Quantities and units in mechanics

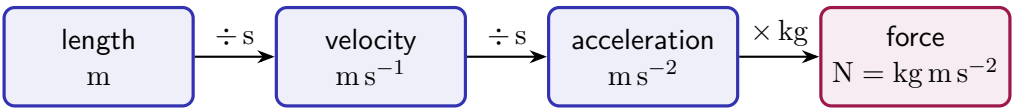
AQA A-level Mathematics 7357, section P: SI base and derived quantities and units

Three base quantities

Quantity	SI unit	Symbol
mass	kilogram	kg
length	metre	m
time	second	s

Building the derived units

- Divide by time for a rate.
- Multiply by mass for a force.



weight is a force too: $W = mg$, in newtons

Scalars and vectors

Scalar (size only)

distance, speed

mass, time

speed $= 5\ \mathrm{m\,s^{-1}}$

Vector (size and direction)

displacement, **velocity**

acceleration, force, weight

velocity $= -5\ \mathrm{m\,s^{-1}}$: 5 m/s backwards

Mass and weight

1. **Given** a student of mass 60 kg
2. **Weight** $W = mg = 60 \times 9.8 = 588$ N
3. **On the Moon** $g \approx 1.6$: $W \approx 96$ N, mass still 60 kg

Decision

Mass is the amount of matter; weight is the pull of gravity on it.

$$W = mg$$

Convert before you combine

Given	Working	SI value
72 km/h	$72 \div 3.6$	$20\ \mathrm{m\,s^{-1}}$
450 g	$450 \div 1000$	0.45 kg
3.2 kN	3.2×1000	3200 N
2 minutes	2×60	120 s
35 cm	$35 \div 100$	0.35 m

Units on both sides must match

Formula	Units of each side
$F = ma$	$\mathrm{N} = \mathrm{kg} \times \mathrm{m\,s^{-2}}$
$v = u + at$	$\mathrm{m\,s^{-1}} = \mathrm{m\,s^{-1}} + \mathrm{m\,s^{-2}} \times \mathrm{s}$
$s = ut + \frac{1}{2}at^2$	$\mathrm{m} = \mathrm{m\,s^{-1}} \times \mathrm{s} + \mathrm{m\,s^{-2}} \times \mathrm{s^2}$

Pure numbers such as $\frac{1}{2}$ have no unit.

Worked: catching a wrong formula

1. **Given** claimed formula $s = ut + at$
2. **First term** ut: $\mathrm{m\,s^{-1}} \times \mathrm{s} = \mathrm{m}$
3. **Second term** at: $\mathrm{m\,s^{-2}} \times \mathrm{s} = \mathrm{m\,s^{-1}}$
4. **Verdict** metres plus metres per second: the formula is wrong

Decision

A unit check cannot catch a wrong number inside a right-shaped formula.

Modelling words

Word	Meaning	What it ignores
particle	all mass at one point	size, rotation, air resistance
light	mass taken as zero	the string's or rod's weight
inextensible	does not stretch	changes in length
smooth	no friction	resistance to sliding
rigid	does not bend	deformation

Further Maths preview: work and power

1. **Given** a cyclist pushes with 80 N over 50 m in 20 s
2. **Work** $W = Fs = 80 \times 50 = 4000$ J
3. **Power** $P = \frac{W}{t} = \frac{4000}{20} = 200$ W

Decision

"300 N of **work**" is nonsense: newtons measure force, joules measure work.

$$W = Fs, \qquad P = \frac{W}{t}$$

Same unit, different quantity

Moment of a force

force $\times$ perpendicular distance

unit: N m

a turning effect (topic 19)

Work done

force $\times$ distance moved

unit: N m, called the joule

energy transferred

Traps with units

Trap	**Fix**
Using 72 in place of $20\ \mathrm{m\,s^{-1}}$	divide km/h by 3.6 first
Mass in grams in $F = ma$	convert to kg
Weight given as 60 kg	weight is mg, in N
Answer “20” with no unit	always write the unit
Speed $-5\ \mathrm{m\,s^{-1}}$	speed is never negative; **velocity** can be

Check yourself

Question	**Answer**
54 km/h in m/s?	$15\ \mathrm{m\,s^{-1}}$
Weight of a 2.5 kg bag?	24.5 N
The **newton** in **base units**?	$\mathrm{kg\,m\,s^{-2}}$
Is **acceleration** a scalar or a vector?	vector
Units of $\frac{1}{2}mv^2$?	$\mathrm{kg\,m^2\,s^{-2}}$, the joule