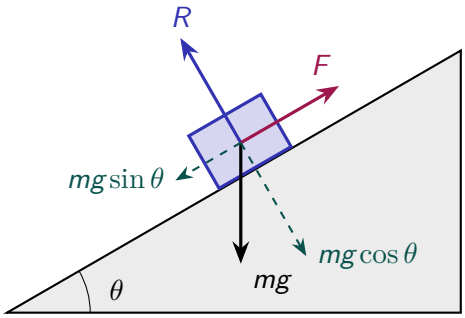
Forces and Newton's laws

AQA A-level Mathematics 7357, section R: Newton's laws, resolving, friction and connected particles

Newton's three laws

Law	Statement	In use
First	zero **resultant force**: constant velocity	equilibrium: forces balance
Second	resultant force $= ma$	one equation per direction
Third	equal and opposite forces on different bodies	never cancel on one diagram

A block on a slope



- One arrow per named force.
- Resolve mg along and into the slope.
- $R = mg\cos\theta$, not mg.

Worked: held by friction

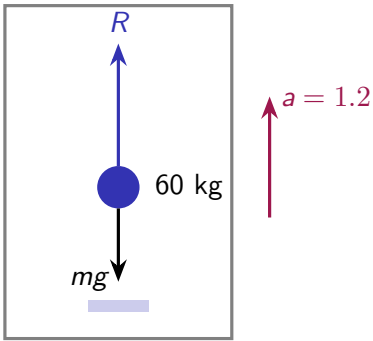
1. **Given** $m = 3$ kg at rest on a 30° slope, $g = 9.8$
2. **Along** $F = mg\sin 30^\circ = 3 \times 9.8 \times 0.5 = 14.7$ N
3. **Perpendicular** $R = mg\cos 30^\circ = 3 \times 9.8 \times \frac{\sqrt{3}}{2} = 25.5$ N
4. **Friction needed** $\mu \geq \frac{F}{R} = \tan 30^\circ = 0.577$

Decision

A coefficient below 0.577 and the block slides.

$$F \leq \mu R$$

Why the lift feels heavier



- The scale reads R, not the weight.
- Up positive: $R - mg = ma$.
- $R = 60(9.8 + 1.2) = 660$ N.

$$R - mg = ma$$

The scale reading in every phase

Motion	a **(up positive)**	**Reading** R
speeding up, going up	$+1.2$	660 N
steady speed, either way	0	588 N
slowing down, going up	-1.2	516 N
cable snaps, free fall	-9.8	0 N

Steady motion is Newton's first law: $R = mg$.

Third-law pairs

A true pair

Earth pulls the book down; the book pulls Earth up.

The table pushes the book up; the book pushes the table down.

Not a pair

The book's weight and the table's reaction on it.

Both act on the book: they balance by the first law, not the third.

Resultant force as a vector

1. **Given** $m = 2$ kg; $\mathbf{F}_1 = (4\mathbf{i} + 3\mathbf{j})$ N, $\mathbf{F}_2 = (2\mathbf{i} + 5\mathbf{j})$ N
2. **Resultant** $\mathbf{F} = 6\mathbf{i} + 8\mathbf{j}$ N
3. **Acceleration** $\mathbf{a} = \frac{\mathbf{F}}{m} = 3\mathbf{i} + 4\mathbf{j}\ \mathrm{m\,s^{-2}}$
4. **Magnitude** $|\mathbf{a}| = \sqrt{3^2 + 4^2} = 5\ \mathrm{m\,s^{-2}}$

$$\mathbf{F} = m\mathbf{a}$$

Friction is a limit

1. **Given** a 5 kg box on a rough floor, $\mu = 0.3$, pulled by 25 N horizontally
2. **Limit** $R = 5 \times 9.8 = 49$ N, so $F_{\max} = 0.3 \times 49 = 14.7$ N
3. **Moves?** $25 > 14.7$: it slides, with $F = 14.7$ N
4. **Acceleration** $25 - 14.7 = 5a$, so $a = 2.06\ \mathrm{m\,s^{-2}}$

Decision

With a 10 N pull, friction would be just 10 N and the box would stay put.

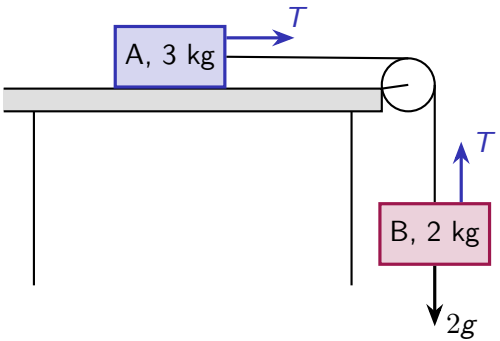
Sliding down a rough slope

1. **Given** $m = 2$ kg slides down a 30° slope, $\mu = 0.2$
2. **Perpendicular** $R = 2 \times 9.8 \cos 30^\circ = 16.97$ N
3. **Friction** $F = 0.2 \times 16.97 = 3.39$ N
4. **Along** $2 \times 9.8 \sin 30^\circ - 3.39 = 2a$, so $a = 3.20\ \mathrm{m\,s^{-2}}$

Decision

Compare with a smooth slope: $a = g \sin 30^\circ = 4.9$.

Connected particles over a pulley



- Light string, smooth pulley; table smooth.
- Same **tension** T throughout; same a.
- One equation per **particle**.

Worked: the pulley system

1. **Given** A 3 kg on a smooth table, B 2 kg hanging, $g = 9.8$
2. **For A** $T = 3a$
3. **For B** $2g - T = 2a$
4. **Add** $2g = 5a$: $a = 3.92\ \mathrm{m\,s^{-2}}$, $T = 11.76$ N

Decision

Check B: $19.6 - 11.76 = 7.84 = 2 \times 3.92$.

Traps with forces

Trap	**Fix**
$R = mg$ on a slope or in a lift	resolve, or include ma
$F = \mu R$ when nothing slides	$F \leq \mu R$; find F from balance
Third-law pair on one diagram	a pair acts on two bodies
$F = ma$ with one force, not the resultant	add every force first
Tension different either side of a smooth pulley	one string, one T

Check yourself

Question	**Answer**
Resultant on a car at steady 30 m/s?	zero
50 kg in a lift with $a = 2$ downwards: R?	390 N
R for 4 kg on a 20° slope?	$4g\cos 20^\circ = 36.8$ N
Max friction, $R = 40$ N, $\mu = 0.25$?	10 N
$\mathbf{F} = (6\mathbf{i} - 8\mathbf{j})$ N on 2 kg: $\lvert\mathbf{a}\rvert$?	$5\ \mathrm{m\,s^{-2}}$