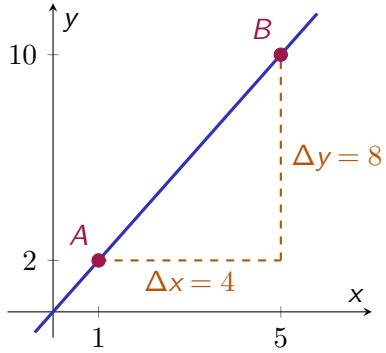
Coordinate geometry

AQA A-level Mathematics 7357, section C: straight lines, circles, parametric equations

Formulae for lines and circles

Quantity	**Formula**
Gradient through two points	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
Line through (x_1, y_1)	$y - y_1 = m(x - x_1)$
Perpendicular gradients	$m_1 m_2 = -1$
Midpoint and distance	$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, $\sqrt{(\Delta x)^2 + (\Delta y)^2}$
Circle, centre (a, b), radius r	$(x - a)^2 + (y - b)^2 = r^2$

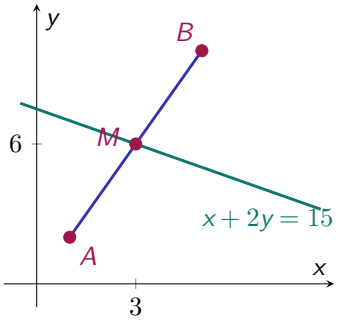
A line through two points



- $A(1,2)$, $B(5,10)$: $m = \frac{10-2}{5-1} = 2$.
- $y - 2 = 2(x-1)$, so $y = 2x$.
- Check B: $2 \times 5 = 10$ ✓

$$y - y_1 = m(x - x_1)$$

The perpendicular bisector

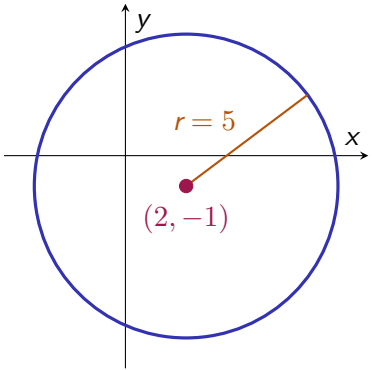


- **Midpoint** of AB: $M(3, 6)$.
- Perpendicular gradient: $-\frac{1}{2}$.
- $y - 6 = -\frac{1}{2}(x - 3)$, so $x + 2y = 15$.

$$m_1 m_2 = -1$$

a perpendicular gradient is the **negative reciprocal**

Reading a circle from its expanded form

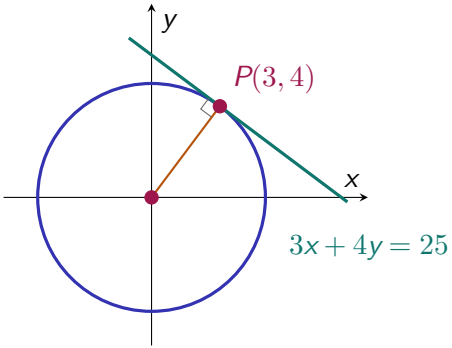


1. **Group** $(x^2 - 4x) + (y^2 + 2y) = 20$
2. **Complete** $(x-2)^2 - 4 + (y+1)^2 - 1 = 20$
3. **Form** $(x-2)^2 + (y+1)^2 = 25$

Decision

Centre $(2, -1)$, radius 5: the signs in the brackets reverse.

The tangent at a point



- $x^2 + y^2 = 25$ at $P(3, 4)$;
  $3^2 + 4^2 = 25$ ✓
- Radius gradient $\frac{4}{3}$; tangent gradient $-\frac{3}{4}$.
- $y - 4 = -\frac{3}{4}(x - 3)$: $3x + 4y = 25$.

$$m_{\text{radius}} \times m_{\text{tangent}} = -1$$

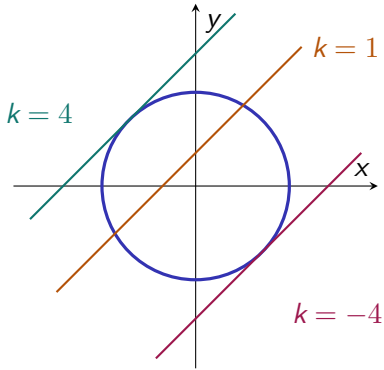
the tangent is perpendicular to the radius at the point of contact

Three circle facts

Fact	**In coordinates**
Tangent is perpendicular to the radius	$m_{\text{tangent}} = -1/m_{\text{radius}}$
The perpendicular from the centre bisects a **chord**	centre lies on the **perpendicular bisector**
The angle in a semicircle is a right angle	AB a **diameter**: centre $=$ midpoint of AB

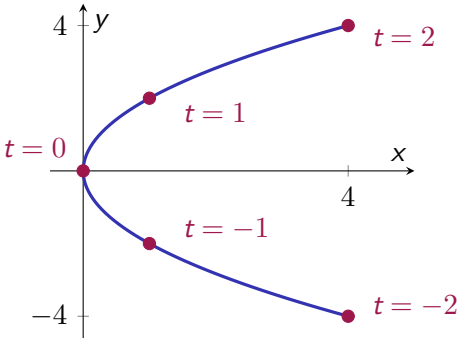
Two perpendicular bisectors of chords meet at the centre.

When does a line touch a circle?



Δ	**Meeting**
> 0	**chord**, two points
$= 0$	tangent, one point
< 0	no meeting

Parametric to Cartesian: $x = t^2$, $y = 2t$



- **Eliminate** t: $t = \frac{y}{2}$.
- $x = \left(\frac{y}{2}\right)^2$, so $y^2 = 4x$.
- t is a label moving along the curve.

$$y^2 = 4x$$

the **Cartesian equation**, with the **parameter** eliminated

Eliminating a trig parameter

1. **Isolate** $\cos t = \frac{x}{3}$, $\sin t = \frac{y}{2}$
2. **Identity** $\cos^2 t + \sin^2 t = 1$
3. **Result** $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

Decision

An ellipse: $-3 \le x \le 3$ and $-2 \le y \le 2$, read from the parametric ranges.

$$\cos^2 t + \sin^2 t = 1$$

A parametric model: a ball's flight

1. **Given** $x = 20t$, $y = 15t - 4.9t^2$ (metres, seconds)
2. **Lands** $y = 0$: $t(15 - 4.9t) = 0$
3. **Time** $t = 0$ (launch) or $t = \frac{15}{4.9} = 3.06$ s
4. **Range** $x = 20 \times 3.06 = 61.2$ m
5. **Top** $\frac{dy}{dt} = 0$ at $t = 1.53$: $y = 11.5$ m

Decision

Range 61.2 m, greatest height 11.5 m; the model ignores air resistance.

Traps in coordinate geometry

Common slips

$(x+3)^2+(y-1)^2=16$ read as centre $(3,-1)$.

Completing the square without adding to the right side.

$m_1 m_2 = -1$ used with a vertical line.

Correct moves

Centre $(-3,1)$: the signs reverse.

Add every constant you created to both sides.

Vertical radius $x=a$: the tangent is horizontal, $y=b\pm r$.

Check yourself

Question	**Answer**
Perpendicular to $y = 3x - 2$?	gradient $-\frac{1}{3}$
Centre and radius of $(x-1)^2 + (y+4)^2 = 9$?	$(1, -4)$, 3
Line meets circle with $\Delta = 0$?	tangent
$x = 2t$, $y = t^2$ as Cartesian?	$y = \frac{x^2}{4}$
Midpoint of $(-2, 5)$ and $(4, 1)$?	$(1, 3)$