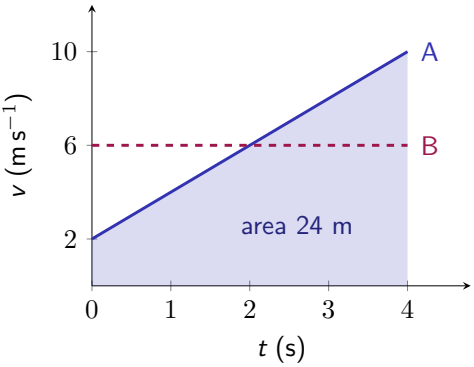
Kinematics

AQA A-level Mathematics 7357, section Q: motion graphs, constant acceleration, calculus and projectiles

What the graphs tell you

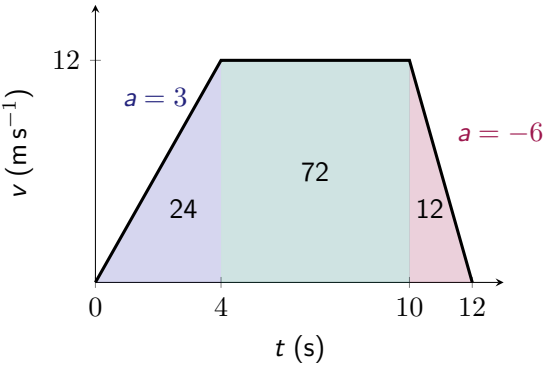
Graph	Gradient	Area under
displacement–time	velocity	—
velocity–time	acceleration	displacement
acceleration–time	—	change in velocity

Same 24 m, different graphs



- Car A: $2 \to 10\ \mathrm{m\,s^{-1}}$ in 4 s.
- Car B: a steady $6\ \mathrm{m\,s^{-1}}$.
- Areas: $\frac{2+10}{2} \times 4 = 24 = 6 \times 4$.

Reading a journey graph



- Gradients: 3, 0, -6 m s^{-2}.
- Area: $24 + 72 + 12 = 108$ m.
- A negative gradient is deceleration.

The five suvat equations

Equation	Leaves out
$v = u + at$	s
$s = ut + \frac{1}{2}at^2$	v
$s = \frac{1}{2}(u + v)t$	a
$v^2 = u^2 + 2as$	t
$s = vt - \frac{1}{2}at^2$	u

List s, u, v, a, t; pick the equation without the one you neither know nor want.

Worked: braking distance

1. **Given** $u = 20$, $v = 0$, $a = -4$; $s = ?$; t not needed
2. **Choose** the equation without t: $v^2 = u^2 + 2as$
3. **Substitute** $0 = 400 - 8s$
4. **Solve** $s = 50$ m

Decision

Deceleration enters as a negative a when the motion is positive.

Worked: the cyclist

1. **Given** $u = 2\ \mathrm{m\,s^{-1}}$, $a = 2\ \mathrm{m\,s^{-2}}$, $t = 4$ s
2. **Final velocity** $v = u + at = 2 + 2 \times 4 = 10\ \mathrm{m\,s^{-1}}$
3. **Displacement** $s = \tfrac{1}{2}(u + v)t = \tfrac{1}{2}(2 + 10) \times 4 = 24$ m

Decision

The trapezium on the graph gives the same 24 m.

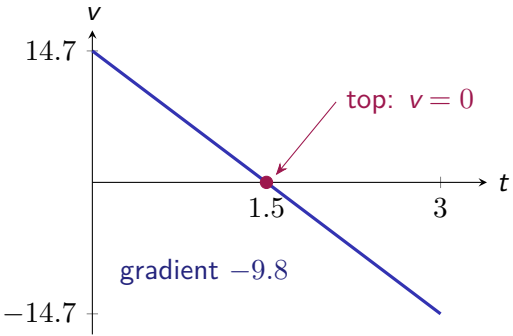
Vertical motion under gravity

1. **Given** a stone dropped from rest; down positive; $a = 9.8$, $t = 3$
2. **Speed** $v = u + at = 0 + 9.8 \times 3 = 29.4\ \mathrm{m\,s^{-1}}$
3. **Distance** $s = ut + \frac{1}{2}at^2 = \frac{1}{2} \times 9.8 \times 9 = 44.1\ \mathrm{m}$

Decision

Air resistance is ignored: the stone is modelled as a particle.

Thrown upwards



- Up positive: $u = 14.7$, $a = -9.8$.
- At the top $v = 0$, but a is still -9.8.
- Top after 1.5 s, at 11.0 m.
- Back down after 3 s, by symmetry.

When the acceleration varies

1. **Given** $s = t^3 - 6t^2 + 9t$ for $t \geq 0$
2. **Velocity** $v = \frac{ds}{dt} = 3t^2 - 12t + 9 = 3(t-1)(t-3)$
3. **At rest** $v = 0$ at $t = 1$ and $t = 3$
4. **Acceleration** $a = \frac{dv}{dt} = 6t - 12$; at $t = 3$, $a = 6$

Decision

Differentiate to go from s to v to a.

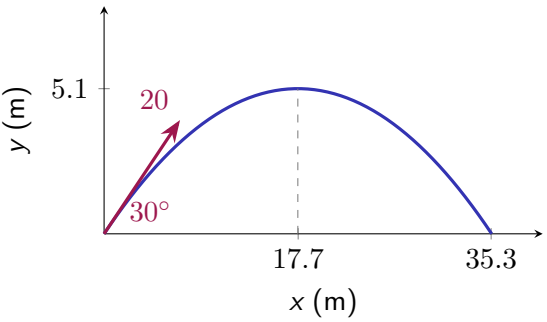
$$v = \frac{ds}{dt}, \qquad a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Integrating back

1. **Given** $a = 4 - 2t$; $v = 3$ and $s = 0$ when $t = 0$
2. **Velocity** $v = \int (4 - 2t)\, dt = 4t - t^2 + c$, $c = 3$
3. **Displacement** $s = \int v\, dt = 3t + 2t^2 - \frac{t^3}{3}$ (constant 0)
4. **At t = 3** $v = 6\ \mathrm{m\,s^{-1}}$, $s = 9 + 18 - 9 = 18$ m

$$v = \int a\, dt, \qquad s = \int v\, dt$$

A projectile, drawn



- $20\ \mathrm{m\,s^{-1}}$ at $30°$ from level ground.
- Vertical: $u_y = 10$, $a = -9.8$; flight 2.04 s.
- Horizontal: $u_x = 17.3$, constant; range 35.3 m.
- Highest point $\frac{10^2}{2\times 9.8} = 5.10$ m.

Traps in kinematics

Trap	**Fix**
suvat on a curved v–t graph	constant a only; else calculus
Mixed signs for up and down	fix one positive direction
$a = 0$ at the top of a throw	$v = 0$ there; a is still -9.8
Distance from negative s	find when $v = 0$ and add the legs
Forgetting the $+c$ when integrating	use the starting values

Check yourself

Question	**Answer**
Area under a v–t graph?	displacement
$u = 5$, $a = 2$, $t = 3$: v?	$11\ \mathrm{m\,s^{-1}}$
Which equation omits t?	$v^2 = u^2 + 2as$
Dropped for 2 s: distance?	19.6 m
$s = 2t^3$: a at $t = 1$?	$12\ \mathrm{m\,s^{-2}}$