Moments

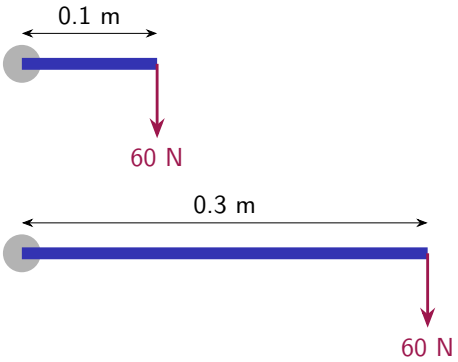
AQA A-level Mathematics 7357, section S: moments, equilibrium of rigid bodies, tipping and ladders

The turning effect of a force

Idea	Meaning
moment about P	force $\times$ perpendicular distance from P to its line
sense	**clockwise** or **anticlockwise**: choose one as positive
force through P	zero distance, so zero moment about P
uniform rod	its weight acts at the midpoint

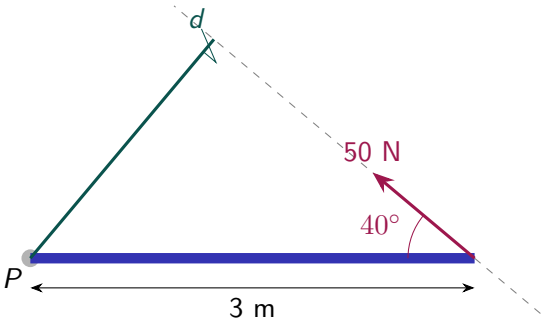
$$M = Fd_{\perp}$$

Why the long spanner wins



- Same 60 N push.
- $60 \times 0.1 = 6$ N m.
- $60 \times 0.3 = 18$ N m: three times the turning effect.

The perpendicular distance



- Measure to the line of the force.
- $d = 3 \sin 40° = 1.93$ m.
- $M = 50 \times 1.93 = 96.4$ N m.

Two conditions for equilibrium

Equilibrium

Forces balance: no translation.

Moments about every point balance: no rotation.

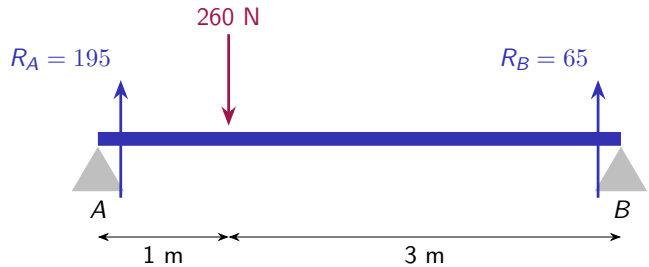
A couple

Two equal, opposite, parallel forces at different points.

Resultant force zero, but the body still spins.

A beam on two supports

- Light 4 m beam, 260 N load 1 m from A.
- Moments about A remove R_A.



Worked: the two reactions

1. **Given** light 4 m beam on end supports A, B; 260 N load 1 m from A
2. **Moments about A** $4R_B = 260 \times 1$, so $R_B = 65$ N
3. **Vertical forces** $R_A + 65 = 260$, so $R_A = 195$ N
4. **Check about B** $195 \times 4 = 780 = 260 \times 3$

Decision

Both reactions positive: the beam stays on both supports.

A uniform plank with a crate

1. **Given** uniform 6 m plank, 200 N, end supports P, Q; 400 N crate 2 m from P
2. **Moments about P** $6R_Q = 400 \times 2 + 200 \times 3 = 1400$, so $R_Q = 233.3$ N
3. **Vertical forces** $R_P = 600 - 233.3 = 366.7$ N
4. **Check about Q** $366.7 \times 6 = 2200 = 400 \times 4 + 200 \times 3$

On the point of tipping

1. **Given** uniform 6 m beam AB, 200 N; supports C, D at 2 m and 5 m from A
2. **Tipping** a 600 N person stands x m beyond D; at the limit $R_C = 0$
3. **Moments about D** $600x = 200 \times (5 - 3)$, so $x = \frac{2}{3}$ m
4. **Answer** the person can stand at most 0.67 m beyond D, 0.33 m from B

Decision

Setting $R_C = 0$ is the condition: the beam has just lifted off C.

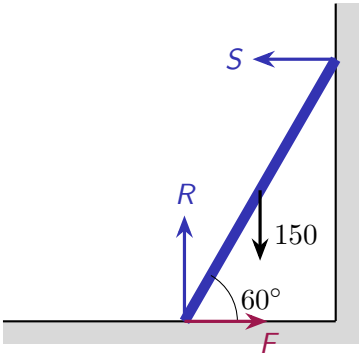
Finding a centre of mass

1. **Given** a 5 m rod, weight 80 N, hangs level from strings at A (30 N) and B (50 N)
2. **Moments about A** $80x = 50 \times 5$, with x from A
3. **Solve** $x = 3.125$ m from A

Decision

The weight sits nearer B, the end with the larger tension.

A ladder against a smooth wall



- Smooth wall; rough ground.
- Moments about the foot:
  $S \times 4 \sin 60° = 150 \times 2 \cos 60°$.
- $S = 43.3$ N $= F$; $R = 150$ N.
- $\mu \geq \frac{43.3}{150} = 0.289$.

Traps with moments

Trap	**Fix**
Distance to where the force acts	distance to its line, perpendicular
Forgetting the rod's own weight	uniform: at the midpoint
Only $\Sigma F = 0$ for **equilibrium**	$\Sigma M = 0$ as well
Mixing **clockwise** and **anticlockwise** signs	one sense positive, stated
Tipping with both reactions kept	set the far reaction to zero

Check yourself

Question	Answer
25 N at 0.4 m, perpendicular: moment?	10 N m
Moment of a force through the **pivot**?	zero
Uniform 2 m rod, 30 N: where is its weight?	at 1 m, the midpoint
Light 3 m beam, 90 N at its centre: each reaction?	45 N
About to tip about D: which reaction is zero?	the other support's