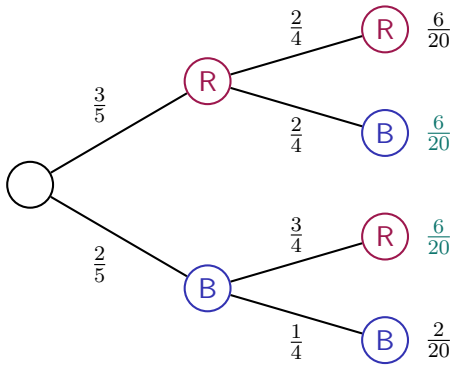
# Probability

AQA A-level Mathematics 7357, section M: tree and Venn diagrams, conditional probability, exclusive and independent events

# The rules of probability

| Rule | Formula |
| --- | --- |
| **Complement** | $P(A') = 1 - P(A)$ |
| Addition | $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ |
| Conditional | $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}, \ P(B) > 0$ |
| Independent | $P(A \cap B) = P(A)P(B)$ |
| **Mutually exclusive** | $P(A \cap B) = 0$ |

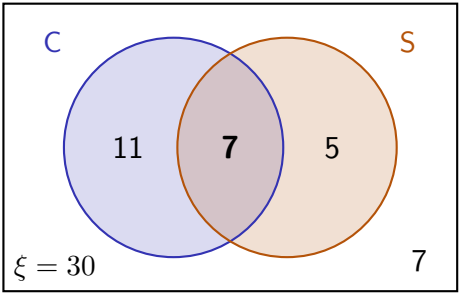
# A tree without replacement




- 3 red, 2 blue; draw two, no replacement.
- The second branch changes with the first.
- Multiply along, add across.

$$P(\text{one of each}) = \frac{6}{20} + \frac{6}{20} = \frac{3}{5}$$

# Fill a Venn diagram from the overlap




- 30 students: 18 cycle, 12 swim, 7 do both.
- Only cycling $18 - 7 = 11$; only swimming $12 - 7 = 5$.
- At least one $23$; neither $7$.

# Conditional probability from the diagram

1. **Given cycling** the space is the 18 cyclists
2. **Who swims** 7 of them
3. **Result** $P(S \mid C) = \frac{7}{18}$, not $\frac{7}{30}$
4. **Reversed** $P(C \mid S) = \frac{7}{12}$: a different number

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{7/30}{18/30} = \frac{7}{18}$$

# Worked: the conditional formula

1. **Given** $P(A \cap B) = 0.12$, $P(B) = 0.3$
2. **Condition** given $B$: divide by $P(B)$
3. **Divide** $P(A \mid B) = \frac{0.12}{0.3} = 0.4$
4. **Rearranged** $P(A \cap B) = P(A \mid B)\, P(B)$: the tree rule

**Decision**

The denominator is the probability of what is given, never the whole space.

# Mutually exclusive is not independent

## Mutually exclusive

$P(A \cap B) = 0$: they cannot both happen.

Rolling a 1 and rolling a 6 on one die.

$P(A \cup B) = P(A) + P(B)$

## Independent

$P(A \cap B) = P(A)P(B)$; $P(A \mid B) = P(A)$.

A head on a coin and a 6 on a die.

Knowing one changes nothing about the other.

> Exclusive events with positive probabilities are never independent: if $A$ happens, $P(B)$ drops to 0.

# Testing for independence

1. **Data** $P(C) = \frac{18}{30} = 0.6,\ P(S) = \frac{12}{30} = 0.4,\ P(C \cap S) = \frac{7}{30}$
2. **Product** $P(C)P(S) = 0.24$
3. **Compare** $\frac{7}{30} = 0.233 \neq 0.24$
4. **Conclude** not independent, though close

$$A, B \text{ independent} \iff P(A \cap B) = P(A)\,P(B)$$

# Two-way tables

| | **Passed** | **Failed** | **Total** |
|---|---|---|---|
| Revised | 42 | 8 | 50 |
| Did not revise | 18 | 12 | 30 |
| Total | 60 | 20 | 80 |

$P(\text{passed} \mid \text{revised}) = \frac{42}{50} = 0.84$; $P(\text{revised} \mid \text{passed}) = \frac{42}{60} = 0.7$.

# At least one

1. **Model** a component fails with probability 0.1, independently, in each of 3 machines
2. **Complement** at least one failure $= 1 -$ P(no failures)
3. **Compute** $1 - 0.9^3 = 1 - 0.729 = 0.271$

## Decision

Listing every route with at least one failure takes seven branches; the **complement** takes one.

# Traps in probability

## Slips

$P(C \cup S) = 0.6 + 0.4 = 1$, forgetting the overlap.

$P(S \mid C)$ over the whole space.

Same second branch without replacement.

## Fixes

Subtract the overlap: $1 - \frac{7}{30} = \frac{23}{30}$.

Divide by the condition's total.

Update the counts after each draw.

# Check yourself

| **Question** | **Answer** |
|---|---|
| $P(A) = 0.3$: $P(A')$? | 0.7 |
| $P(A) = 0.5$, $P(B) = 0.4$, independent: $P(A \cap B)$? | 0.2 |
| $P(A \cup B)$ for those? | $0.5 + 0.4 - 0.2 = 0.7$ |
| 2 reds from 4 red, 1 blue, no replacement? | $\frac{4}{5} \cdot \frac{3}{4} = \frac{3}{5}$ |
| Can exclusive events be independent? | only if one has probability 0 |