $c = \sqrt{3^2 + 4^2} = 5$ m. Its angle to the ground satisfies $\tan\theta = 4/3$, so $\theta \approx 53.1^\circ$. With two sides 6 and 8 enclosing 60° , $c^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \cos 60^\circ = 52$.

Label the known sides before choosing a rule



Where it goes wrong

- Label sides relative to the chosen angle.
- Pythagoras needs a right angle.
- A calculator angle mode error can produce a plausible but wrong result.
- Keep unrounded values for later steps.

■ Watch out

Not true: Pythagoras applies to every triangle, including triangles without a right angle. Pythagoras needs a right angle; for other triangles use the cosine rule.

Applying it

- Use a plan or elevation for a three-dimensional problem before applying a triangle rule.
- Explain why the chosen triangle contains the required length or angle.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Hypotenuse:** The side opposite the right angle in a right-angled triangle.
- In a right triangle $a^2+b^2=c^2$; $\sin\theta=\text{opposite/hypotenuse}$, $\cos\theta=\text{adjacent/hypotenuse}$ and $\tan\theta=\text{opposite/adjacent}$.
- Watch for: label sides relative to the chosen angle.

3.4 · Vectors and transformation geometry · Foundation

Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
- This lesson studies **resultant** : The vector sum representing the combined displacement or force.

The method

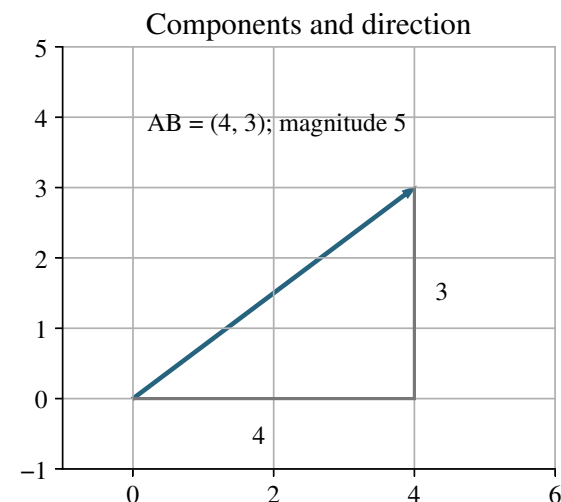
- Add corresponding vector components and subtract position vectors to find a displacement.
- A translation moves every point by the same vector; a scalar multiple changes length and possibly direction.

A worked case

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

■ Worked example

With $a=(4,1)$ and $b=(1,3)$, $a+b=(5,4)$ and $2a-b=(7,-1)$. From $A=(1,2)$ to $B=(5,5)$, $AB=(4,3)$ and its length is 5. Write each 2D vector as a column with the horizontal component above the vertical component: the translation AB has top entry 4 and bottom entry 3. A negative horizontal entry moves left; a negative vertical entry moves down. Drawing b from the head of a gives the head-to-tail diagram for $a+b$. Subtracting b adds its opposite $-b$.



Where it goes wrong

- The order of subtraction matters: $BA=-AB$.
- Proving parallelism needs a scalar-multiple relation; a sketch alone is insufficient.
- Negative enlargement reverses position about its centre.

■ Watch out

Not true: AB and BA have the same components. BA goes the opposite way to AB , so $BA=-AB$: every component changes sign.

Applying it

- Draw arrows with direction and identify the starting and ending points.
- Scalar products, spatial line equations and advanced angle calculations are excluded from this Foundation/Core lesson.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Resultant:** The vector sum representing the combined displacement or force.
- Add corresponding vector components and subtract position vectors to find a displacement.
- Watch for: the order of subtraction matters: $BA = -AB$.

3.4 · Vectors and transformation geometry · Higher

Why is displacement shorter than the walk?

- Walking 4 m east and 3 m north gives a displacement of 5 m, even though the travelled distance is 7 m.
- This lesson studies **resultant** : The vector sum representing the combined displacement or force.

The method

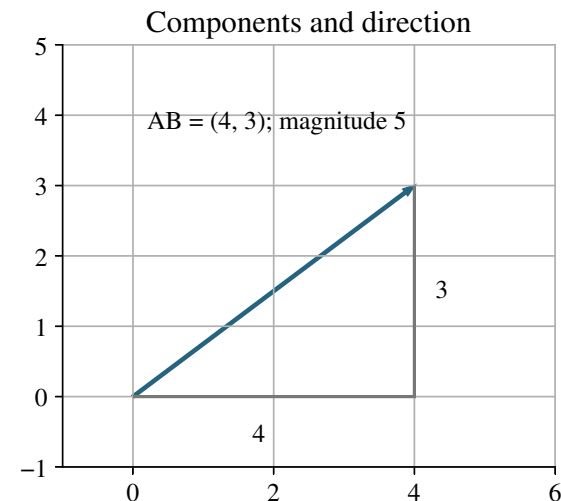
- Add corresponding vector components and subtract position vectors to find a displacement.
- A translation moves every point by the same vector; a scalar multiple changes length and possibly direction.

A worked case

$$\begin{pmatrix} 4 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

■ Worked example

With $a=(4,1)$ and $b=(1,3)$, $a+b=(5,4)$ and $2a-b=(7,-1)$. From $A=(1,2)$ to $B=(5,5)$, $AB=(4,3)$ and its length is 5. Write each 2D vector as a column with the horizontal component above the vertical component: the translation AB has top entry 4 and bottom entry 3. A negative horizontal entry moves left; a negative vertical entry moves down. Drawing b from the head of a gives the head-to-tail diagram for $a+b$. Subtracting b adds its opposite $-b$.



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- Negative enlargement reverses position about its centre.

■ Watch out

Not true: AB and BA have the same components. BA goes the opposite way to AB , so $BA = -AB$: every component changes sign.

Applying it

- Draw arrows with direction and identify the starting and ending points.
- Scalar products, spatial line equations and advanced angle calculations are excluded from this Foundation/Core lesson.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Resultant:** The vector sum representing the combined displacement or force.
- Add corresponding vector components and subtract position vectors to find a displacement.
- Watch for: the order of subtraction matters: $BA = -AB$.

3.4 · Reflections, rotations and enlargements · Foundation

Where is the enlargement centre?

- A logo is enlarged around a point away from the origin. Multiplying its coordinates alone puts it in the wrong place.
- This lesson studies **centre of enlargement** : The point from which each point's displacement is multiplied by a scale factor.

The method

- A translation adds a vector.
- A reflection reverses signed perpendicular distance from a mirror line.
- A rotation needs a centre, angle and direction.
- For enlargement from C, use new $P = C + k(P - C)$.

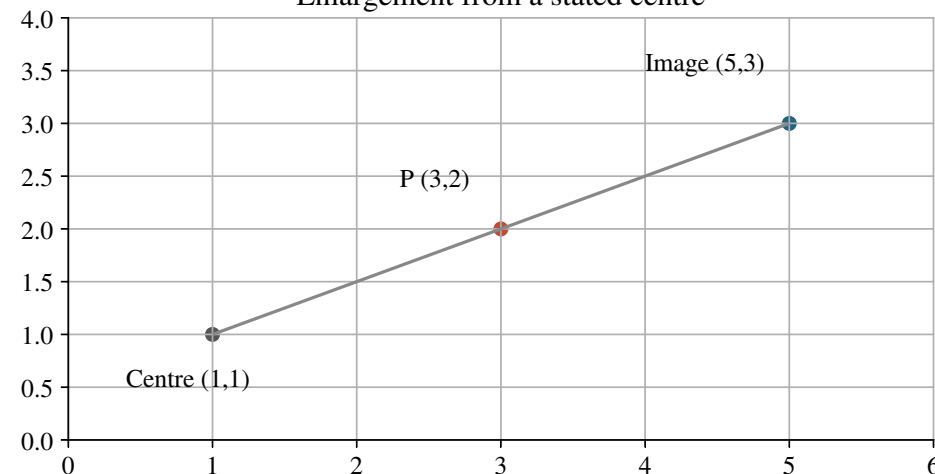
A worked case

$$\mathbf{p}_{\text{image}} = \mathbf{c} + k(\mathbf{p} - \mathbf{c})$$

■ Worked example

For centre $C = (1,1)$, point $P = (3,2)$ and factor 2, the image is $(5,3)$. At factor $1/2$ the image is $C + (1/2)(P - C) = (2,1.5)$, halfway from C to P. Reflecting $(3,2)$ in the y-axis gives $(-3,2)$. Rotating it 90° anticlockwise about the origin gives $(-2,3)$. Translation by $(2,-1)$ gives $(5,1)$. State the centre, line or vector as appropriate.

Enlargement from a stated centre



Where it goes wrong

- Specify a reflection line, rotation centre/angle/direction or enlargement centre/factor.
- Fractional positive enlargement factors reduce a shape without reversing its position about the centre.

■ Watch out

Not true: Every enlargement is centred at the origin unless its scale factor is negative. An enlargement can have any centre; a negative scale factor puts the image on the other side of the centre.

Applying it

- AQA G7 Foundation includes single reflections, rotations, translations and enlargements with positive integer/fractional factors.
- Negative factors and combinations of isometries belong to separate Higher teaching.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Centre of enlargement:** The point from which each point's displacement is multiplied by a scale factor.
- A translation adds a vector.
- Watch for: specify a reflection line, rotation centre/angle/direction or enlargement centre/factor.

3.4 · Reflections, rotations and enlargements · Higher

Where is the enlargement centre?

- A logo is enlarged around a point away from the origin. Multiplying its coordinates alone puts it in the wrong place.
- This lesson studies **centre of enlargement**: The point from which each point's displacement is multiplied by a scale factor.

The method

- A translation adds a vector.
- A reflection reverses signed perpendicular distance from a mirror line.
- A rotation needs a centre, angle and direction.
- For enlargement from C, use new $P = C + k(P - C)$.

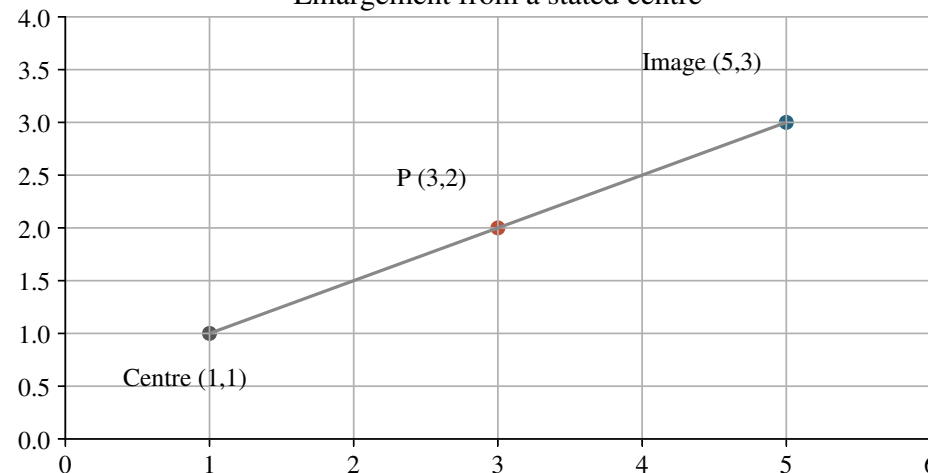
A worked case

$$\mathbf{p}_{\text{image}} = \mathbf{c} + k(\mathbf{p} - \mathbf{c})$$

■ Worked example

For $C=(1,1)$, $P=(3,2)$, $k=2$, the image is $C+2(P-C)=(5,3)$. With $k=1/2$ it is $(2,1.5)$; with $k=-2$ it is $(-3,-1)$, on the opposite side of the centre. Reflecting $(3,2)$ in the y-axis gives $(-3,2)$, and a 90° anticlockwise rotation about the origin gives $(-2,3)$. Identify centre, factor, line, angle and direction as applicable.

Enlargement from a stated centre



Where it goes wrong

- A rotation needs its centre and direction, not just an angle.
- A negative enlargement factor places the image on the opposite side of the centre.
- A translation does not change orientation or size.

■ Watch out

Not true: Every enlargement is centred at the origin unless its scale factor is negative. An enlargement can have any centre; a negative scale factor puts the image on the other side of the centre.

Applying it

- Describe a transformation completely before constructing the image.
- Check corresponding distances and angles.
- For combined transformations, apply them in the stated order; they usually do not commute.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Centre of enlargement:** The point from which each point's displacement is multiplied by a scale factor.
- A translation adds a vector.
- Watch for: A rotation needs its centre and direction, not just an angle.

3.4 · Constructions, loci and geometric conditions · Foundation

Where can both conditions hold?

- A router must be equally far from two rooms and within reach of a power point. Each condition creates a different set of possible positions.
- This lesson studies **locus** : The set of all points satisfying a stated geometric condition.

The method

- Points equally distant from A and B lie on the perpendicular bisector of AB.
- Points at fixed distance r from C lie on a circle.
- Points equally distant from two intersecting lines lie on their angle bisectors.

A worked case

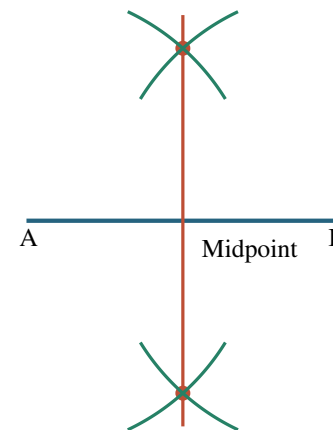
$$PA = PB, \quad h^2 + 3^2 = 5^2$$

■ Worked example

For endpoints A and B 6 cm apart, draw equal-radius arcs above and below AB, with compass opening greater than 3 cm. Join the arc intersections to obtain the perpendicular bisector, crossing AB at its midpoint 3 cm from each end. To construct a perpendicular from P to a line, use a circle centred at P to mark two line points, then bisect their segment. For a perpendicular at P on the line, mark equal distances on each side of P and use equal arcs. For an angle bisector, draw one vertex-centred arc meeting both arms, then equal arcs from those two points; join their intersection

to the vertex. Equal-radius circles centred at the endpoints of a segment construct an equilateral triangle and hence a 60° angle. For a point equally distant from A and B and 5 cm from A, intersect the bisector with a 5 cm circle centred at A. Each intersection is 4 cm perpendicular to AB by a 3–4–5 triangle. The shortest point-to-line distance follows a perpendicular, since any slanted route is a longer hypotenuse.

Equal compass radii: join the two arc intersections



Where it goes wrong

- The perpendicular bisector concerns distance to two points; the angle bisector concerns distance to two lines.
- A sketch is not a ruler-and-compass construction: preserve arcs as evidence of the method.

■ Watch out

Not true: Points equally distant from two points lie on their angle bisector. Points equidistant from two points lie on the perpendicular bisector of the segment joining them; the angle bisector is equidistant from two lines.

Applying it

- AQA G2 uses ruler-and-compass constructions, including a 60° angle, perpendiculars, bisectors and intersections of loci.
- Preserve construction arcs and justify the equidistance condition.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Locus:** The set of all points satisfying a stated geometric condition.
- Points equally distant from A and B lie on the perpendicular bisector of AB.
- Watch for: the perpendicular bisector concerns distance to two points; the angle bisector concerns distance to two lines.

3.4 · Constructions, loci and geometric conditions · Higher

Where can both conditions hold?

- A router must be equally far from two rooms and within reach of a power point. Each condition creates a different set of possible positions.
- This lesson studies **locus** : The set of all points satisfying a stated geometric condition.

The method

- Points equally distant from A and B lie on the perpendicular bisector of AB.
- Points at fixed distance r from C lie on a circle.
- Points equally distant from two intersecting lines lie on their angle bisectors.

A worked case

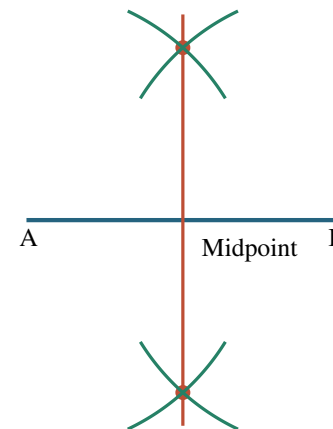
$$PA = PB, \quad h^2 + 3^2 = 5^2$$

■ Worked example

For endpoints A and B 6 cm apart, draw equal-radius arcs above and below AB, with compass opening greater than 3 cm. Join the arc intersections to obtain the perpendicular bisector, crossing AB at its midpoint 3 cm from each end. To construct a perpendicular from P to a line, use a circle centred at P to mark two line points, then bisect their segment. For a perpendicular at P on the line, mark equal distances on each side of P and use equal arcs. For an angle bisector, draw one vertex-centred arc meeting both arms, then equal arcs from those two points; join their intersection to the vertex. Equal-radius circles centred at the endpoints of a segment construct

an equilateral triangle and hence a 60° angle. For a point equally distant from A and B and 5 cm from A, intersect the bisector with a 5 cm circle centred at A. Each intersection is 4 cm perpendicular to AB by a 3–4–5 triangle. The shortest point-to-line distance follows a perpendicular, since any slanted route is a longer hypotenuse.

Equal compass radii: join the two arc intersections



Where it goes wrong

- The perpendicular bisector concerns distance to two points; the angle bisector concerns distance to two lines.
- A sketch is not a ruler-and-compass construction: preserve arcs as evidence of the method.

■ Watch out

Not true: Points equally distant from two points lie on their angle bisector. Points equidistant from two points lie on the perpendicular bisector of the segment joining them; the angle bisector is equidistant from two lines.

Applying it

- AQA G2 uses ruler-and-compass constructions, including a 60° angle, perpendiculars, bisectors and intersections of loci.
- Preserve construction arcs and justify the equidistance condition.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Locus:** The set of all points satisfying a stated geometric condition.
- Points equally distant from A and B lie on the perpendicular bisector of AB.
- Watch for: the perpendicular bisector concerns distance to two points; the angle bisector concerns distance to two lines.

3.4 · Circle theorems and reasoned proofs · Higher

Which angles see the same chord?

- Two observers see the same chord from the circle. Their angles are linked by a theorem rather than by the apparent size of the drawing.
- This lesson studies **cyclic quadrilateral**: A quadrilateral whose four vertices lie on one circle.

The method

- The angle at the centre is twice the angle at the circumference on the same arc.
- Angles in the same segment are equal.
- Opposite angles of a cyclic quadrilateral sum to 180° .
- A radius is perpendicular to a tangent at contact.

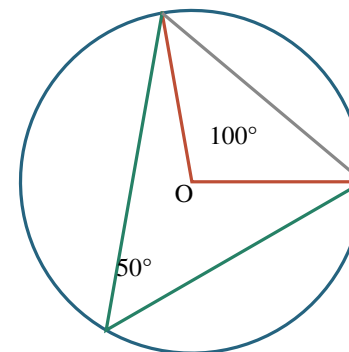
A worked case

$$\theta_{\text{centre}} = 2\theta_{\text{circumference}}$$

■ Worked example

If a central angle is 100° , the corresponding angle at the circumference is 50° . In a cyclic quadrilateral with one angle 112° , its opposite angle is $180-112=68^\circ$. A radius meeting a tangent gives 90° , even if the drawing looks oblique.

The centre angle is twice the circumference angle



Where it goes wrong

- Identify the same chord and the correct arc before using a theorem.
- Two visible right angles do not prove a quadrilateral cyclic without a valid converse argument.
- A diagram need not be to scale.

■ Watch out

Not true: Every quadrilateral has opposite angles summing to 180° . Opposite angles sum to 180° only in a cyclic quadrilateral, one whose vertices lie on a circle.

Applying it

- Write one reason alongside each angle calculation.
- For the alternate-segment theorem, name the tangent and chord, then identify the angle in the opposite segment.
- Use auxiliary radii only when they help the proof.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Cyclic quadrilateral:** A quadrilateral whose four vertices lie on one circle.
- The angle at the centre is twice the angle at the circumference on the same arc.
- Watch for: identify the same chord and the correct arc before using a theorem.

3.4 · Shape vocabulary, properties and symmetry · Foundation

A shape may satisfy more than one definition, so the name alone does not always give every property.

- A square tile is also a rectangle and a rhombus.
- This lesson studies **line of symmetry** : A mirror line that maps a whole shape onto itself.

The method

- A point marks a position; a line extends in both directions and a segment has two endpoints.
- A plane is a flat two-dimensional surface.
- A vertex is a corner; an edge is a boundary segment where solid faces meet.
- A polygon is a closed plane shape made of straight sides; regular means all sides and all interior angles are equal.
- Points label vertices; AB names a side and angle ABC has vertex B.
- Parallel lines have the same direction; perpendicular lines meet at 90° .
- Reflection symmetry uses a mirror line; rotational symmetry counts matches during one full turn, including 360° .

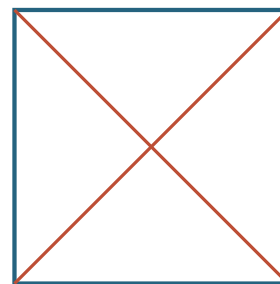
A worked case

$$\text{order} = \frac{360^\circ}{\text{smallest matching rotation}}$$

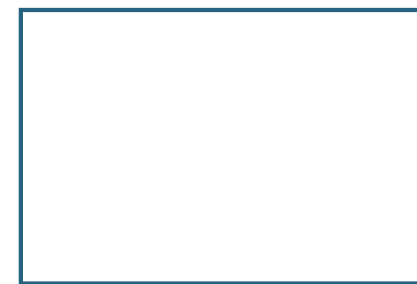
■ Worked example

A square has four equal sides, four right angles, four reflection axes and rotational order 4. A non-square rectangle has opposite sides equal, four right angles, two axes and order 2. A non-square rhombus has four equal sides, opposite angles equal, two diagonal axes and order 2. A parallelogram has two parallel side pairs and opposite angles equal; a general one has no reflection axis. A kite has two pairs of adjacent equal sides; a trapezium has a pair of parallel sides. An equilateral triangle has three equal sides/angles and three reflection axes; an isosceles triangle has two equal sides and equal base angles; scalene means no equal sides. Acute, right and obtuse classify triangles by their largest angle. Pentagons, hexagons, octagons and decagons have 5,6,8,10 sides. A rectangle has equal diagonals that bisect each other. A rhombus has perpendicular bisecting diagonals; a square has both properties. For a parallelogram, a diagonal splits it into triangles: alternate angles on the two parallel side pairs agree and the diagonal is shared, so ASA establishes congruence and opposite sides are equal. Opposite-angle equality also follows from parallel-line angle facts.

Square diagonals are symmetry axes



Square



Rectangle

Where it goes wrong

- Equal-looking lengths need stated equal-length marks or a deduction.
- Regular does not mean equal sides alone.
- An axis of symmetry is a full line, not just an internal diagonal.

■ Watch out

Not true: Every rectangle has four equal sides. A rectangle has four right angles; only a square also has four equal sides.

Applying it

- AQA G1/G4 requires conventional names, notation and derived shape properties.
- State the property that justifies a classification and allow overlapping classes, such as square and rectangle.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Line of symmetry:** A mirror line that maps a whole shape onto itself.
- A point marks a position; a line extends in both directions and a segment has two endpoints.
- Watch for: equal-looking lengths need stated equal-length marks or a deduction.

3.4 · Shape vocabulary, properties and symmetry · Higher

A shape may satisfy more than one definition, so the name alone does not always give every property.

- A square tile is also a rectangle and a rhombus.
- This lesson studies **line of symmetry** : A mirror line that maps a whole shape onto itself.

The method

- A point marks a position; a line extends in both directions and a segment has two endpoints.
- A plane is a flat two-dimensional surface.
- A vertex is a corner; an edge is a boundary segment where solid faces meet.

- A polygon is a closed plane shape made of straight sides; regular means all sides and all interior angles are equal.
- Points label vertices; AB names a side and angle ABC has vertex B.
- Parallel lines have the same direction; perpendicular lines meet at 90° .
- Reflection symmetry uses a mirror line; rotational symmetry counts matches during one full turn, including 360° .

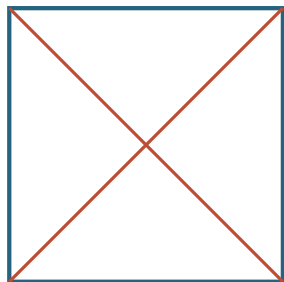
A worked case

$$\text{order} = \frac{360^\circ}{\text{smallest matching rotation}}$$

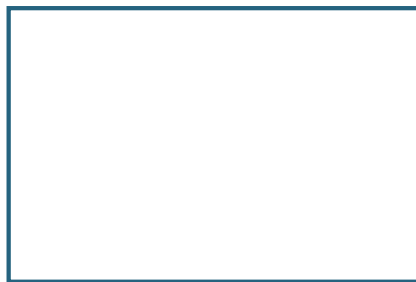
■ Worked example

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Square diagonals are symmetry axes



Square



Rectangle

Where it goes wrong

- Equal-looking lengths need stated equal-length marks or a deduction.
- Regular does not mean equal sides alone.
- An axis of symmetry is a full line, not just an internal diagonal.

■ Watch out

Not true: Every rectangle has four equal sides. A rectangle has four right angles; only a square also has four equal sides.

Applying it

- AQA G1/G4 requires conventional names, notation and derived shape properties.
- State the property that justifies a classification and allow overlapping classes, such as square and rectangle.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Line of symmetry:** A mirror line that maps a whole shape onto itself.
- A point marks a position; a line extends in both directions and a segment has two endpoints.
- Watch for: equal-looking lengths need stated equal-length marks or a deduction.

3.4 · Angle facts and polygon reasoning · Foundation

Parallelism allows another angle to be deduced without measuring it.

- A surveyor measures one angle where a road crosses two parallel boundaries.
- This lesson studies **corresponding angles** : Angles in matching positions at a transversal crossing two lines.

The method

- Angles around one point sum to 360° ; angles on a straight line sum to 180° ; vertically opposite angles are equal.
- When the crossed lines are parallel, corresponding and alternate angles are equal and co-interior angles sum to 180° .
- A triangle has angle sum 180° .
- Split an n -sided polygon into $n-2$ triangles to derive its interior sum $(n-2) \times 180^\circ$.

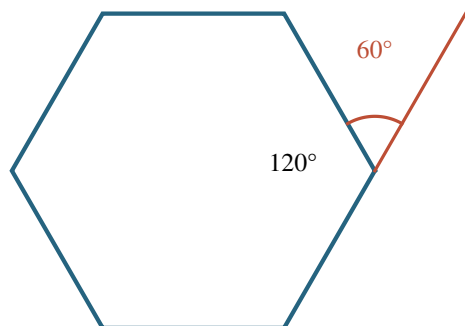
A worked case

$$S_n = (n-2)180^\circ, \quad E_{\text{regular}} = \frac{360^\circ}{n}$$

■ Worked example

If a straight-line angle is 68° , its neighbour is 112° ; its vertically opposite angle is 68° . A parallel-line corresponding angle is also 68° , with the reason named. Drawing a line through a triangle vertex parallel to the opposite side transfers its two base angles by alternate-angle equality; the three angles then form a straight line and sum to 180° . A pentagon splits into three triangles, giving 540° . A regular hexagon has total 720° , each interior angle 120° and each exterior turn 60° . Exterior turns of a convex polygon sum to one full turn, 360° ; a regular polygon with turn 45° has eight sides.

Regular hexagon: interior 120° , exterior 60°



Where it goes wrong

- Corresponding/alternate equalities require parallel lines.
- Name the theorem rather than using informal letter-shape labels.
- An interior angle is not the same as an exterior turning angle.

■ Watch out

Not true: Corresponding angles are equal even when the crossed lines are not parallel. Corresponding angles are equal only when the lines are parallel.

Applying it

- AQA G3/G6 expects reasons and derivations.
- Mark the given parallelism, identify the angle positions and build a chain of justified equalities rather than reading angles from a sketch.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Corresponding angles:** Angles in matching positions at a transversal crossing two lines.
- Angles around one point sum to 360° ; angles on a straight line sum to 180° ; vertically opposite angles are equal.
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3.4 · Angle facts and polygon reasoning · Higher

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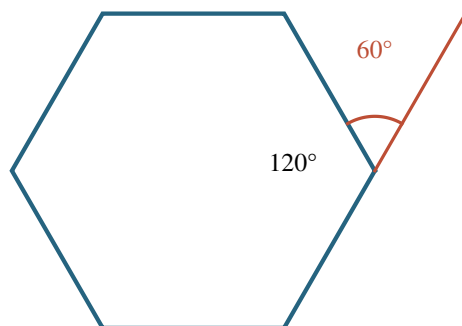
A worked case

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If a straight-line angle is 68° , its neighbour is 112° ; its vertically opposite angle is 68° . A parallel-line corresponding angle is also 68° , with the reason named. Drawing a line through a triangle vertex parallel to the opposite side transfers its two base angles by alternate-angle equality; the three angles then form a straight line and sum to 180° . A pentagon splits into three triangles, giving 540° . A regular hexagon has total 720° , each interior angle 120° and each exterior turn 60° . Exterior turns of a convex polygon sum to one full turn, 360° ; a regular polygon with turn 45° has eight sides.

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Not true: Corresponding angles are equal even when the crossed lines are not parallel. Corresponding angles are equal only when the lines are parallel.

Applying it

- AQA G3/G6 expects reasons and derivations.
- Mark the given parallelism, identify the angle positions and build a chain of justified equalities rather than reading angles from a sketch.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Corresponding angles:** Angles in matching positions at a transversal crossing two lines.
- Angles around one point sum to 360° ; angles on a straight line sum to 180° ; vertically opposite angles are equal.
- Watch for: corresponding/alternate equalities require parallel lines.

3.4 · Triangle congruence and geometric proof · Foundation

Matching three angles fixes their shape but does not guarantee that their sizes agree.

- Two triangular braces need to fit the same frame.
- This lesson studies **congruence** : Equal shape and size, with matching lengths and angles.

The method

- Use SSS (three sides), SAS (two sides and their included angle), ASA (two angles and the corresponding side), or RHS (right angle, hypotenuse and one other side).
- Match vertices in the same order.
- AAA establishes similarity, not congruence.
- SSA generally permits more than one triangle.
- A proof needs a given fact, a valid criterion and a matching-part conclusion.

A worked case

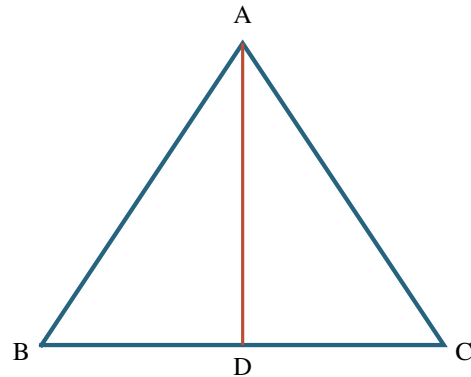
$$(a+b)^2 = 4\frac{ab}{2} + c^2 \implies a^2 + b^2 = c^2$$

■ Worked example

For an isosceles triangle ABC with $AB=AC$, let D be the midpoint of BC. Triangles ABD and ACD have $AB=AC$, $BD=DC$ and shared AD, so SSS gives congruence. Matching base angles ABC and BCA are therefore equal; the two angles at D are equal and form 180° , so each is 90° . To derive Pythagoras, arrange four congruent right triangles with legs a,b around a tilted square of side c inside a square of side $a+b$. Area gives $(a+b)^2 = 4(ab/2) + c^2$, hence $a^2 + b^2 = c^2$. For $a=3, b=4$, $c^2 = 9+16=25$, so $c=5$. These arguments establish results independently of a scale drawing.

Congruence proves equal base angles

$$AB = AC$$



BD = DC; AD shared → SSS

Where it goes wrong

- The SAS angle must lie between the named sides.
- RHS uses the hypotenuse, not two arbitrary sides.
- A proof diagram supports the argument; it cannot establish equality just by appearance.

■ Watch out

Not true: Two triangles with equal angles must be congruent. Equal angles show similarity only; the triangles may be different sizes.

Applying it

- AQA G5/G6 uses basic congruence criteria and simple geometric proofs, including isosceles base angles and Pythagoras.
- State every matching pair and explain which criterion applies.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Congruence:** Equal shape and size, with matching lengths and angles.
- Use SSS (three sides), SAS (two sides and their included angle), ASA (two angles and the corresponding side), or RHS (right angle, hypotenuse and one other side).
- Watch for: the SAS angle must lie between the named sides.

3.4 · Triangle congruence and geometric proof · Higher

Matching three angles fixes their shape but does not guarantee that their sizes agree.

- Two triangular braces need to fit the same frame.
- This lesson studies **congruence** : Equal shape and size, with matching lengths and angles.

The method

- Use SSS (three sides), SAS (two sides and their included angle), ASA (two angles and the corresponding side), or RHS (right angle, hypotenuse and one other side).
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- AAA establishes similarity, not congruence.
- SSA generally permits more than one triangle.
- A proof needs a given fact, a valid criterion and a matching-part conclusion.

A worked case

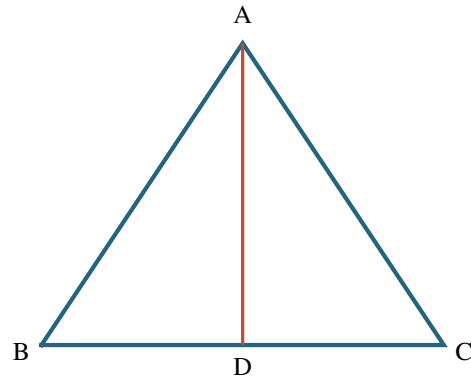
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Applying it

- AQA G5/G6 uses basic congruence criteria and simple geometric proofs, including isosceles base angles and Pythagoras.
- State every matching pair and explain which criterion applies.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Congruence:** Equal shape and size, with matching lengths and angles.
- Use SSS (three sides), SAS (two sides and their included angle), ASA (two angles and the corresponding side), or RHS (right angle, hypotenuse and one other side).
- Watch for: the SAS angle must lie between the named sides.

3.4 · Circle parts and geometric definitions · Foundation

These cuts create different regions, even though both use a curved boundary.

- A round window can be split by a straight chord or by two radii.
- This lesson studies **segment** : A region bounded by a chord and its corresponding arc.

The method

- A circle consists of points a fixed radius from its centre.
- A diameter is a chord through the centre and has length $2r$.
- A chord joins two circumference points; an arc is part of the circumference.
- A sector lies between two radii and an arc.
- A segment lies between a chord and an arc.
- A tangent touches at one point and is perpendicular to the radius there.

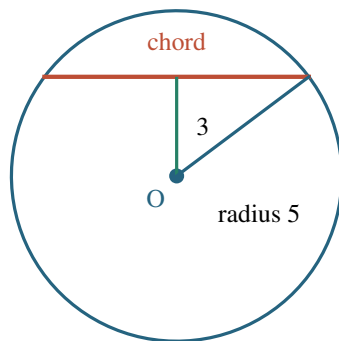
A worked case

$$d = 2r, \quad \left(\frac{c}{2}\right)^2 + h^2 = r^2$$

■ Worked example

For centre O and radius 5 cm, every circumference point is 5 cm from O and the diameter is 10 cm. A chord 3 cm from O has half-length $\sqrt{(5^2 - 3^2)} = 4$ cm, so its whole length is 8 cm. The perpendicular from O meets the chord at its midpoint. Joining the two chord endpoints to O creates a sector; the smaller region between chord and arc is a segment. At the rightmost circumference point, the vertical touching line is tangent and the horizontal radius is perpendicular to it. The circumference is a length, $2\pi r = 10\pi$ cm, rather than an area.

Circle of radius 5: distances use one consistent scale



Perpendicular bisects chord: half-length 4

Where it goes wrong

- A chord need not pass through the centre; only a diameter must.
- Sector and segment have different straight boundaries.
- Do not confuse circumference length with the shaded area inside a circle.

■ Watch out

Not true: A sector is bounded by a chord and an arc. A sector is bounded by two radii and an arc; a chord and an arc bound a segment.

Applying it

- AQA G9 includes all named circle parts at Foundation.
- Use the given radius and position labels to identify the part; Higher circle-theorem proofs are developed separately.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Segment:** A region bounded by a chord and its corresponding arc.
- A circle consists of points a fixed radius from its centre.
- Watch for: A chord need not pass through the centre; only a diameter must.

3.4 · Circle parts and geometric definitions · Higher

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- This lesson studies **segment** : A region bounded by a chord and its corresponding arc.

The method

- A circle consists of points a fixed radius from its centre.
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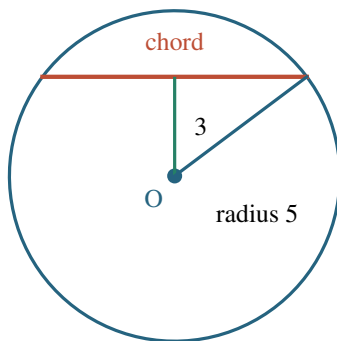
A worked case

$$d = 2r, \quad \left(\frac{c}{2}\right)^2 + h^2 = r^2$$

■ Worked example

For centre O and radius 5 cm, every circumference point is 5 cm from O and the diameter is 10 cm. A chord 3 cm from O has half-length $\sqrt{(5^2-3^2)}=4$ cm, so its whole length is 8 cm. The perpendicular from O meets the chord at its midpoint. Joining the two chord endpoints to O creates a sector; the smaller region between chord and arc is a segment. At the rightmost circumference point, the vertical touching line is tangent and the horizontal radius is perpendicular to it. The circumference is a length, $2\pi r=10\pi$ cm, rather than an area.

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■ Watch out

Not true: A sector is bounded by a chord and an arc. A segment is bounded by two radii and an arc; a chord and an arc bound a segment.

Applying it

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Use the given radius and position labels to identify the part; Higher circle-theorem proofs are developed separately.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Segment:** A region bounded by a chord and its corresponding arc.
- A circle consists of points a fixed radius from its centre.
- Watch for: A chord need not pass through the centre; only a diameter must.

3.4 · Geometric reasoning on coordinate axes · Foundation

The axes allow lengths and perpendicular edges to be checked even if the sketch looks distorted.

- A rectangular garden is drawn using corner coordinates.
- This lesson studies **midpoint** : The point halfway between the endpoints of a segment.

The method

- Find horizontal or vertical length by subtracting coordinates; use Pythagoras for a diagonal.
- Midpoint coordinates are averages of the endpoints.
- Equal coordinate changes identify translations.
- To prove a quadrilateral property, justify side directions and lengths, rather than naming a shape from its appearance.

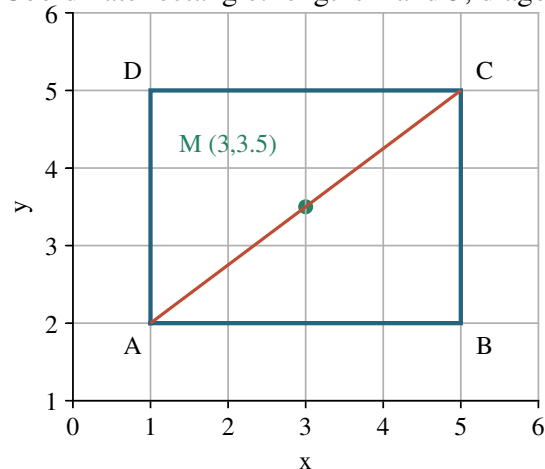
A worked case

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

■ Worked example

For $A=(1,2)$, $B=(5,2)$, $C=(5,5)$, $D=(1,5)$, $AB=4$ and $BC=3$. AB is horizontal, BC vertical, so they are perpendicular; opposite sides have matching directions and lengths, establishing a rectangle. Diagonal AC has length $\sqrt{4^2+3^2}=5$. Its midpoint is $((1+5)/2, (2+5)/2)=(3,3.5)$. Diagonal BD has the same midpoint, confirming that the diagonals bisect each other. A translation by $(2,-1)$ sends A to $(3,1)$ and C to $(7,4)$, preserving the diagonal length. A square would additionally need adjacent side lengths equal; these lengths 4 and 3 exclude it.

Coordinate rectangle: lengths 4 and 3; diagonal 5



Where it goes wrong

- A coordinate difference can be negative even though a length is nonnegative.
- The diagonal is not the sum of the two edge lengths.
- One pair of equal sides alone does not establish a rectangle.

■ Watch out

Not true: Every quadrilateral with two equal diagonals is a square. A rectangle and an isosceles trapezium also have equal diagonals; a square also needs perpendicular diagonals of equal length that bisect each other.

Applying it

- AQA G11 solves geometric problems on axes.
- Keep point labels, coordinate arithmetic and geometric reasons linked in the written argument; use the diagram to choose a method, then verify it numerically.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Midpoint:** The point halfway between the endpoints of a segment.
- Find horizontal or vertical length by subtracting coordinates; use Pythagoras for a diagonal.
- Watch for: A coordinate difference can be negative even though a length is non-negative.

3.4 · Geometric reasoning on coordinate axes · Higher

The axes allow lengths and perpendicular edges to be checked even if the sketch looks distorted.

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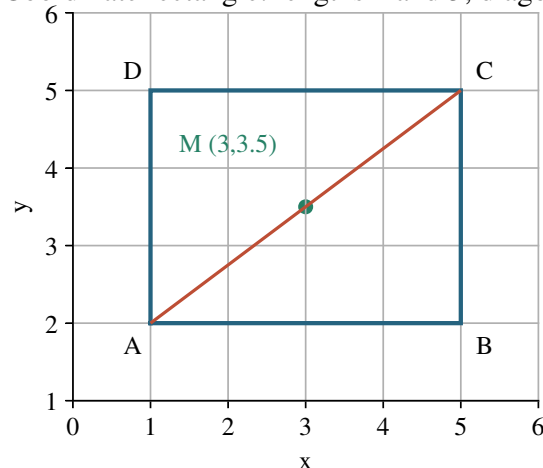
A worked case

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

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Not true: Every quadrilateral with two equal diagonals is a square. A rectangle and an isosceles trapezium also have equal diagonals; a square also needs perpendicular diagonals of equal length that bisect each other.

Applying it

- AQA G11 solves geometric problems on axes.
- Keep point labels, coordinate arithmetic and geometric reasons linked in the written argument; use the diagram to choose a method, then verify it numerically.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Midpoint:** The point halfway between the endpoints of a segment.
- Find horizontal or vertical length by subtracting coordinates; use Pythagoras for a diagonal.
- Watch for: A coordinate difference can be negative even though a length is non-negative.

3.4 · Solid properties, plans and elevations · Foundation

Front and side elevations add information that a single view cannot show.

- A building plan shows the footprint but hides its height.
- This lesson studies **elevation**: An orthographic view from a stated horizontal direction.

The method

- A face is a flat boundary polygon; curved surfaces must be named separately.
- Edges join faces and vertices are corners.
- A prism has two congruent parallel end faces and a constant cross-section.
- A pyramid joins one polygon base to an apex.
- Plan is the view from above; front and side elevations are direct views without perspective.
- Give the viewing direction and align corresponding widths, depths and heights.

A worked case

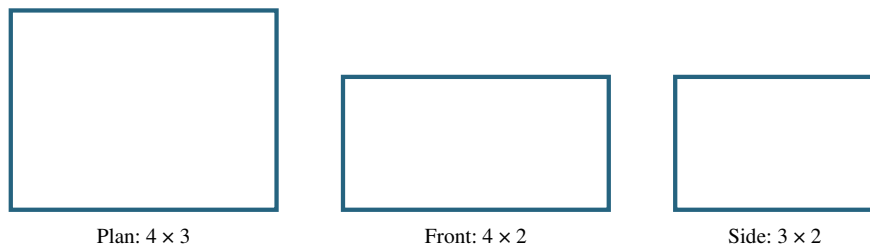
$$A_{\text{plan}} = wd, \quad A_{\text{front}} = wh$$

■ Worked example

A cube or cuboid has 6 faces, 12 edges and 8 vertices. A triangular prism has 5 faces, 9 edges and 6 vertices; a square pyramid has 5 faces, 8 edges and 5 vertices. A cylinder has two circular flat faces and one curved surface, with no vertices. A cone has one flat circular face, one curved surface and one apex; a sphere has only a curved surface. For a cuboid of width 4, depth 3 and height 2 units, the plan is 4 by 3, front elevation 4 by 2 and side elevation 3 by 2. A stack with front-row column heights 1 and 3, and back-row heights 2 and 1, occupies four cells. The front elevation has column maxima 2 and 3; the side elevation has depth-row maxima 3 and 2. These views do

not determine every hidden cube uniquely.

Cuboid width 4, depth 3, height 2



Where it goes wrong

- Do not draw perspective diagonals in an orthographic elevation.
- A plan alone cannot give height.
- Different hidden arrangements can share the same plan and elevations; do not claim a unique reconstruction without enough information.

■ Watch out

Not true: A plan view determines the height of a solid. A plan view shows the solid from above, so it hides the height; you need a side or front elevation.

Applying it

- AQA G12/G13 covers the named solids and construction/interpretation of views.
- Label dimensions and directions, preserve alignment between views and explain any hidden-space ambiguity.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Elevation:** An orthographic view from a stated horizontal direction.
- A face is a flat boundary polygon; curved surfaces must be named separately.
- Watch for: do not draw perspective diagonals in an orthographic elevation.

3.4 · Solid properties, plans and elevations · Higher

Front and side elevations add information that a single view cannot show.

- A building plan shows the footprint but hides its height.
- This lesson studies **elevation** : An orthographic view from a stated horizontal direction.

The method

- A face is a flat boundary polygon; curved surfaces must be named separately.
- Edges join faces and vertices are corners.
- A prism has two congruent parallel end faces and a constant cross-section.
- A pyramid joins one polygon base to an apex.
- Plan is the view from above; front and side elevations are direct views without perspective.
- Give the viewing direction and align corresponding widths, depths and heights.

A worked case

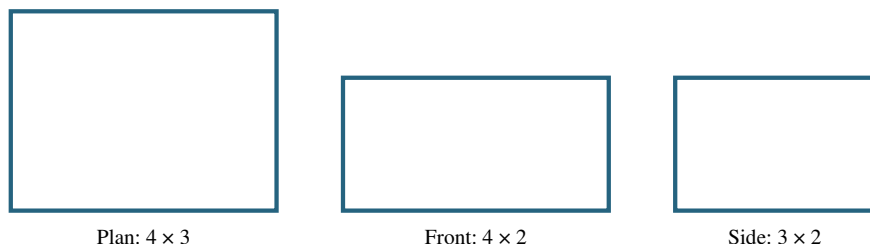
$$A_{\text{plan}} = wd, \quad A_{\text{front}} = wh$$

■ Worked example

A cube or cuboid has 6 faces, 12 edges and 8 vertices. A triangular prism has 5 faces, 9 edges and 6 vertices; a square pyramid has 5 faces, 8 edges and 5 vertices. A cylinder has two circular flat faces and one curved surface, with no vertices. A cone has one flat circular face, one curved surface and one apex; a sphere has only a curved surface. For a cuboid of width 4, depth 3 and height 2 units, the plan is 4 by 3, front elevation

4 by 2 and side elevation 3 by 2. A stack with front-row column heights 1 and 3, and back-row heights 2 and 1, occupies four cells. The front elevation has column maxima 2 and 3; the side elevation has depth-row maxima 3 and 2. These views do not determine every hidden cube uniquely.

Cuboid width 4, depth 3, height 2



Where it goes wrong

- Do not draw perspective diagonals in an orthographic elevation.
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Not true: A plan view determines the height of a solid. A plan view shows the solid from above, so it hides the height; you need a side or front elevation.

Applying it

- AQA G12/G13 covers the named solids and construction/interpretation of views.
- Label dimensions and directions, preserve alignment between views and explain any hidden-space ambiguity.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Elevation:** An orthographic view from a stated horizontal direction.
- A face is a flat boundary polygon; curved surfaces must be named separately.
- Watch for: do not draw perspective diagonals in an orthographic elevation.

3.4 · Measuring angles and three-figure bearings · Foundation

Turning around changes the reference point and reverses the direction; it does not keep the same bearing.

- A walking route has bearing 070° from A to B.
- This lesson studies **bearing**: A direction measured clockwise from north at the starting point.

The method

- Draw a north line at the starting point, then measure clockwise to the route.
- Write three digits: east 090° , south 180° , west 270° , north 000° .
- NE, SE, SW and NW are $045^\circ, 135^\circ, 225^\circ, 315^\circ$.
- For the reverse bearing add 180° and reduce modulo 360° .
- Use a ruler for a stated-scale length and the correct protractor scale for a stated angle.

A worked case

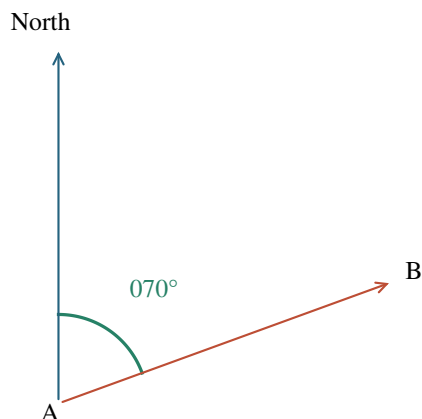
$$b_{\text{reverse}} = (b + 180^\circ) \bmod 360^\circ$$

■ Worked example

From A to B the bearing 070° is 70° clockwise from north. From B to A it is $070+180=250^\circ$. A bearing of 320° reverses to $500-360=140^\circ$. East is 090° , not 90 without its three-digit form. A route bearing 120° points southeast of the starting point, making 30° below east. On a 1:10000 map, a 3 cm route represents 300 m; to construct bearing 120° , place the protractor centre at the route start, align its

zero with north and measure clockwise. Keep the ruler scale and angular direction as separate decisions.

Clockwise from north at A; reverse bearing at B is 250°



Where it goes wrong

- A bearing is measured at the departure point, not the destination.
- Read clockwise from north rather than the acute angle to the nearest compass axis.
- The back bearing differs by 180° , even when the diagram is oblique.

■ Watch out

Not true: The reverse bearing of 070° is 290° . Add or subtract 180° : the reverse bearing of 070° is 250° .

Applying it

- AQA G15 includes measured lengths/angles, maps, the eight compass directions and three-figure bearings.
- State which point supplies north and distinguish a numerical bearing from a measured scale distance.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Bearing:** A direction measured clockwise from north at the starting point.
- Draw a north line at the starting point, then measure clockwise to the route.
- Watch for: A bearing is measured at the departure point, not the destination.

3.4 · Measuring angles and three-figure bearings · Higher

Turning around changes the reference point and reverses the direction; it does not keep the same bearing.

- A walking route has bearing 070° from A to B.
- This lesson studies **bearing** : A direction measured clockwise from north at the starting point.

The method

- Draw a north line at the starting point, then measure clockwise to the route.
- Write three digits: east 090° , south 180° , west 270° , north 000° .
- NE, SE, SW and NW are $045^\circ, 135^\circ, 225^\circ, 315^\circ$.
- For the reverse bearing add 180° and reduce modulo 360° .
- Use a ruler for a stated-scale length and the correct protractor scale for a stated angle.

A worked case

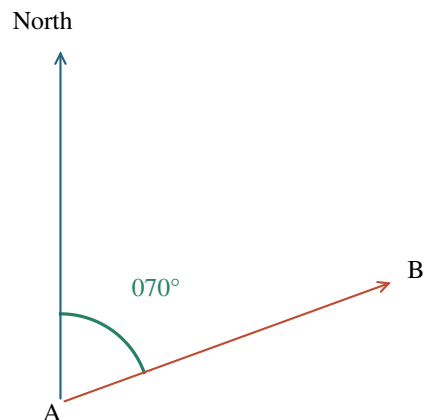
$$b_{\text{reverse}} = (b + 180^\circ) \bmod 360^\circ$$

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From A to B the bearing 070° is 70° clockwise from north. From B to A it is $070+180=250^\circ$. A bearing of 320° reverses to $500-360=140^\circ$. East is 090° , not 90 without its three-digit form. A route bearing 120° points southeast of the starting point, making 30° below east. On a 1:10000 map, a 3 cm route represents 300 m; to construct bearing 120° , place the protractor centre at the route start, align its zero with north and measure clockwise. Keep the ruler scale and angular direction as

separate decisions.

Clockwise from north at A; reverse bearing at B is 250°



Where it goes wrong

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- Read clockwise from north rather than the acute angle to the nearest compass axis.
- The back bearing differs by 180° , even when the diagram is oblique.

■ Watch out

Not true: The reverse bearing of 070° is 290° . Add or subtract 180° : the reverse bearing of 070° is 250° .

Applying it

- AQA G15 includes measured lengths/angles, maps, the eight compass directions and three-figure bearings.
- State which point supplies north and distinguish a numerical bearing from a measured scale distance.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Bearing:** A direction measured clockwise from north at the starting point.
- Draw a north line at the starting point, then measure clockwise to the route.
- Watch for: A bearing is measured at the departure point, not the destination.

3.4 · Combined isometries and invariants · Higher

The order of instructions is part of the transformation.

- Reflecting a logo and then sliding it can differ from sliding it and then reflecting it.
- This lesson studies **invariant**: A property preserved by a specified transformation.

The method

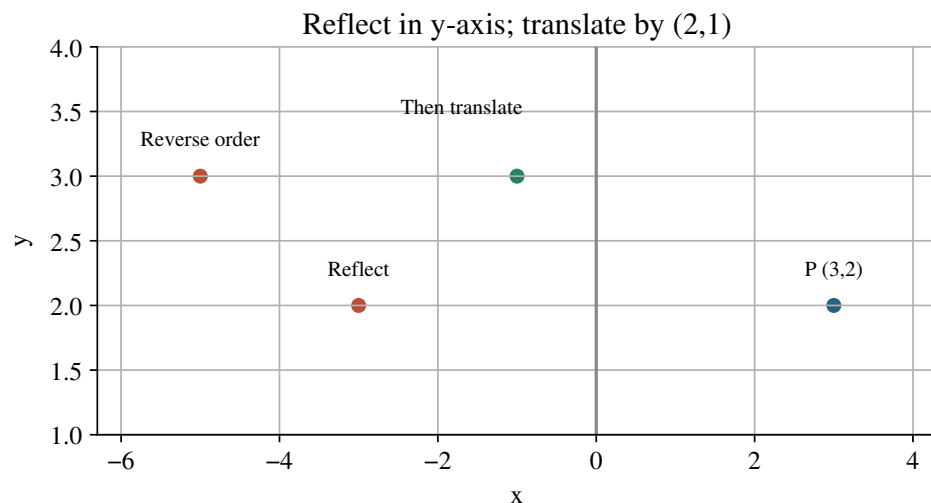
- Apply each transformation to the current image in the stated order.
- Rotations, reflections and translations preserve lengths and angles, so their combinations also preserve them.
- Reflections reverse orientation; rotations and translations preserve it.
- Two reflections in parallel lines give a translation; in intersecting lines they give a rotation through twice the directed angle between the mirrors.

A worked case

$$T \circ R \neq R \circ T \quad \text{in general}$$

■ Worked example

Start with $P=(3,2)$. Reflect in the y-axis to get $(-3,2)$, then translate by $(2,1)$ to get $(-1,3)$. Reversing the order gives $(5,3)$ then $(-5,3)$, a different point. Reflecting in $x=0$ followed by $x=2$ maps $(3,2)$ to $(-3,2)$ then $(7,2)$, equivalent to translation by $(4,0)$. Two reflections reverse orientation twice, restoring it. Reflecting in the x-axis then y-axis sends $(3,2)$ to $(-3,-2)$, a 180° rotation about the origin. Every pairwise length and angle is unchanged, but position generally changes.



Where it goes wrong

- Preserved length does not mean every point stays fixed.
- Do not commute transformations unless a checked argument permits it.
- A single reflection reverses orientation, while two reflections restore it.

■ Watch out

Not true: Changing the order of transformations never changes the final image. The order can matter: reflecting then rotating can give a different image from rotating then reflecting.

Applying it

- AQA G8 Higher requires changes and invariance under combinations of rigid transformations.
- Describe the resulting map completely and test it on more than one point before making a whole-shape claim.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Invariant:** A property preserved by a specified transformation.
- Apply each transformation to the current image in the stated order.
- Watch for: preserved length does not mean every point stays fixed.

3.4 · Circle angles and proof chains · Higher

The same-looking angle on the other side of the chord can instead be supplementary.

- A circle theorem connects angles standing on one chord.
- This lesson studies **angle at the centre** : An angle formed by two radii meeting at the centre.

The method

- The central angle on an arc is twice a circumference angle standing on that same arc.
- A diameter therefore gives a 90° circumference angle.
- Angles in the same segment are equal.
- Opposite angles of a cyclic quadrilateral sum to 180° .
- State the chord, arc and segment before applying a theorem.

A worked case

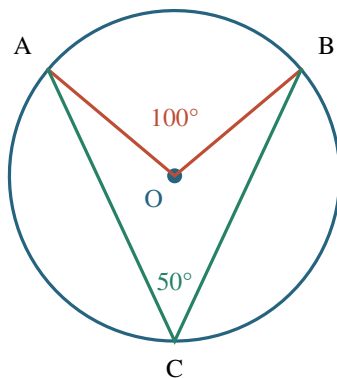
$$\angle AOB = 2\angle ACB, \quad \angle A + \angle C = 180^\circ$$

■ Worked example

Let A,B,C lie on the circle and let O be the centre inside angle ACB. Put α =angle ACO and β =angle OCB. Equal radii make triangles AOC and BOC isosceles. Thus angle AOC= $180-2\alpha$ and angle COB= $180-2\beta$. Angles around O give the remaining angle AOB= $360-(180-2\alpha)-(180-2\beta)=2(\alpha+\beta)=2$ angle ACB. Other centre positions need the corresponding subtraction of isosceles angles, with the same result for the chosen arc. If AB is a diameter, angle AOB= 180° , so angle ACB= 90° . Two circumference angles

on the same chord in the same segment each equal half the same central angle, so they agree. For opposite cyclic angles, their arcs together make 360° ; half-arc angles therefore sum to 180° . A central angle 100° gives 50° at the circumference; an angle opposite 112° in a cyclic quadrilateral is 68° .

Angles on the same upper arc: centre = twice circumference



Where it goes wrong

- Distinguish the reflex central angle from the smaller one and identify which arc does not contain the circumference vertex.
- Opposite segments can give supplementary rather than equal angles.
- A theorem must be tied to named points.

■ Watch out

Not true: Any two circumference angles are equal regardless of their chords and segments. Angles at the circumference are equal only when they stand on the same arc, in the same segment.

Applying it

- AQA G10 Higher requires application and proof.
- Draw auxiliary radii, use isosceles base angles and point sums, then extend the proof to the intended configuration rather than inferring equality from the drawing.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Angle at the centre:** An angle formed by two radii meeting at the centre.
- The central angle on an arc is twice a circumference angle standing on that same arc.
- Watch for: distinguish the reflex central angle from the smaller one and identify which arc does not contain the circumference vertex.

3.4 · Tangents, chords and the alternate segment · Higher

Their equal lengths follow from right-triangle congruence, not from a symmetric-looking sketch.

- Two paths from an outside point just touch a circular pond.
- This lesson studies **alternate segment theorem**: The tangent-chord angle equals the angle subtended by that chord in the opposite segment.

The method

- A tangent is perpendicular to its contact radius.
- Tangents from one external point are equal.
- The perpendicular from the centre to a chord bisects it.
- The tangent-chord angle equals the angle on that chord in the alternate segment.
- Use equal radii, right triangles and the central-angle theorem to prove these relationships.

A worked case

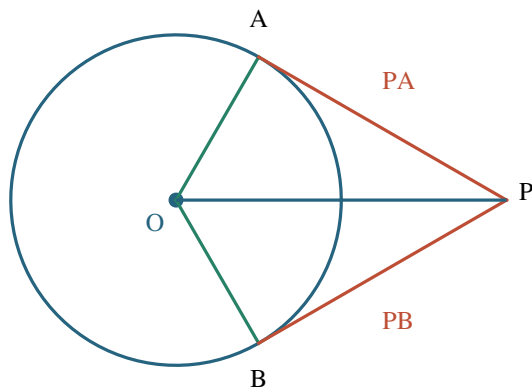
$$PA = PB, \quad OM \perp AB \implies AM = MB$$

■ Worked example

At contact T, the radius OT is perpendicular to the tangent: a non-perpendicular line through T would have a smaller centre-to-line distance than the radius and cut the circle twice. For tangents PA and PB, $OA=OB$, OP is shared and both contact angles are 90° . RHS makes OAP and OBP congruent, so $PA=PB$. For a centre perpendicular OM

to chord AB, $OA=OB$ and OM is shared; RHS gives $AM=MB$. If $OA=5$ and $OM=3$, $AM=4$, hence $AB=8$. For a chord AB with minor central angle φ , triangle OAB has base angle $(180-\varphi)/2$. The adjacent tangent-chord angle is $90-(180-\varphi)/2=\varphi/2$, equal to the circumference angle in the alternate segment. Thus a tangent-chord angle of 35° gives 35° in that segment. Reflex/supplementary configurations require the matching arc and angle.

Shared OP and equal radii $\rightarrow$ RHS $\rightarrow PA = PB$



Where it goes wrong

- A tangent-chord angle and a radius-chord angle are different.
- Equal tangent lengths refer to one external point.
- The chord is bisected by a perpendicular from the centre, not by any line that happens to cross it.

■ Watch out

Not true: Any line through a circle centre bisects every chord it meets. The perpendicular from the centre bisects a chord; a line through the centre at another angle does not.

Applying it

- AQA G10 Higher includes these theorem proofs and related results.
- Give the congruence criterion or angle chain explicitly and identify the relevant chord, contact point and alternate segment.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Alternate segment theorem:** The tangent-chord angle equals the angle subtended by that chord in the opposite segment.
- A tangent is perpendicular to its contact radius.
- Watch for: A tangent-chord angle and a radius-chord angle are different.

3.4 · Areas, perimeters and composite plane shapes · Foundation

The side length and the height serve different jobs in an area formula.

- A sloping garden fence can have a long side but a small perpendicular height.
- This lesson studies **perpendicular height** : The distance between a base and its parallel opposite level, measured at right angles.

The method

- Triangle area is $bh/2$, parallelogram area bh and trapezium area $(a+b)h/2$ for parallel sides a, b .
- Heights are perpendicular to the selected base.
- Perimeter adds only the outside boundary.
- Split a composite shape into non-overlapping parts, or subtract a missing region from a containing shape.

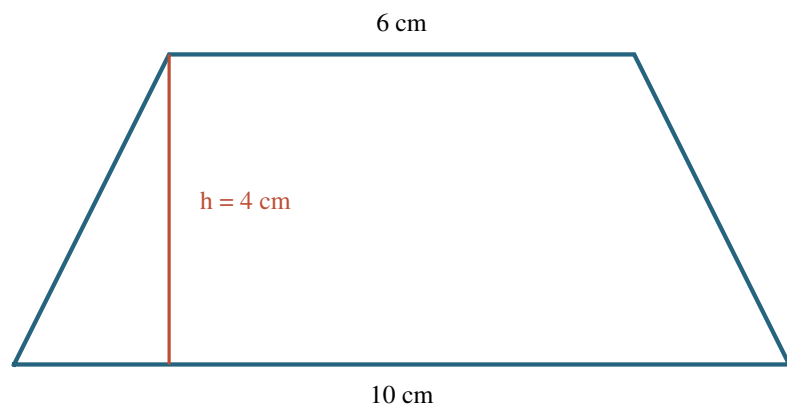
A worked case

$$A_{\triangle} = \frac{1}{2}bh, \quad A_{\text{trap}} = \frac{1}{2}(a+b)h$$

■ Worked example

A triangle with base 8 cm and height 5 cm has area 20 cm^2 ; a parallelogram with the same base/height has area 40 cm^2 . A trapezium with parallel sides 6 and 10 cm and height 4 cm has area $(6+10) \times 4/2 = 32 \text{ cm}^2$. A rectangular 8 by 6 cm sheet with a 3 by 2 cm corner removed has area $48 - 6 = 42 \text{ cm}^2$. Its perimeter is still 28 cm: the two removed outside segments total 5 cm and the two new notch edges also total 5 cm. This perimeter equality depends on a corner rectangular cut; an internal hole adds a separate boundary. Rearranging triangle area gives $h = 2A/b$.

Trapezium area = average parallel sides $\times$ height



Where it goes wrong

- Use perpendicular height rather than a sloping side.
- A shared internal division line is not part of the perimeter.
- An area answer has squared units; a perimeter answer has length units.

■ Watch out

Not true: The sloping side of a parallelogram is its perpendicular height. The height must be perpendicular to the base; the sloping side is longer than the height.

Applying it

- AQA G16/G17 includes triangle/parallelogram/trapezium area and composite perimeter/area.
- Mark the parallel bases, perpendicular height and counted outside edges before calculating.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Perpendicular height:** The distance between a base and its parallel opposite level, measured at right angles.
- Triangle area is $bh/2$, parallelogram area bh and trapezium area $(a+b)h/2$ for parallel sides a, b .
- Watch for: use perpendicular height rather than a sloping side.

3.4 · Areas, perimeters and composite plane shapes · Higher

The side length and the height serve different jobs in an area formula.

- A sloping garden fence can have a long side but a small perpendicular height.
- This lesson studies **perpendicular height**: The distance between a base and its parallel opposite level, measured at right angles.

The method

- Triangle area is $bh/2$, parallelogram area bh and trapezium area $(a+b)h/2$ for parallel sides a, b .
- Heights are perpendicular to the selected base.
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- Split a composite shape into non-overlapping parts, or subtract a missing region from a containing shape.

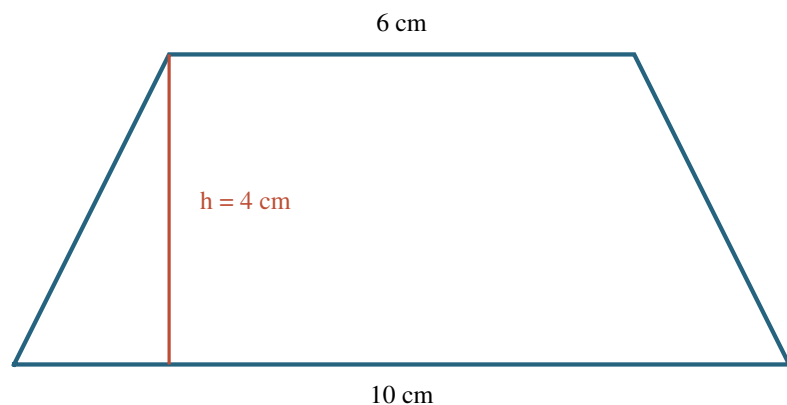
A worked case

$$A_{\triangle} = \frac{1}{2}bh, \quad A_{\text{trap}} = \frac{1}{2}(a+b)h$$

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A triangle with base 8 cm and height 5 cm has area 20 cm^2 ; a parallelogram with the same base/height has area 40 cm^2 . A trapezium with parallel sides 6 and 10 cm and height 4 cm has area $(6+10) \times 4/2 = 32 \text{ cm}^2$. A rectangular 8 by 6 cm sheet with a 3 by 2 cm corner removed has area $48 - 6 = 42 \text{ cm}^2$. Its perimeter is still 28 cm: the two removed outside segments total 5 cm and the two new notch edges also total 5 cm. This perimeter equality depends on a corner rectangular cut; an internal hole adds a separate boundary. Rearranging triangle area gives $h = 2A/b$.

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Not true: The sloping side of a parallelogram is its perpendicular height. The height must be perpendicular to the base; the sloping side is longer than the height.

Applying it

- AQA G16/G17 includes triangle/parallelogram/trapezium area and composite perimeter/area.
- Mark the parallel bases, perpendicular height and counted outside edges before calculating.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Perpendicular height:** The distance between a base and its parallel opposite level, measured at right angles.
- Triangle area is $bh/2$, parallelogram area bh and trapezium area $(a+b)h/2$ for parallel sides a, b .
- Watch for: use perpendicular height rather than a sloping side.

3.4 · Prism volume and cylinder measurement · Foundation

Its volume depends on that end area and the distance the end shape extends.

- A tank has the same triangular end shape all along its length.
- This lesson studies **cross-section**: A slice perpendicular to a prism's length that remains constant along it.

The method

- For a right prism, volume is constant cross-sectional area times perpendicular length.
- Cuboid volume is lwh .
- A cylinder is a circular prism: $V = \pi r^2 h$.
- Total closed-cylinder surface area includes two circular ends and the curved surface $2\pi rh$.
- Open containers omit the specified faces.

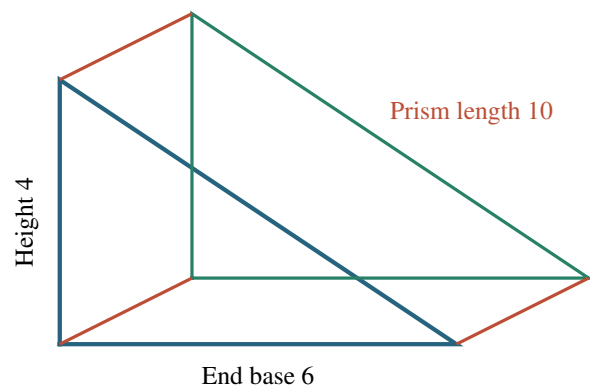
A worked case

$$V = A_{\text{cross}}L, \quad V_{\text{cylinder}} = \pi r^2 h$$

■ Worked example

A triangular prism with end base 6 cm, end height 4 cm and length 10 cm has cross-section 12 cm^2 and volume 120 cm^3 . A cylinder of radius 3 cm and height 5 cm has volume $45\pi \text{ cm}^3$. Its curved surface unwraps to a rectangle of width $2\pi r = 6\pi$ and height 5, so curved area is $30\pi \text{ cm}^2$. Adding two ends gives total area $30\pi + 18\pi = 48\pi \text{ cm}^2$. An open-top tank of those dimensions has surface area $39\pi \text{ cm}^2$ because it keeps only one end. If a prism has volume 180 cm^3 and cross-section 15 cm^2 , its length is 12 cm. Convert 2000 cm^3 to 2 litres, keeping volume conversion separate from surface area.

Constant triangular end area 12 $\rightarrow$ volume 120



Where it goes wrong

- Use cross-sectional area, not perimeter, in the volume formula.
- Distinguish cylinder radius from diameter and identify whether end faces are present.
- A length times an area gives cubic units.

■ Watch out

Not true: Prism volume equals cross-section perimeter times length. Volume equals cross-sectional area times length; perimeter times length gives the area of the side faces.

Applying it

- AQA G16/G17 includes cuboids, right prisms and cylinders.
- Draw the constant end shape, calculate it first and label the extrusion length.
- Preserve exact multiples of pi when asked.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Cross-section:** A slice perpendicular to a prism's length that remains constant along it.
- For a right prism, volume is constant cross-sectional area times perpendicular length.
- Watch for: use cross-sectional area, not perimeter, in the volume formula.

3.4 · Prism volume and cylinder measurement · Higher

Its volume depends on that end area and the distance the end shape extends.

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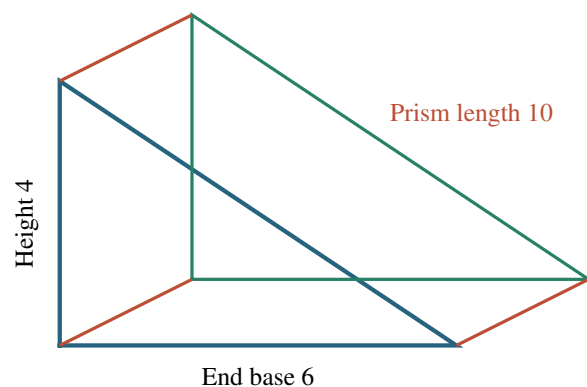
A worked case

$$V = A_{\text{cross}} L, \quad V_{\text{cylinder}} = \pi r^2 h$$

■ Worked example

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Applying it

- AQA G16/G17 includes cuboids, right prisms and cylinders.
- Draw the constant end shape, calculate it first and label the extrusion length.
- Preserve exact multiples of pi when asked.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Cross-section:** A slice perpendicular to a prism's length that remains constant along it.
- For a right prism, volume is constant cross-sectional area times perpendicular length.
- Watch for: use cross-sectional area, not perimeter, in the volume formula.

3.4 · Circle lengths, areas and composite boundaries · Foundation

Glass uses area; the frame includes a curved arc and the straight diameter.

- A semicircular window needs glass and a frame.
- This lesson studies **circumference**: The total length of a circle's boundary.

The method

- Circle circumference is $2\pi r = \pi d$ and area is πr^2 .
- A semicircle has half the circle area, but its complete perimeter includes the diameter as well as half the circumference.
- For composite shapes count every exposed boundary once, and separate straight edges from arcs.

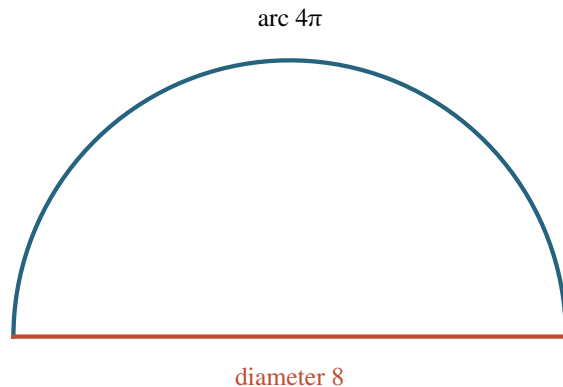
A worked case

$$C = 2\pi r, \quad A = \pi r^2, \quad P_{\text{semi}} = \pi r + 2r$$

■ Worked example

For radius 4 cm, circumference is 8π cm and area 16π cm². A semicircle of that radius has area 8π cm² and perimeter $4\pi + 8$ cm. A rectangular 8 by 3 cm window topped by this semicircle has area $24 + 8\pi$ cm². Its perimeter is the bottom 8, two vertical sides totalling 6 and the curved top 4π : $14 + 4\pi$ cm. The diameter across the join is internal and is not counted. An annulus with outer radius 5 and inner radius 3 has area $\pi(25 - 9) = 16\pi$ cm². Its two circular boundaries together have length $10\pi + 6\pi = 16\pi$ cm, despite this accidental equality of coefficients; length and area still have different units.

Semicircle perimeter: curved arc + straight diameter



Where it goes wrong

- Radius must be halved from a given diameter before squaring.
- Half a circle's circumference is only its arc, not the complete semicircle perimeter.
- An internal join is not exposed boundary.

■ Watch out

Not true: The complete perimeter of a semicircle is half a circle's circumference. The perimeter of a semicircle is half the circumference plus the diameter.

Applying it

- AQA G17 includes exact pi answers and composite circle perimeters/areas.
- Give both the exact expression and the required final rounded value, keeping the meaning and units clear.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Circumference:** The total length of a circle's boundary.
- Circle circumference is $2\pi r = \pi d$ and area is πr^2 .
- Watch for: radius must be halved from a given diameter before squaring.

3.4 · Circle lengths, areas and composite boundaries · Higher

Glass uses area; the frame includes a curved arc and the straight diameter.

- A semicircular window needs glass and a frame.
- This lesson studies **circumference** : The total length of a circle's boundary.

The method

- Circle circumference is $2\pi r = \pi d$ and area is πr^2 .
- A semicircle has half the circle area, but its complete perimeter includes the diameter as well as half the circumference.
- For composite shapes count every exposed boundary once, and separate straight edges from arcs.

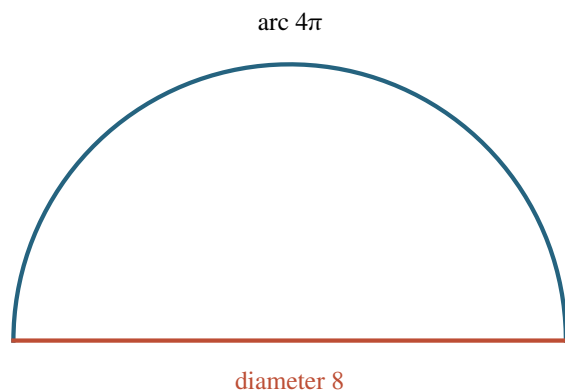
A worked case

$$C = 2\pi r, \quad A = \pi r^2, \quad P_{\text{semi}} = \pi r + 2r$$

■ Worked example

For radius 4 cm, circumference is 8π cm and area 16π cm². A semicircle of that radius has area 8π cm² and perimeter $4\pi + 8$ cm. A rectangular 8 by 3 cm window topped by this semicircle has area $24 + 8\pi$ cm². Its perimeter is the bottom 8, two vertical sides totalling 6 and the curved top 4π : $14 + 4\pi$ cm. The diameter across the join is internal and is not counted. An annulus with outer radius 5 and inner radius 3 has area $\pi(25 - 9) = 16\pi$ cm². Its two circular boundaries together have length $10\pi + 6\pi = 16\pi$ cm, despite this accidental equality of coefficients; length and area still have different units.

Semicircle perimeter: curved arc + straight diameter



Where it goes wrong

- Radius must be halved from a given diameter before squaring.
- Half a circle's circumference is only its arc, not the complete semicircle perimeter.
- An internal join is not exposed boundary.

■ Watch out

Not true: The complete perimeter of a semicircle is half a circle's circumference. The perimeter of a semicircle is half the circumference plus the diameter.

Applying it

- AQA G17 includes exact pi answers and composite circle perimeters/areas.
- Give both the exact expression and the required final rounded value, keeping the meaning and units clear.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Circumference:** The total length of a circle's boundary.
- Circle circumference is $2\pi r = \pi d$ and area is πr^2 .
- Watch for: radius must be halved from a given diameter before squaring.

3.4 · Spheres, cones, pyramids and frustums · Foundation

The missing small cone must be removed from the original cone before the capacity is known.

- A cone-shaped cup is cut flat at its top.
- This lesson studies **frustum**: The remaining solid after a cone or pyramid is cut parallel to its base.

The method

- Pyramid and cone volumes are one third of base area times perpendicular height.
- A sphere has volume $4\pi r^3/3$ and area $4\pi r^2$.
- A cone's curved area is $\pi r l$ using slant height l ; its volume uses perpendicular height h .
- For a frustum subtract the removed similar solid.
- Composite surface area counts only exposed faces; joined faces are hidden.

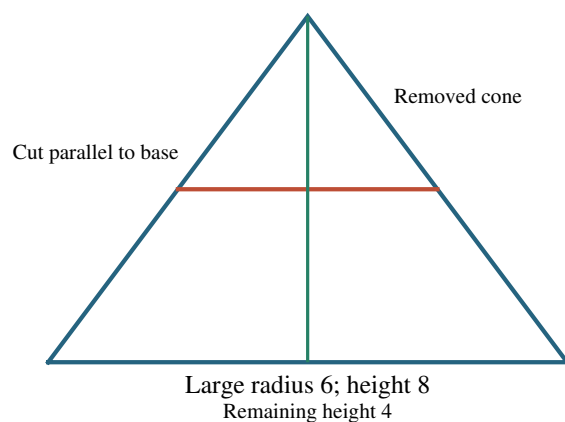
A worked case

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h, \quad V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

■ Worked example

A cone with radius 3 cm and perpendicular height 4 cm has slant height 5 cm. Volume is $12\pi \text{ cm}^3$, curved area $15\pi \text{ cm}^2$ and total closed area $24\pi \text{ cm}^2$. A sphere of radius 3 has volume $36\pi \text{ cm}^3$ and area $36\pi \text{ cm}^2$, with different units. A square pyramid of base side 6 and height 4 has volume $6^2 \times 4/3 = 48 \text{ cm}^3$; each triangular face has slant height $\sqrt{4^2 + 3^2} = 5$, so lateral area is $4 \times (6 \times 5/2) = 60 \text{ cm}^2$ and total area 96 cm^2 . A large cone $r=6, h=8$ loses a similar top cone $r=3, h=4$: frustum volume is $96\pi - 12\pi = 84\pi \text{ cm}^3$. Its slant height is $10-5=5$; curved area is $60\pi - 15\pi = 45\pi$, and two circular ends add $36\pi + 9\pi$ for total $90\pi \text{ cm}^2$.

Frustum volume = whole cone – removed similar cone



Where it goes wrong

- Perpendicular height and slant height are not interchangeable.
- A frustum is not a full cone of the leftover height.
- Shared composite faces do not contribute exposed area.

■ Watch out

Not true: A cone's slant height can replace its perpendicular height in the volume formula. Cone volume uses the perpendicular height; the slant height is longer and gives too large a volume.

Applying it

- AQA G17 additional Foundation includes spheres, pyramids, cones, composite solids and frustums.
- Use similarity to find missing removed dimensions, then subtract volumes or exposed areas with matching units.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Frustum:** The remaining solid after a cone or pyramid is cut parallel to its base.
- Pyramid and cone volumes are one third of base area times perpendicular height.
- Watch for: perpendicular height and slant height are not interchangeable.

3.4 · Spheres, cones, pyramids and frustums · Higher

The missing small cone must be removed from the original cone before the capacity is known.

- A cone-shaped cup is cut flat at its top.
- This lesson studies **frustum**: The remaining solid after a cone or pyramid is cut parallel to its base.

The method

- Pyramid and cone volumes are one third of base area times perpendicular height.
- A sphere has volume $4\pi r^3/3$ and area $4\pi r^2$.
- A cone's curved area is $\pi r l$ using slant height l ; its volume uses perpendicular height h .
- For a frustum subtract the removed similar solid.
- Composite surface area counts only exposed faces; joined faces are hidden.

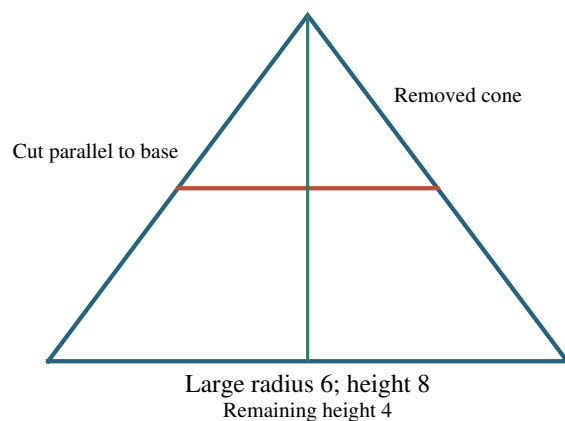
A worked case

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h, \quad V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

■ Worked example

A cone with radius 3 cm and perpendicular height 4 cm has slant height 5 cm. Volume is $12\pi \text{ cm}^3$, curved area $15\pi \text{ cm}^2$ and total closed area $24\pi \text{ cm}^2$. A sphere of radius 3 has volume $36\pi \text{ cm}^3$ and area $36\pi \text{ cm}^2$, with different units. A square pyramid of base side 6 and height 4 has volume $6^2 \times 4/3 = 48 \text{ cm}^3$; each triangular face has slant height $\sqrt{4^2 + 3^2} = 5$, so lateral area is $4 \times (6 \times 5/2) = 60 \text{ cm}^2$ and total area 96 cm^2 . A large cone $r=6, h=8$ loses a similar top cone $r=3, h=4$: frustum volume is $96\pi - 12\pi = 84\pi \text{ cm}^3$. Its slant height is $10-5=5$; curved area is $60\pi - 15\pi = 45\pi$, and two circular ends add $36\pi + 9\pi$ for total $90\pi \text{ cm}^2$.

Frustum volume = whole cone – removed similar cone



Where it goes wrong

- Perpendicular height and slant height are not interchangeable.
- A frustum is not a full cone of the leftover height.
- Shared composite faces do not contribute exposed area.

■ Watch out

Not true: A cone's slant height can replace its perpendicular height in the volume formula. Cone volume uses the perpendicular height; the slant height is longer and gives too large a volume.

Applying it

- AQA G17 additional Foundation includes spheres, pyramids, cones, composite solids and frustums.
- Use similarity to find missing removed dimensions, then subtract volumes or exposed areas with matching units.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Frustum:** The remaining solid after a cone or pyramid is cut parallel to its base.
- Pyramid and cone volumes are one third of base area times perpendicular height.
- Watch for: perpendicular height and slant height are not interchangeable.

3.4 · Arcs, sectors and reverse angle calculations · Foundation

It covers one third of a full turn, so its arc and area are each one third of the corresponding whole circle.

- A circular fan opens through 120° .
- This lesson studies **sector**: A circle region bounded by two radii and their intervening arc.

The method

- For an angle θ in degrees, fraction of a turn is $\theta/360$.
- Arc length is this fraction of $2\pi r$ and sector area is this fraction of πr^2 .
- Sector perimeter adds the two radii.
- Rearrange the same fraction to recover an angle from an arc or an area.
- Keep degree and length units separate.

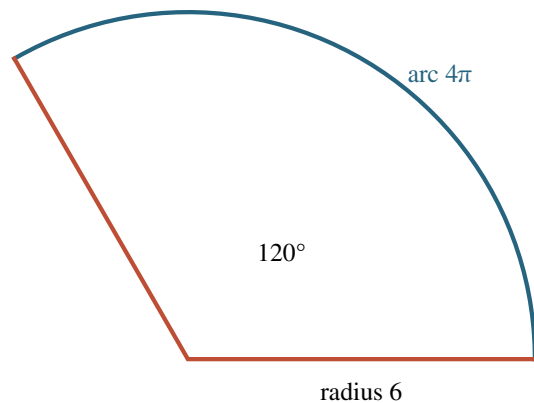
A worked case

$$s = \frac{\theta}{360^\circ} 2\pi r, \quad A = \frac{\theta}{360^\circ} \pi r^2$$

■ Worked example

At $r=6$ cm and $\theta=120^\circ$, the fraction is $1/3$: arc is 4π cm, area is 12π cm² and perimeter is $4\pi+12$ cm. If another $r=6$ sector has area 9π cm², its fraction is $9\pi/36\pi=1/4$ and angle is 90° . If its arc instead measures 3π cm, its fraction is $3\pi/12\pi=1/4$, giving the same angle. A full 360° sector has the whole circle area, while the circle boundary has no extra radii. A 60° sector of radius 3 has area $(1/6)\times 9\pi=1.5\pi$ cm².

One third of a turn: sector area 12π



Where it goes wrong

- An arc length is not a sector perimeter.
- Use the angle as a fraction of 360° , not 180° .
- Radius is squared only for area, not arc length.

■ Watch out

Not true: A sector's arc length and perimeter are the same. The perimeter also includes the two radii: perimeter = arc + $2r$.

Applying it

- AQA G18 includes arc lengths, sector angles and areas in degree-based geometry.
- Show the full-circle quantity and fraction before multiplying, then check that the result fits the angle.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Sector:** A circle region bounded by two radii and their intervening arc.
- For an angle θ in degrees, fraction of a turn is $\theta/360$.
- Watch for: an arc length is not a sector perimeter.

3.4 · Arcs, sectors and reverse angle calculations · Higher

It covers one third of a full turn, so its arc and area are each one third of the corresponding whole circle.

- A circular fan opens through 120° .
- This lesson studies **sector** : A circle region bounded by two radii and their intervening arc.

The method

- For an angle θ in degrees, fraction of a turn is $\theta/360$.
- Arc length is this fraction of $2\pi r$ and sector area is this fraction of πr^2 .
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- Rearrange the same fraction to recover an angle from an arc or an area.
- Keep degree and length units separate.

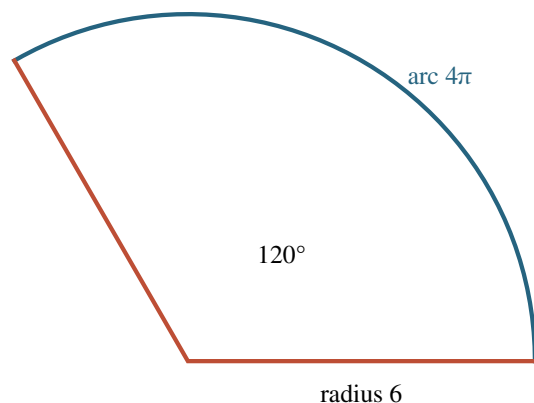
A worked case

$$s = \frac{\theta}{360^\circ} 2\pi r, \quad A = \frac{\theta}{360^\circ} \pi r^2$$

■ Worked example

At $r=6$ cm and $\theta=120^\circ$, the fraction is $1/3$: arc is 4π cm, area is 12π cm² and perimeter is $4\pi+12$ cm. If another $r=6$ sector has area 9π cm², its fraction is $9\pi/36\pi=1/4$ and angle is 90° . If its arc instead measures 3π cm, its fraction is $3\pi/12\pi=1/4$, giving the same angle. A full 360° sector has the whole circle area, while the circle boundary has no extra radii. A 60° sector of radius 3 has area $(1/6)\times 9\pi=1.5\pi$ cm².

One third of a turn: sector area 12π



Where it goes wrong

- An arc length is not a sector perimeter.
- Use the angle as a fraction of 360° , not 180° .
- Radius is squared only for area, not arc length.

■ Watch out

Not true: A sector's arc length and perimeter are the same. The perimeter also includes the two radii: perimeter = arc + $2r$.

Applying it

- AQA G18 includes arc lengths, sector angles and areas in degree-based geometry.
- Show the full-circle quantity and fraction before multiplying, then check that the result fits the angle.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Sector:** A circle region bounded by two radii and their intervening arc.
- For an angle θ in degrees, fraction of a turn is $\theta/360$.
- Watch for: an arc length is not a sector perimeter.

3.4 · Exact trigonometric values from special triangles · Foundation

Why is $\sin 30^\circ$ exactly one half?

- A calculator shows $\sin 30^\circ$ as 0.5, but a special triangle explains why the value is exactly one half and how the other values connect.
- This lesson studies **exact value** : A value retained as a fraction or surd rather than a rounded decimal.

The method

- Bisect an equilateral triangle of side 2 to obtain a 30–60–90 triangle with sides 1, $\sqrt{3}$, 2.
- A right isosceles triangle has sides 1, 1, $\sqrt{2}$.
- Apply opposite/hypotenuse, adjacent/hypotenuse and opposite/adjacent to derive sine, cosine and tangent.
- Use degree angles and retain surds exactly; $\tan 90^\circ$ is undefined.

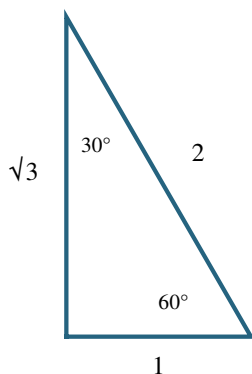
A worked case

$$\sin 30^\circ = \frac{1}{2}, \quad \cos 45^\circ = \frac{\sqrt{2}}{2}, \quad \tan 60^\circ = \sqrt{3}$$

■ Worked example

For angles $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$, sine values are $0, 1/2, \sqrt{2}/2, \sqrt{3}/2, 1$; cosine values are $1, \sqrt{3}/2, \sqrt{2}/2, 1/2, 0$. For $0^\circ, 30^\circ, 45^\circ, 60^\circ$, tangent values are $0, 1/\sqrt{3}, 1, \sqrt{3}$. The side 1 opposite 30° in the bisected equilateral triangle gives $\sin 30^\circ = 1/2$; the adjacent $\sqrt{3}$ gives $\cos 30^\circ = \sqrt{3}/2$ and $\tan 30^\circ = 1/\sqrt{3}$. In the isosceles right triangle, $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2} = \sqrt{2}/2$. A right triangle with hypotenuse 10 and angle 30° has opposite side 5 and adjacent side $5\sqrt{3}$. Since $\sin 90^\circ = 1$ and $\cos 90^\circ = 0$, tangent at 90° would divide by zero.

Bisected equilateral triangle: sides 1, $\sqrt{3}$, 2



Where it goes wrong

- Sine and cosine interchange when the chosen acute angle changes to its complement.
- Do not turn $\sqrt{3}$ into a rounded decimal when an exact answer is required.
- Tangent at 90° is not zero.

■ Watch out

Not true: All trigonometric functions have finite values at 90° . $\tan 90^\circ$ is undefined, because $\cos 90^\circ = 0$.

Applying it

- AQA G21 is additional Foundation and includes the listed exact values.
- Derive them from labelled special triangles, then use them in G20 right-triangle calculations.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Exact value:** A value retained as a fraction or surd rather than a rounded decimal.
- Bisect an equilateral triangle of side 2 to obtain a 30–60–90 triangle with sides 1, $\sqrt{3}$, 2.
- Watch for: sine and cosine interchange when the chosen acute angle changes to its complement.

3.4 · Exact trigonometric values from special triangles · Higher

Why is $\sin 30^\circ$ exactly one half?

- A calculator shows $\sin 30^\circ$ as 0.5, but a special triangle explains why the value is exactly one half and how the other values connect.
- This lesson studies **exact value**: A value retained as a fraction or surd rather than a rounded decimal.

The method

- Bisect an equilateral triangle of side 2 to obtain a 30–60–90 triangle with sides $1, \sqrt{3}, 2$.
- A right isosceles triangle has sides $1, 1, \sqrt{2}$.
- Apply opposite/hypotenuse, adjacent/hypotenuse and opposite/adjacent to derive sine, cosine and tangent.
- Use degree angles and retain surds exactly; $\tan 90^\circ$ is undefined.

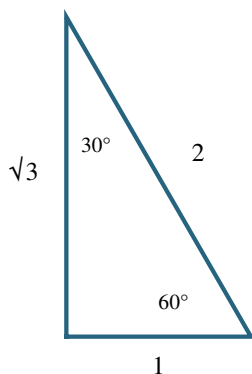
A worked case

$$\sin 30^\circ = \frac{1}{2}, \quad \cos 45^\circ = \frac{\sqrt{2}}{2}, \quad \tan 60^\circ = \sqrt{3}$$

■ Worked example

For angles $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$, sine values are $0, 1/2, \sqrt{2}/2, \sqrt{3}/2, 1$; cosine values are $1, \sqrt{3}/2, \sqrt{2}/2, 1/2, 0$. For $0^\circ, 30^\circ, 45^\circ, 60^\circ$, tangent values are $0, 1/\sqrt{3}, 1, \sqrt{3}$. The side 1 opposite 30° in the bisected equilateral triangle gives $\sin 30^\circ = 1/2$; the adjacent $\sqrt{3}$ gives $\cos 30^\circ = \sqrt{3}/2$ and $\tan 30^\circ = 1/\sqrt{3}$. In the isosceles right triangle, $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2} = \sqrt{2}/2$. A right triangle with hypotenuse 10 and angle 30° has opposite side 5 and adjacent side $5\sqrt{3}$. Since $\sin 90^\circ = 1$ and $\cos 90^\circ = 0$, tangent at 90° would divide by zero.

Bisected equilateral triangle: sides $1, \sqrt{3}, 2$



Where it goes wrong

- Sine and cosine interchange when the chosen acute angle changes to its complement.
- Do not turn $\sqrt{3}$ into a rounded decimal when an exact answer is required.
- Tangent at 90° is not zero.

■ Watch out

Not true: All trigonometric functions have finite values at 90° . $\tan 90^\circ$ is undefined, because $\cos 90^\circ = 0$.

Applying it

- AQA G21 is additional Foundation and includes the listed exact values.
- Derive them from labelled special triangles, then use them in G20 right-triangle calculations.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Exact value:** A value retained as a fraction or surd rather than a rounded decimal.
- Bisect an equilateral triangle of side 2 to obtain a 30–60–90 triangle with sides $1, \sqrt{3}, 2$.
- Watch for: sine and cosine interchange when the chosen acute angle changes to its complement.

3.4 · Sine rule, cosine rule and triangle area · Higher

Two known sides and their included angle can determine the remaining side without inventing a perpendicular side length.

- A triangular plot has no right angle.
- This lesson studies **included angle**: The angle between the two named sides.

The method

- Sine rule pairs opposite sides/angles: $a/\sin A = b/\sin B = c/\sin C$.
- Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$ uses the angle opposite a .
- Area is $ab \sin C / 2$ when C lies between a and b .
- Choose the rule from the known information.
- An inverse sine may give an acute angle and an obtuse supplement; check the angle sum and supplied sides before accepting either.

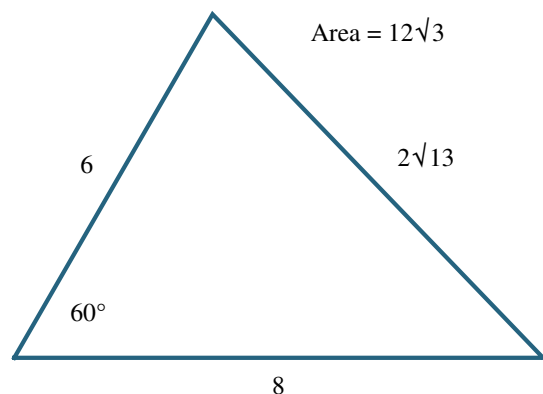
A worked case

$$a^2 = b^2 + c^2 - 2bc \cos A, \quad A_{\Delta} = \frac{1}{2}ab \sin C$$

■ Worked example

With sides 6 and 8 enclosing 60° , $c^2 = 36 + 64 - 96 \times (1/2) = 52$, hence $c = 2\sqrt{13}$. Its area is $(1/2) \times 6 \times 8 \times \sin 60^\circ = 12\sqrt{3}$. For $a=4$ opposite $A=30^\circ$ and $B=45^\circ$, $b=4 \sin 45^\circ / \sin 30^\circ = 4\sqrt{2}$. If sides $a=7, b=5, c=6$, $\cos A = (25 + 36 - 49) / (2 \times 5 \times 6) = 1/5$, so $A \approx 78.5^\circ$. To find an angle from area 12 with enclosing sides 6 and 8, $\sin C = 24/48 = 1/2$; C could be 30° or 150° until the remaining data selects a shape. Label opposite pairs and check triangle inequalities to reject impossible side combinations.

Two sides and their included angle



Where it goes wrong

- Do not pair a side with its adjacent angle in the sine rule.
- The area angle must be included.
- A calculator's first inverse-sine answer need not be the only possible triangle.

■ Watch out

Not true: The sine rule pairs each side with any angle in the triangle. Each side is paired with the angle opposite it: $a/\sin A = b/\sin B$.

Applying it

- AQA G22/G23 Higher includes unknown sides/angles and areas of general triangles.
- Write the chosen rule before substitution and state any second possible configuration.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Included angle:** The angle between the two named sides.
- Sine rule pairs opposite sides/angles: $a/\sin A = b/\sin B = c/\sin C$.
- Watch for: do not pair a side with its adjacent angle in the sine rule.

3.4 · Right triangles in three-dimensional shapes · Higher

Two different right triangles are needed to connect its width, depth and height.

- A cuboid's space diagonal is longer than its base diagonal.
- This lesson studies **projection**: The view or component of a spatial length on a selected plane.

The method

- Identify a plane containing the wanted length or angle.
- Calculate a base diagonal first, then combine it with the perpendicular height.
- For an angle between a line and a plane, use the angle between the line and its orthogonal projection onto that plane.
- Mark right angles; general-triangle rules are used only when the selected triangle is not right-angled.

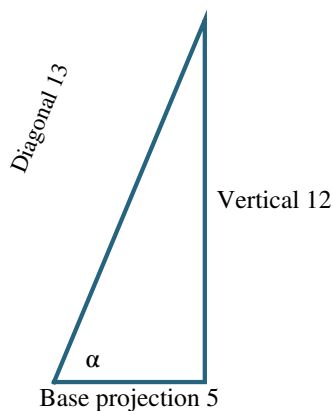
A worked case

$$d = \sqrt{l^2 + w^2 + h^2}, \quad \tan \alpha = \frac{h}{\sqrt{l^2 + w^2}}$$

■ Worked example

A cuboid of width 3, depth 4 and height 12 has base diagonal $\sqrt{9+16}=5$ and space diagonal $\sqrt{25+144}=13$. The angle α of the space diagonal to the horizontal base satisfies $\tan \alpha = 12/5$, giving about 67.4° . Its sine is $12/13$; its cosine is $5/13$. In a square-based pyramid of side 6 and vertical height 4, the base centre-to-side-midpoint distance is 3, giving face slant height 5. The centre-to-corner distance is $3\sqrt{2}$, giving edge length $\sqrt{18+16}=\sqrt{34}$. The face slant and edge lengths differ because their base projections differ.

Cuboid section: first $\sqrt{3^2+4^2}=5$, then $\sqrt{5^2+12^2}=13$



Where it goes wrong

- Do not combine unrelated lengths as if they met at a right angle.
- The angle to a plane uses the base projection, not an arbitrary base edge.
- A pyramid's face slant is different from its sloping edge.

■ Watch out

Not true: A cuboid's space diagonal can be found from just two of its three edge lengths. The space diagonal is $\sqrt{a^2+b^2+c^2}$, so it needs all three edges.

Applying it

- AQA G20 Higher extends right-triangle and, where possible, general-triangle reasoning into 3D.
- Draw the relevant section triangle separately and name which spatial points it represents.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Projection:** The view or component of a spatial length on a selected plane.
- Identify a plane containing the wanted length or angle.
- Watch for: do not combine unrelated lengths as if they met at a right angle.

3.4 · Vector geometry and midpoint arguments · Higher

Vectors can prove that it is parallel to the third side and exactly half its length.

- A triangle's two side midpoints create a new segment.
- This lesson studies **collinear** : Lying on one straight line.

The method

- Add/subtract components and multiply a vector by a scalar.
- Position vectors locate points from one origin; displacement $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ joins two points.
- A scalar multiple gives parallel directions; to establish collinearity, also connect the displacements to a common point.
- Midpoints average position vectors.
- Give a chain of vector equalities with clear start and end points.

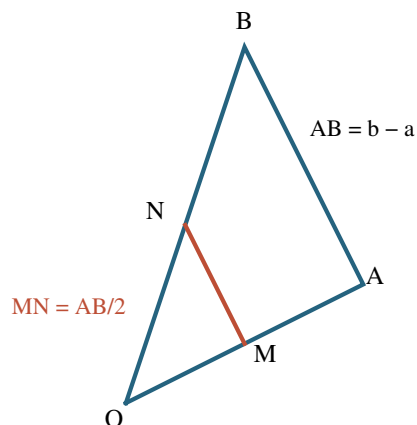
A worked case

$$\overrightarrow{MN} = \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}\overrightarrow{AB}$$

■ Worked example

Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, with M midpoint of OA and N midpoint of OB. Then $OM = a/2$ and $ON = b/2$, so $MN = b/2 - a/2 = (b-a)/2 = AB/2$. Thus MN is parallel to AB and half as long. For $\mathbf{a} = (4, 2)$, $\mathbf{b} = (2, 6)$, $M = (2, 1)$, $N = (1, 3)$, $\overrightarrow{AB} = (-2, 4)$ and $\overrightarrow{MN} = (-1, 2)$, verifying the general result. In parallelogram OACB with $\overrightarrow{OC} = \mathbf{a} + \mathbf{b}$, the midpoint of OC is $(\mathbf{a} + \mathbf{b})/2$; the midpoint of AB is the same, so the diagonals bisect each other. If $AP = 3AB/2$, P lies on line AB beyond B; if $AP = AB/2$, P is its midpoint. A parallel vector at another location alone does not prove three specified points collinear.

Midpoint segment: half length, parallel direction



Where it goes wrong

- Keep $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, not $\mathbf{a} - \mathbf{b}$.
- Parallelism alone does not locate a line.
- A numerical example can check the algebra but cannot replace the general midpoint proof.

■ Watch out

Not true: If two vectors are parallel, any three points used in their diagrams must be collinear. Parallel vectors give collinear points only when the vectors also share a point.

Applying it

- AQA G25 Higher uses vectors for geometric arguments/proofs, while G24 and the basic G25 operations also belong to Foundation.
- This proof lesson avoids scalar products and spatial line equations.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.4.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Collinear:** Lying on one straight line.
- Add/subtract components and multiply a vector by a scalar.
- Watch for: keep $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, not $\mathbf{a} - \mathbf{b}$.

3.5 · Probability trees and outcomes · Foundation

What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability**: The probability of an event after restricting the sample space to a stated condition.

The method

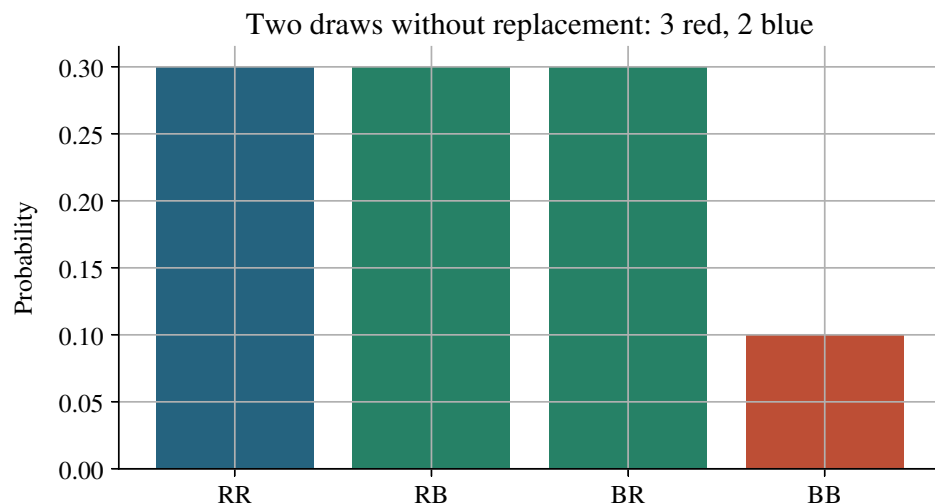
- A probability lies between 0 and 1.
- Exhaustive, mutually exclusive outcomes have probabilities summing to 1.
- Multiply successive branch probabilities and add separate routes to an outcome.

A worked case

$$P(RR) = P(R_1)P(R_2 | R_1)$$

■ Worked example

A bag contains 3 red and 2 blue counters. With replacement, $P(\text{two red}) = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25} = 0.36$. Without replacement, the red-red branch is $\frac{3}{5} \times \frac{2}{4} = 0.3$. Label each branch before multiplying.



Where it goes wrong

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

■ Watch out

Not true: Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so $P(A \text{ and } B) = 0 \neq P(A)P(B)$.

Applying it

- Use a frequency table or a simple tree before calculating.
- Formal conditional probability formulae are outside this Foundation/Core support lesson.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.5.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Conditional probability:** The probability of an event after restricting the sample space to a stated condition.
- A probability lies between 0 and 1.
- Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.

3.5 · Probability, trees and conditional reasoning · Higher

What changes after the first draw?

- A bag has 3 red and 2 blue counters. Taking two without replacement changes the chance of the second colour.
- This lesson studies **conditional probability**: The probability of an event after restricting the sample space to a stated condition.

The method

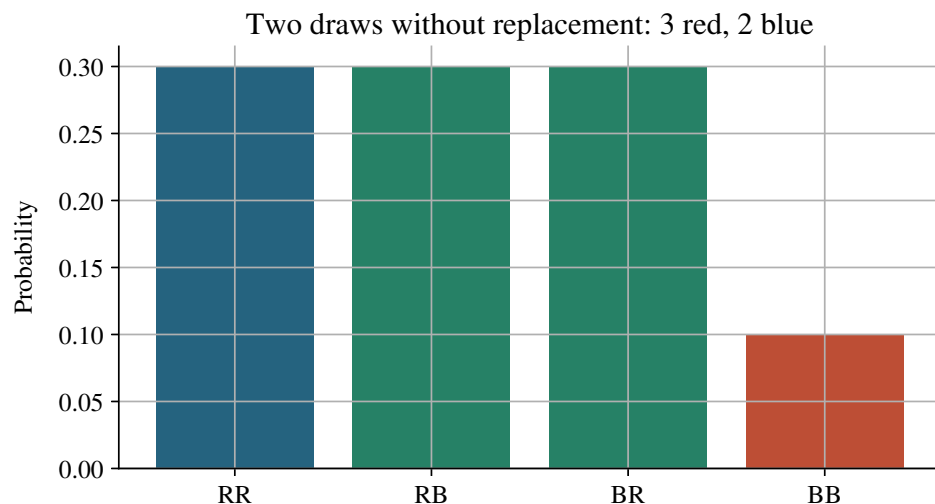
- Multiply along a tree branch and add disjoint branches.
- With replacement, the composition stays fixed.
- Conditional probability is $P(A \text{ given } B) = P(A \cap B) / P(B)$, for $P(B) > 0$.

A worked case

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

■ Worked example

Without replacement, $P(\text{two red}) = 3/5 \times 2/4 = 3/10$. $P(\text{one of each}) = 3/5 \times 2/4 + 2/5 \times 3/4 = 3/5$. If $P(A \cap B) = 0.12$ and $P(B) = 0.3$, $P(A \text{ given } B) = 0.4$.



Where it goes wrong

- Mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.
- Two disjoint events with positive probability are not independent.

■ Watch out

Not true: Mutually exclusive events with positive probabilities must be independent. Mutually exclusive events with positive probabilities are dependent: if one happens, the other cannot, so $P(A \text{ and } B) = 0 \neq P(A)P(B)$.

Applying it

- A two-way table makes the restricted denominator visible.
- Before using a product $P(A)P(B)$, justify independence from the context or the supplied information.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.5.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Conditional probability:** The probability of an event after restricting the sample space to a stated condition.
- Multiply along a tree branch and add disjoint branches.
- Watch for: mutually exclusive means no overlap; independent means that knowing one event does not change the other's probability.

3.5 · Experimental probability, fairness and expected outcomes · Foundation

This does not mean it must give four tails next, or prove that the coin is unfair.

- A coin gives six heads in its first ten tosses.
- This lesson studies **relative frequency**: The observed count of an outcome divided by the number of trials.

The method

- Record each trial consistently and total the counts.
- Relative frequency estimates probability as outcome frequency/trials.
- For a stated probability p , expected count in n future trials is np ; it is a long-run average, not a guarantee.
- Unbiased trials with larger samples usually give more stable estimates, but cannot force exact agreement with theory.
- Probabilities lie from 0 to 1 and a mutually exclusive exhaustive list sums to 1.

A worked case

$$\hat{p} = \frac{f}{n}, \quad E = np$$

■ Worked example

A spinner lands red 18 times in 60 spins, giving estimated $P(\text{red}) = 18/60 = 0.3 = 30\%$. Using this estimate predicts $0.3 \times 200 = 60$ red results in 200 future spins. If red, blue and green are exhaustive with probabilities 0.3, 0.45 and p , then $p = 1 - 0.75 = 0.25$. A fair coin has theoretical $P(\text{head}) = 0.5$; 100 tosses give expected heads 50, but 48 or 54 is possible. An experiment with 6 heads in 10 tosses estimates 0.6; one with 502 heads in 1000 estimates 0.502. These illustrative runs are not proof that error falls at every stage. Spin using the same method and record all results rather than stopping when a favourite outcome appears.



Where it goes wrong

- A larger biased sample can still be misleading.
- Expected does not mean certain.
- Mutually exclusive events cannot happen together; if categories overlap, do not simply add their probabilities as separate outcomes.

■ Watch out

Not true: Expected count 50 means exactly 50 successes must occur. The expected count is a long-run average; the actual count varies around it.

Applying it

- AQA P1–P5 uses frequency tables/trees, randomness/fairness, expected counts, the probability scale and empirical/theoretical comparison.
- State whether a value is observed, estimated or theoretical.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.5.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Relative frequency:** The observed count of an outcome divided by the number of trials.
- Record each trial consistently and total the counts.
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3.5 · Experimental probability, fairness and expected outcomes · Higher

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In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.5.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Relative frequency:** The observed count of an outcome divided by the number of trials.
- Record each trial consistently and total the counts.
- Watch for: A larger biased sample can still be misleading.

3.5 · Systematic possibilities and equally likely outcomes · Foundation

The possible totals are not equally likely.

- Two fair dice can total seven in six different ways, but total two in only one way.
- This lesson studies **sample space**: The complete set of outcomes in an experiment.

The method

- List outcomes systematically using a grid, table or tree.
- For equally likely outcomes, probability is favourable outcomes/total outcomes.
- Keep ordered outcomes distinct when the experiments have labelled first and second stages.
- A grid shows completeness and prevents duplicate counting; unequal probabilities need weights rather than a simple count.

A worked case

$$P(E) = \frac{\text{favourable equally likely outcomes}}{\text{all equally likely outcomes}}$$

■ Worked example

A coin and a fair die have 12 equally likely ordered outcomes: H1 to H6 and T1 to T6. Heads with an even die result has three outcomes, so probability is $3/12=1/4$. Two dice have 36 ordered pairs. Total seven arises from (1,6),(2,5),(3,4),(4,3),(5,2),(6,1), so probability is $6/36=1/6$. A double has six outcomes and probability $1/6$. Total at least eleven occurs in (5,6),(6,5),(6,6), giving $1/12$. With a spinner divided into unequal sectors, the named colours are not automatically equally likely; use the sector proportions or a justified experimental estimate.

36 equally likely cells; six sums equal seven

Second die	6	7	8	9	10	11	12
	5	6	7	8	9	10	11
	4	5	6	7	8	9	10
	3	4	5	6	7	8	9
	2	3	4	5	6	7	8
	1	2	3	4	5	6	7
		1	2	3	4	5	6
		First die					

Where it goes wrong

- Counting totals rather than equally likely dice pairs gives wrong weights.
- State whether order matters.
- A possibility table lists outcomes; its entries are not automatically equiprobable.

■ Watch out

Not true: All possible totals of two fair dice are equally likely. A total of 7 has six ways, but 2 has only one, so the totals are not equally likely.

Applying it

- AQA P6/P7 uses tables, grids and trees to enumerate theoretical possibilities.
- Explain why the selected elementary outcomes have equal probability before dividing counts.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.5.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Sample space:** The complete set of outcomes in an experiment.
- List outcomes systematically using a grid, table or tree.
- Watch for: counting totals rather than equally likely dice pairs gives wrong weights.

3.5 · Systematic possibilities and equally likely outcomes · Higher

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- Explain why the selected elementary outcomes have equal probability before dividing counts.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.5.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Sample space:** The complete set of outcomes in an experiment.
- List outcomes systematically using a grid, table or tree.
- Watch for: counting totals rather than equally likely dice pairs gives wrong weights.

3.5 · Venn diagrams, unions and two-way counts · Foundation

Some students do both, so adding the two group totals counts those students twice.

- A class survey asks about cycling and swimming.
- This lesson studies **intersection** : The outcomes belonging to both named sets.

The method

- Place the overlap in a Venn diagram first, then fill the only-regions and neither-region.
- Union means at least one named set; intersection means both; complement means outside a named set in the stated universal group.
- Two-way tables classify every observation by one category from each of two variables.
- Check row, column and grand totals.

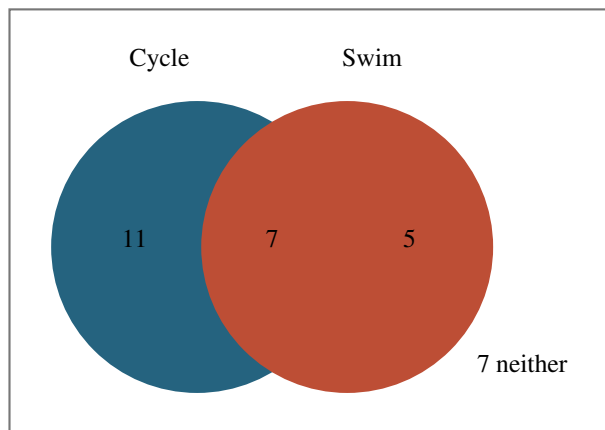
A worked case

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

■ Worked example

In a class of 30, 18 cycle, 12 swim and 7 do both. Cycling only is 11 and swimming only 5; at least one is $11+7+5=23$, leaving 7 neither. A random student has probability $7/30$ of both and $23/30$ of at least one. The two-way table has cycle-and-swim 7, cycle-not-swim 11, not-cycle-swim 5 and neither 7. The cycle row totals 18, swim column totals 12 and grand total 30. The outcomes both, cycling only, swimming only and neither are mutually exclusive and exhaustive, so their probabilities sum to 1. A frequency tree starts at 30, branches to cycle 18 and not-cycle 12, then to swim/not-swim counts $7/11$ and $5/7$. Each pair of terminal counts sums back to its parent.

30 students: overlap first, then only and neither



Where it goes wrong

- Do not add the overlap twice.
- Neither is outside both circles, not just outside their overlap.
- A universal group must be stated before taking a complement.

■ Watch out

Not true: The union count is the sum of the two set counts without adjustment. Elements in both sets are counted twice, so subtract the intersection: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Applying it

- AQA P4/P6 uses exhaustive events, systematic sets and Venn/table representations.
- Translate the words both, either/at least one, only and neither into the correct counted regions.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.5.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Intersection:** The outcomes belonging to both named sets.
- Place the overlap in a Venn diagram first, then fill the only-regions and neither-region.
- Watch for: do not add the overlap twice.

3.5 · Venn diagrams, unions and two-way counts · Higher

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- A class survey asks about cycling and swimming.
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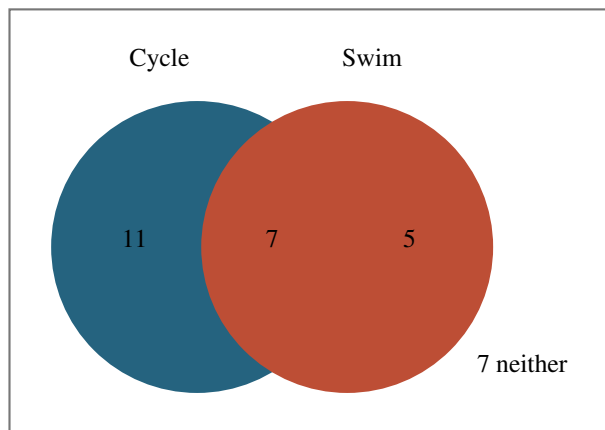
A worked case

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Not true: The union count is the sum of the two set counts without adjustment. Elements in both sets are counted twice, so subtract the intersection: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Applying it

- AQA P4/P6 uses exhaustive events, systematic sets and Venn/table representations.
- Translate the words both, either/at least one, only and neither into the correct counted regions.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.5.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Intersection:** The outcomes belonging to both named sets.
- Place the overlap in a Venn diagram first, then fill the only-regions and neither-region.
- Watch for: do not add the overlap twice.

3.5 · Conditional probability with tables and expected frequencies · Higher

The given condition changes which students are possible.

- The probability a randomly selected student swims changes when we learn that the student cycles.

- This lesson studies **conditional probability** : A probability within the group selected by given information.

The method

- Conditioning restricts the denominator to the given group.
- In a table, divide the intersection count by the condition's row/column total.
- $P(A \text{ given } B) = P(A \text{ and } B) / P(B)$ when $P(B) > 0$.
- Reversing the condition usually changes the denominator.
- Expected-frequency trees help interpret percentages without treating the two conditions as interchangeable.

A worked case

$$P(A | B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) > 0$$

■ Worked example

Using a class of 30 with 18 cyclists, 12 swimmers and 7 doing both, $P(\text{swim given cycle}) = 7/18$, while $P(\text{cycle given swim}) = 7/12$. The unconditional swim probability is $12/30 = 0.4$. For an expected cohort of 1000, suppose 20% have a condition; a test is positive for 90% with it and 10% without it. Expected positive counts are 180 from 200 with the condition and 80 from 800 without it. Of 260 positive tests, the conditional proportion with the condition is $180/260 = 9/13 \approx 0.6923$, not 90%. These are stated illustrative model rates, not claims about a real diagnostic test.

Given positive: use 180 out of 260, not 180 out of 200

	Positive	Negative	Total
Condition	180	20	200
No condition	80	720	800
Total	260	740	1000

Where it goes wrong

- Given positive and positive given condition are different questions.
- Use the conditioned group total, not the grand total.
- A rare starting category can make false-positive counts significant even when the detection rate is high.

■ Watch out

Not true: $P(A \text{ given } B)$ equals $P(B \text{ given } A)$. The two conditional probabilities have different denominators, so they are usually different.

Applying it

- AQA P9 Higher requires conditional calculations and interpretation using two-way tables, trees and Venn diagrams.
- Name the restricted group and retain expected counts until the final ratio.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.5.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Conditional probability:** A probability within the group selected by given information.
- Conditioning restricts the denominator to the given group.
- Watch for: given positive and positive given condition are different questions.

3.6 · Centre, spread and data displays · Foundation

Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
- This lesson studies **range** : The maximum observed value minus the minimum observed value.

The method

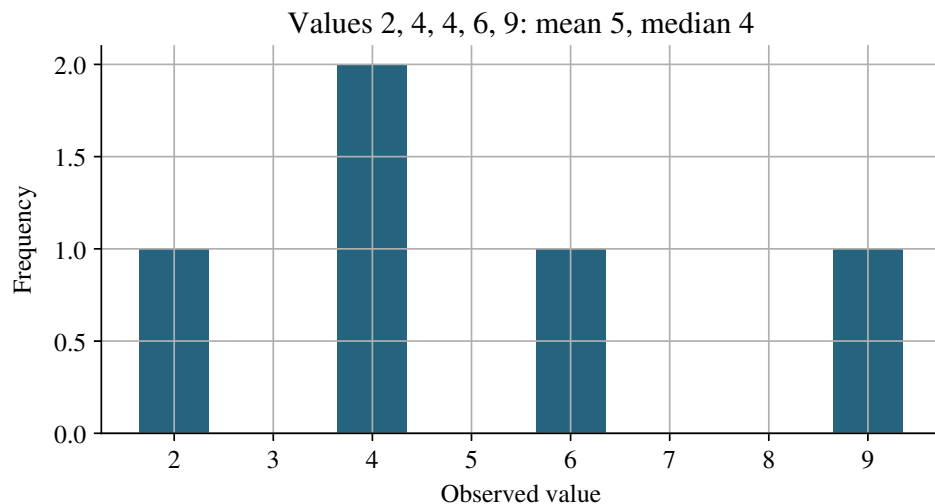
- The mean is total divided by count.
- The median is the central value after sorting.
- The range is maximum minus minimum.
- Use frequency tables, bar charts and suitable comparisons.

A worked case

$$\bar{x} = \frac{\sum x_i}{n}$$

■ Worked example

For 2,4,4,6,9, total=25 and count=5, so mean=5. The central value is 4, so median=4. The range is 9-2=7. Explain both a typical value and the spread.



Where it goes wrong

- The tallest histogram bar need not contain the most observations.
- A grouped mean is an estimate.
- Correlation does not prove causation, and extrapolation extends beyond the observed range.

■ Watch out

Not true: The mean of a list must be one of its observed values. With unequal class widths the height is frequency density; frequency is the area of the bar.

Applying it

- A bar chart uses separate bars for categories.
- Unequal-class-width histograms and formal density calculations are outside this Foundation/Core lesson.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.6.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Range:** The maximum observed value minus the minimum observed value.
- The mean is total divided by count.
- Watch for: the tallest histogram bar need not contain the most observations.

3.6 · Data summaries, histograms and interpretation · Higher

Can one average tell the whole story?

- Two groups have the same median but different spread. One summary cannot describe both location and consistency.
- This lesson studies **frequency density** : Frequency divided by class width, used as histogram height.

The method

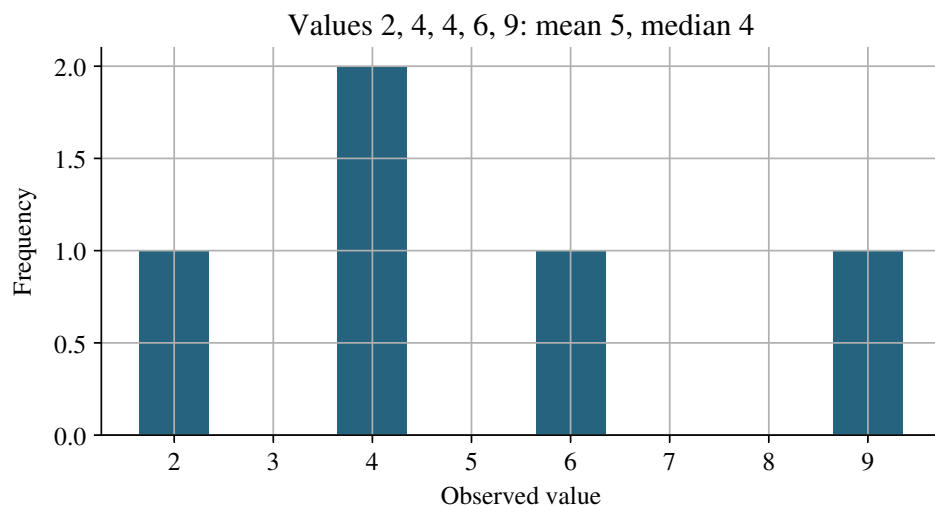
- Compare an appropriate average and spread in context.
- A histogram uses area for frequency, so height=frequency/class width.
- Grouped estimates assume representative values within intervals.

A worked case

$$\bar{x} = \frac{\sum x_i}{n}, \quad \text{density} = \frac{\text{frequency}}{\text{class width}}$$

■ Worked example

A class from 10 to 20 with frequency 30 has density $30/10=3$. A class from 20 to 40 with frequency 20 has density $20/20=1$. Its wider bar must not be mistaken for a larger density. For values 2, 4, 4, 6, 9, the median is 4 and mean is 5.



Where it goes wrong

- The tallest histogram bar need not contain the most observations.
- A grouped mean is an estimate.
- Correlation does not prove causation, and extrapolation extends beyond the observed range.

■ Watch out

Not true: A histogram bar's height equals its frequency, even with unequal class widths. With unequal class widths the height is frequency density; frequency is the area of the bar.

Applying it

- Choose a display that fits the data type.
- Give both a numerical comparison and what it means for the population; do not infer more precision than the sample supports.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.6.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Frequency density:** Frequency divided by class width, used as histogram height.
- Compare an appropriate average and spread in context.
- Watch for: the tallest histogram bar need not contain the most observations.

3.6 · Cumulative frequency and box plots · Higher

Which route is more consistent?

- Two bus routes have similar typical times but different reliability. Quartiles show the middle half of the journeys.
- This lesson studies **interquartile range** : The difference between the upper and lower quartiles.

The method

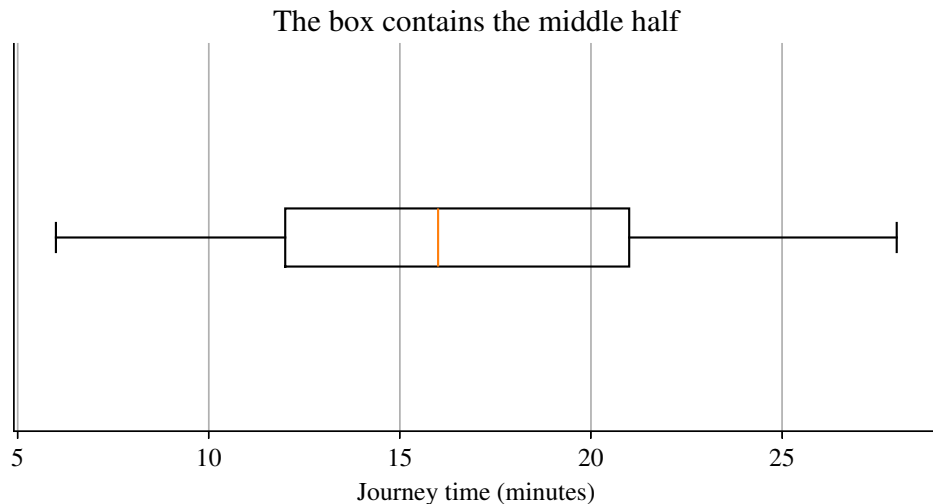
- A cumulative frequency counts observations below successive class boundaries.
- Read quartiles at one quarter, one half and three quarters of the total frequency.
- A box plot represents minimum, lower quartile, median, upper quartile and maximum.

A worked case

$$\text{IQR} = Q_3 - Q_1$$

■ Worked example

For 80 observations, read Q1 at cumulative frequency 20, median at 40 and Q3 at 60. If $Q1=12, Q3=21$, then $IQR=9$. Compare the medians for typical journey time and the IQRs for consistency.



Where it goes wrong

- Plot against class boundaries rather than midpoints.
- Grouped quartiles are estimates.
- The range is sensitive to extremes; the IQR describes only the middle half.

■ Watch out

Not true: The interquartile range equals the maximum minus the minimum. The interquartile range is $Q3-Q1$; maximum minus minimum is the range.

Applying it

- Explain a comparison in the context of the measured quantity.
- An outlier rule may use $Q1-1.5IQR$ and $Q3+1.5IQR$; use the rule specified in the task rather than assuming every graph follows it.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.6.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Interquartile range:** The difference between the upper and lower quartiles.
- A cumulative frequency counts observations below successive class boundaries.
- Watch for: plot against class boundaries rather than midpoints.

3.6 · Samples, populations and justified comparisons · Foundation

A large share of active answers may describe the club rather than the whole school.

- A school asks the first twenty students leaving a sports club about exercise.
- This lesson studies **population** : The complete group about which a statistical claim is made.

The method

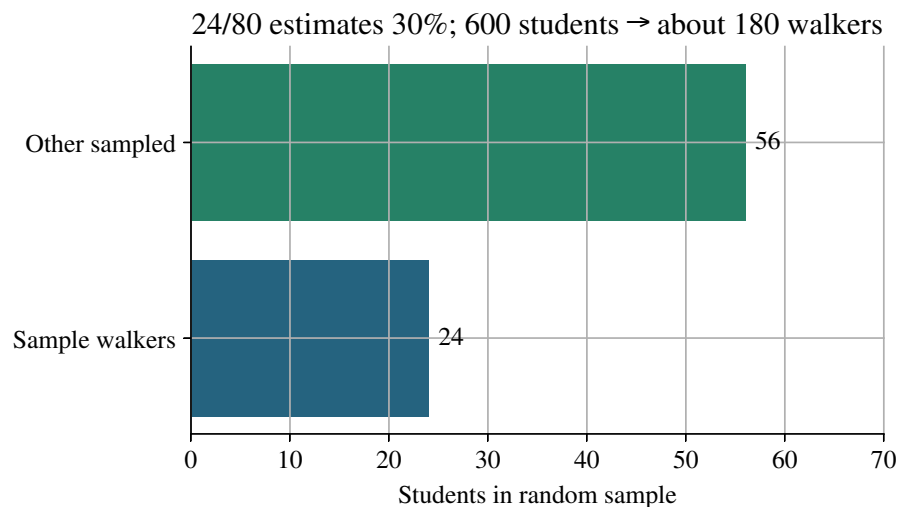
- Define the target population and variables before sampling.
- A sample is a subset; a census includes the whole population.
- Random selection reduces systematic selection bias but does not eliminate sampling variability or non-response.
- Primary data is collected for the present investigation; secondary data was collected by another source or purpose.
- Discrete data is counted; continuous data is measured.

A worked case

$$\hat{N} = N \frac{f}{n}$$

■ Worked example

In a random sample of 80 students, 24 walk to school, so the observed proportion is $24/80=0.3$. Applied to a school of 600 it suggests about 180 walkers, with sampling uncertainty: it is an estimate rather than an exact count. A sports-club convenience sample may overrepresent active students. A voluntary online poll can miss people who do not respond; adding responses does not necessarily remove that bias. A study should record who could be selected, missing responses and the question wording. Number of siblings is discrete; travel time is continuous even if recorded to whole minutes. A school's published attendance records are secondary data for a new project; measuring new travel times produces primary data.



Where it goes wrong

- Sample size alone cannot fix biased selection.
- A precise calculated estimate need not be accurate for the target population.
- Recording continuous measurements as integers does not change the underlying variable type.

■ Watch out

Not true: A sample result gives an exact population count whenever the arithmetic is correct. A sample gives an estimate with sampling error, not an exact population count.

Applying it

- AQA S1/S4/S5 uses samples to describe populations and recognises limitations/data types.
- State what the sample supports and what might make extrapolating to the whole population unreliable.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.6.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Population:** The complete group about which a statistical claim is made.
- Define the target population and variables before sampling.
- Watch for: sample size alone cannot fix biased selection.

3.6 · Samples, populations and justified comparisons · Higher

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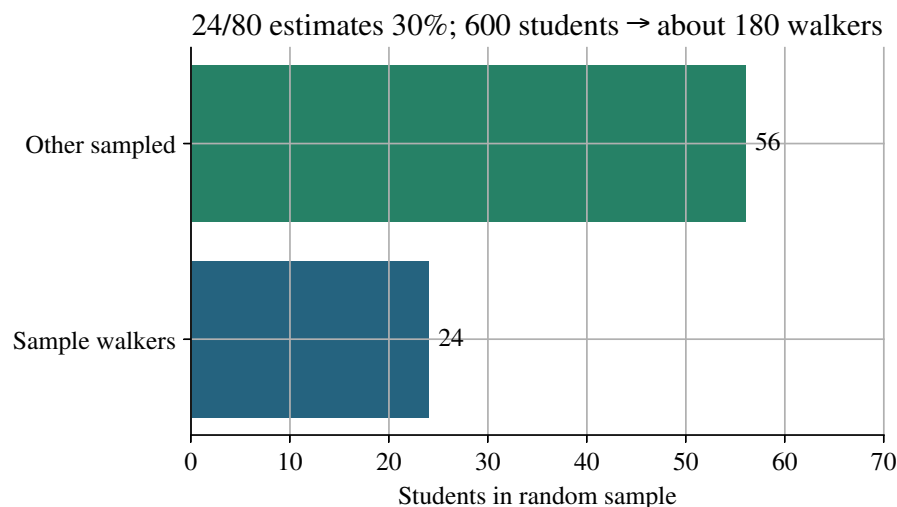
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- Discrete data is counted; continuous data is measured.

A worked case

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■ Key idea

- **Population:** The complete group about which a statistical claim is made.
- Define the target population and variables before sampling.
- Watch for: sample size alone cannot fix biased selection.

3.6 · Choosing displays, pie charts and time series · Foundation

Joining unrelated categories with a line suggests a false progression.

- A bar chart compares travel categories, while a line graph shows attendance changing through the week.
- This lesson studies **time series** : Measurements recorded in time order.

The method

- Use separate bars for categories, vertical line charts for discrete numerical values and time-ordered lines for a time series.
- Pie-chart sector angle is $\text{frequency}/\text{total} \times 360^\circ$.

- A pictogram needs a stated key and honest fractional symbols.
- Label axes/units and choose a scale that does not conceal the relevant variation.
- Continuous grouped distributions need histograms rather than categorical bars.

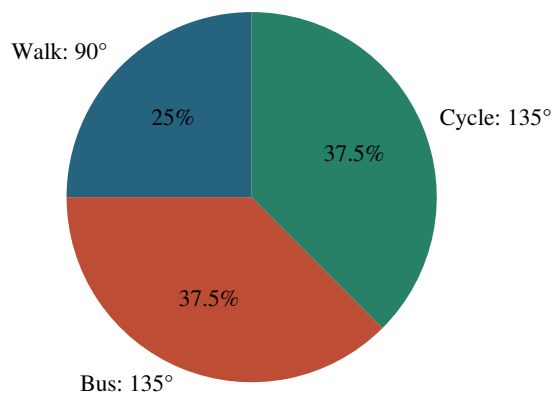
A worked case

$$\theta = \frac{f}{n} 360^\circ$$

■ Worked example

Of 40 students, 10 walk, 15 take the bus and 15 cycle. Pie angles are $90^\circ, 135^\circ, 135^\circ$, summing to 360° . With one pictogram symbol representing five students, the groups use 2,3,3 symbols. If one symbol instead means four students, ten walkers need 2.5 symbols. A daily attendance time series 32,35,33,36,34 can be joined in weekday order; the largest count is 36 and its range is 4. A frequency table for number of siblings 0,1,2,3 uses those numbers as positions on a vertical line chart, rather than treating widths as probabilities.

40 students: frequencies determine sector angles



Where it goes wrong

- A pie chart must represent a complete total with non-overlapping categories.
- Pictogram keys control the count, not decorative icon size.

- A truncated axis may exaggerate a small difference; read the scale before comparing.

■ Watch out

Not true: A line graph is the best display for unrelated categories. Unrelated categories suit a bar chart; a line graph suggests a continuous change, such as over time.

Applying it

- AQA S2 covers tables, categorical bars/pies/pictograms, discrete vertical lines and time-series tables/graphs.
- Explain why the representation fits the variable and the question.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.6.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Time series:** Measurements recorded in time order.
- Use separate bars for categories, vertical line charts for discrete numerical values and time-ordered lines for a time series.
- Watch for: A pie chart must represent a complete total with non-overlapping categories.

3.6 · Choosing displays, pie charts and time series · Higher

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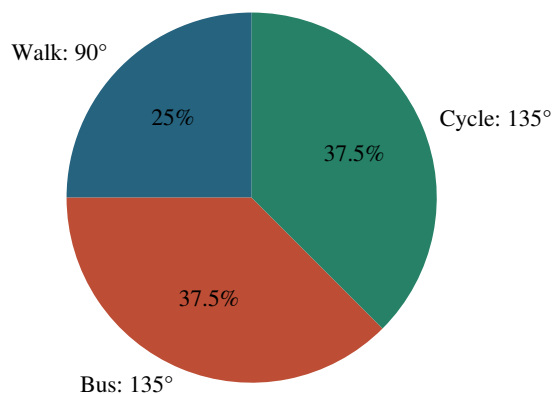
A worked case

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■ Key idea

- **Time series:** Measurements recorded in time order.
- Use separate bars for categories, vertical line charts for discrete numerical values and time-ordered lines for a time series.
- Watch for: A pie chart must represent a complete total with non-overlapping categories.

3.6 · Frequency summaries and grouped estimates · Foundation

Its mean needs the frequencies as weights; averaging the listed values alone loses the repeated observations.

- A table stores how often each value occurs.
- This lesson studies **modal class**: The class interval containing the greatest frequency.

The method

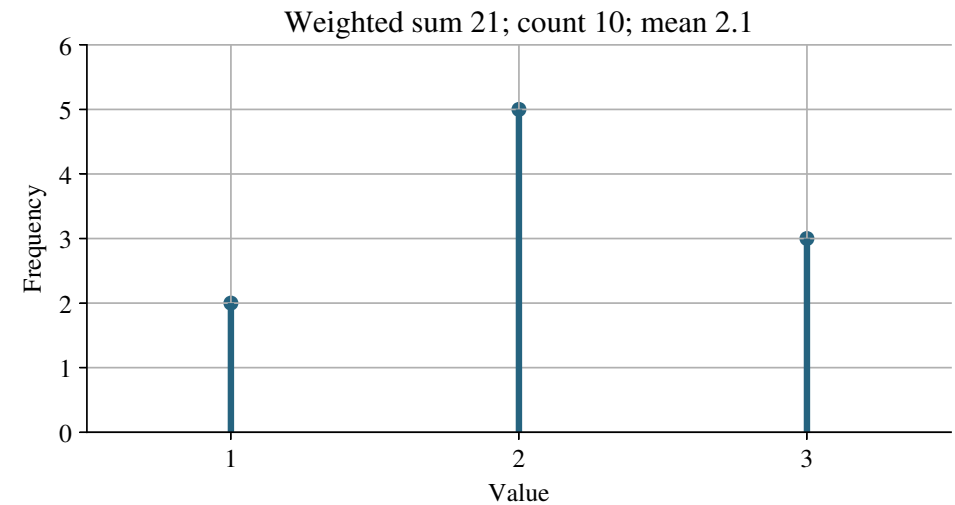
- For exact value frequencies, mean is $\text{sum}(\text{value} \times \text{frequency}) / \text{total frequency}$.
 - Locate the median using cumulative counts.
 - For interval data, use class midpoints to estimate a mean; the exact values are unknown.
 - The modal class has greatest frequency, which need not be the tallest unequal-width histogram bar.
 - Compare a suitable average and spread, in context, and state the limitations of grouping or outliers.
-

A worked case

$$\bar{x} \approx \frac{\sum fm}{\sum f}$$

■ Worked example

Values 1,2,3 with frequencies 2,5,3 give total 10 and weighted sum $2+10+9=21$, so mean is 2.1. The fifth and sixth observations are both 2, giving median 2 and mode 2. For continuous classes $0 \leq x < 10, 10 \leq x < 20, 20 \leq x < 30$ with frequencies 2,5,3, midpoints 5,15,25 give estimated sum $10+75+75=160$ and estimated mean 16. The modal class is $10 \leq x < 20$. The range of individual observations cannot be recovered exactly from these intervals. For values 2,4,4,6,24, mean is 8 and median 4; the unusually large value raises the mean. Comparing two groups should describe both a typical value and variation, rather than selecting whichever summary favours a claim.



Where it goes wrong

- The grouped mean is an estimate because all members are represented by a midpoint.
- A median is not found by averaging the class labels.
- Outliers can alter the mean/range substantially.

■ Watch out

Not true: A grouped midpoint mean is the exact mean of the original measurements. Midpoints assume values sit in the middle of each class, so the result is only an estimate.

Applying it

- AQA S4/S5 Foundation includes appropriate mean/median/mode/modal class and range, with grouped data and outlier awareness.
 - Higher quartiles/box plots are taught separately.
-

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.6.
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- For exact value frequencies, mean is $\text{sum}(\text{value} \times \text{frequency}) / \text{total frequency}$.
- Watch for: the grouped mean is an estimate because all members are represented by a midpoint.

3.6 · Frequency summaries and grouped estimates · Higher

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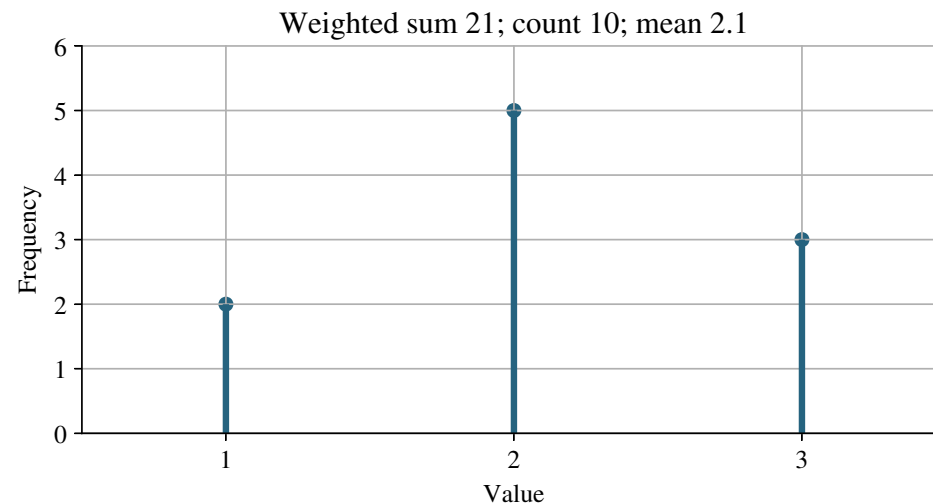
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3.6 · Scatter graphs, correlation and cautious predictions · Foundation

A plotted association does not prove that changing someone's height would directly change their shoe size.

- Taller students may tend to have larger shoes.
- This lesson studies **correlation**: A pattern of association between two measured variables.

The method

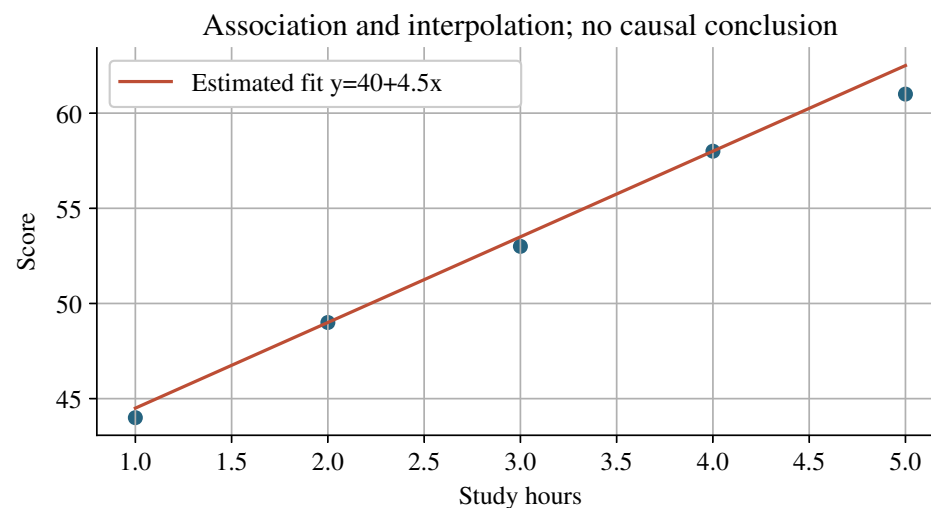
- Plot paired observations as points, with one variable on each axis.
- Positive correlation rises, negative falls and no correlation has no clear trend.
- Strong/weak describes how tightly points follow a trend, not its steepness.
- Draw an estimated line of best fit through the middle of the pattern, balancing points around it; it need not pass through the origin.
- Interpolation stays within observed inputs; extrapolation goes outside them.

A worked case

$$\hat{y} = 40 + 4.5x$$

■ Worked example

Observed study hours 1,2,3,4,5 with scores 44,49,53,58,61 show a positive trend. An estimated line $y=40+4.5x$ predicts 53.5 at $x=3$ and 56.2 at $x=3.6$. These inputs are inside 1–5, so the predictions interpolate. At $x=10$ the line gives 85, but this extrapolation may fail if gains flatten or the group differs. Sleep hours and tiredness may show a negative correlation; an almost horizontal cloud with no ordered pattern may show little correlation. A shared cause such as prior preparation can affect both study time and score; the plot alone does not establish causation.



Where it goes wrong

- Do not join scatter points in observation order as though they formed a time series.
- A steep line need not imply strong correlation.
- An extrapolated formula value is a prediction whose assumptions need scrutiny.

■ Watch out

Not true: A positive correlation proves that one variable causes the other. Correlation does not prove causation; a third variable may affect both.

Applying it

- AQA S6 Foundation includes correlation, estimated best-fit lines, predictions and dangers of extrapolation.
- State the observed input range, association direction/strength and the limits of a causal claim.

In your course

- AQA GCSE Mathematics (8300), Foundation tier, specification section 3.6.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Correlation:** A pattern of association between two measured variables.
- Plot paired observations as points, with one variable on each axis.
- Watch for: do not join scatter points in observation order as though they formed a time series.

3.6 · Scatter graphs, correlation and cautious predictions · Higher

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- Taller students may tend to have larger shoes.
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The method

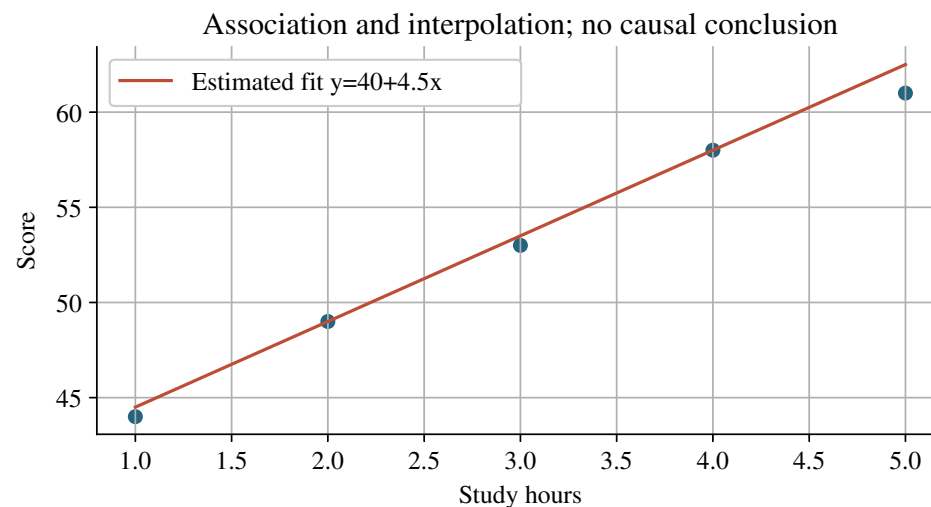
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■ Key idea

- **Correlation:** A pattern of association between two measured variables.
- Plot paired observations as points, with one variable on each axis.
- Watch for: do not join scatter points in observation order as though they formed a time series.

3.6 · Histogram density and cumulative-frequency construction · Higher

Frequency is represented by bar area, not height alone.

- A wide class can contain many observations even if its histogram bar is short.
- This lesson studies **frequency density** : Frequency divided by class width, used as histogram height.

The method

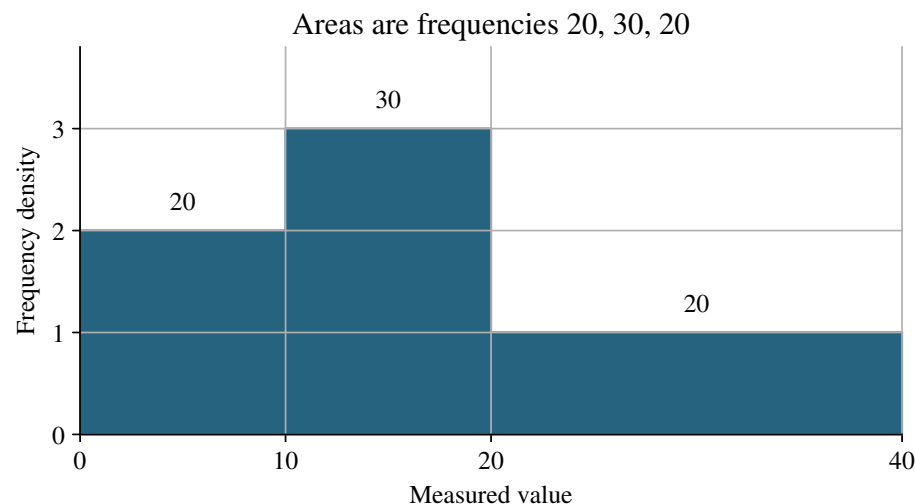
- Use continuous adjoining class boundaries on the horizontal axis.
- Density=frequency/class width, so bar area recovers frequency.
- Unequal widths require this adjustment.
- A cumulative-frequency graph plots each upper class boundary against the running total, beginning at the first lower boundary with zero.
- Read percentiles at fractions of the total; values within classes are estimates.

A worked case

$$d = \frac{f}{w}, \quad f = dw$$

■ Worked example

For classes $0 \leq x < 10, 10 \leq x < 20, 20 \leq x < 40$ with frequencies 20, 30, 20, widths are 10, 10, 20 and densities 2, 3, 1. The last two bars have different heights but areas 30 and 20. Cumulative points are (0,0), (10,20), (20,50), (40,70). The median is at cumulative count 35; a straight-line estimate in the second class gives $10 + (35 - 20) / 30 \times 10 = 15$. Q1 is at 17.5 and Q3 at 52.5, giving corresponding interpolated estimates 8.75 and 22.5. Class midpoints 5, 15, 30 give estimated mean $(100 + 450 + 600) / 70 \approx 16.43$. A sketch that joins upper boundaries with a smooth curve can give slightly different readings, so use the stated precision and graph.



Where it goes wrong

- A histogram uses density on its vertical axis when widths differ.
- Plot cumulative totals at boundaries, not midpoints.
- Grouped percentiles depend on interpolation assumptions and are not exact individual measurements.

■ Watch out

Not true: Histogram bar height equals the number of observations. Bar height is frequency density; the area of the bar shows the frequency.

Applying it

- AQA S3 Higher includes equal/unequal-class histograms and cumulative-frequency graphs, with suitable interpretation.
- Explain why bar area represents frequency and label the graph axes/units.

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.6.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Frequency density:** Frequency divided by class width, used as histogram height.
- Use continuous adjoining class boundaries on the horizontal axis.
- Watch for: A histogram uses density on its vertical axis when widths differ.

3.6 · Quartiles, box plots and distribution comparisons · Higher

A box plot shows the middle half separately from the extreme observations.

- Two classes can have the same median but different consistency.
- This lesson studies **interquartile range**: The upper quartile minus the lower quartile.

The method

- A box plot marks minimum, lower quartile Q_1 , median, upper quartile Q_3 and maximum, or identifies separately shown outliers according to the stated convention.
- $IQR = Q_3 - Q_1$ describes the middle half.

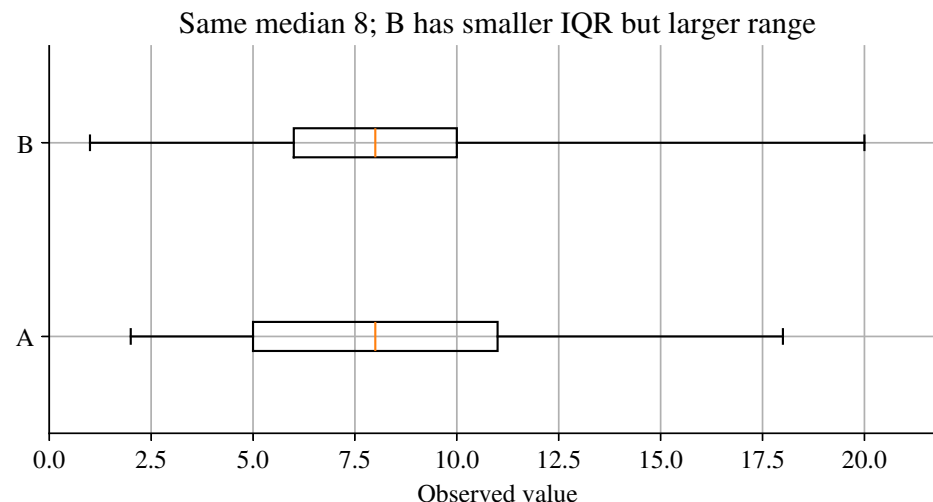
- Compare median and IQR in context; a smaller IQR suggests less middle-half variation, not necessarily a smaller full range.
- Quartile conventions vary for a finite raw list, so follow the task or supplied summaries.

A worked case

$$IQR = Q_3 - Q_1$$

■ Worked example

Group A has summary 2,5,8,11,18 and group B has 1,6,8,10,20. Both medians are 8. A has $IQR=6$ and $range=16$; B has $IQR=4$ and $range=19$. B's middle half is more consistent although its full range is larger. Under a stated $1.5 \times IQR$ outlier rule, A's fences are $5-9=-4$ and $11+9=20$; a new value 23 lies above the upper fence. This rule is a specified convention, not a compulsory rule for every box plot. When using cumulative frequency, read Q_1 at $N/4$ and Q_3 at $3N/4$, then subtract; small reading errors affect the estimated IQR.



Where it goes wrong

- The median line need not be halfway along the box.
- Equal medians do not mean identical distributions.

- Whisker meanings depend on whether outliers are drawn separately; read the stated convention.

■ Watch out

Not true: Equal medians prove that two groups have the same spread. Equal medians say nothing about spread; compare the IQR or the range.

Applying it

- AQA S4 Higher adds box plots, quartiles and IQR to distribution comparisons.
 - Name both centre and spread, with units and context, and avoid claims about all observations from just the box.
-

In your course

- AQA GCSE Mathematics (8300), Higher tier, specification section 3.6.
- Paper 1 is non-calculator: keep fractions, surds and π exact and show each step. On calculator papers, still write the expression before its value.

■ Key idea

- **Interquartile range:** The upper quartile minus the lower quartile.
- A box plot marks minimum, lower quartile Q1, median, upper quartile Q3 and maximum, or identifies separately shown outliers according to the stated convention.
- Watch for: the median line need not be halfway along the box.